Ralph J. Lawrence.
527 E. Gates St.
Roxborough
Phila.

REINFORCED CONCRETE CONSTRUCTION

VOLUME I

FUNDAMENTAL PRINCIPLES

ENGINEERING EDUCATION SERIES

REINFORCED CONCRETE CONSTRUCTION

VOLUME I. FUNDAMENTAL PRINCIPLES

INCLUDING

NUMEROUS TABLES AND DIAGRAMS TO FACILITATE THE CALCULATION AND DESIGN OF REINFORCED CONCRETE STRUCTURES

PREPARED IN THE

EXTENSION DIVISION OF

THE UNIVERSITY OF WISCONSIN

BY

GEORGE A. HOOL, S. B.

ASSOCIATE PROFESSOR OF STRUCTURAL ENGINEERING
THE UNIVERSITY OF WISCONSIN

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PREFACE

This volume forms the first part of the regular course on Reinforced Concrete Construction offered by the Extension Division of The University of Wisconsin and, in common with a number of the other structural engineering courses offered, presupposes a knowledge of the elements of structures. It has been written primarily to meet the needs of those who desire to take up the study of this subject by correspondence, but the author sees no reason why a text of this nature may not be employed for other purposes.

The complete text for the course in Reinforced Concrete Construction is in three volumes: one on the fundamentals, one on the design and construction of retaining walls and buildings, and one on the design and construction of bridges and miscellaneous structures. The present volume on fundamentals omits, for simplicity, the flat-slab type of floor construction—a subject reserved for thorough treatment under the heading of concrete floors in Volume II. The text is intended to be supplemented with such material as is suited to the special needs of the individual student.

Information on the subject has been drawn from many sources but it should be stated that the text-books, "Principles of Reinforced Concrete Construction" by Turneaure and Maurer (Copyright, 1907, 1909 by F. E. Turneaure and E. R. Maurer) and "Concrete Plain and Reinforced" by Taylor and Thompson (Copyright, 1905, 1909 by Frederick W. Taylor) have been referred to constantly.

The author wishes to express his indebtedness to Mr. F. C. Thiessen for his excellent work in preparing the illustrations and for his help with the computations.

The photographs of the beam and column tests were kindly lent by Mr. M. O. Withey, Assistant Professor of Mechanics in The University of Wisconsin.

G. A. H.

The University of Wisconsin,
Madison, Wisconsin,
June 1, 1912.

TABLE OF CONTENTS

PART I

Properties of the Material

CHAPTER I

Concrete

CHAPTER II

Steel

CHAPTER III

Concrete and Steel in Combination

PART II

THE THEORY AND DESIGN OF SLABS, BEAMS, AND COLUMNS

CHAPTER IV

RECTANGULAR BEAMS

CHAPTER V

SLABS, CROSS-BEAMS, AND GIRDERS

Tables

Diagrams

REINFORCED CONCRETE CONSTRUCTION

PART I

PROPERTIES OF THE MATERIAL

Reinforced Concrete is concrete which is strengthened by having embedded in it some metal, usually steel.

The component materials should separately possess certain properties, if satisfactory strength and durability are to be obtained in the structures having these materials in combination. The properties of each material will now be discussed, and those properties in particular will be emphasized which have the most to do with the safe and economic designing of structures.

CHAPTER I

CONCRETE

1. General Requirements.—Concrete used in reinforced concrete construction should be strong, of uniform quality, free from voids, and thoroughly sound. These qualities are required even more than in massive concrete, as the sections in reinforced concrete structures are comparatively small and the stability of a given structure depends upon the strength and durability of every part.

The proportions commonly used in American practice vary from about 1:1 1/2:3 to 1:3:6, using either crushed stone or gravel. The rich mixture is usually required in structural parts subjected to high stresses or where exceptional water-tightness is desired. On the other hand, the use of a 1:3:6 concrete requires careful grading of the materials to produce satisfactory results, even for ordinary work.

2. Cement.—The cement employed in reinforced concrete construction should be of high grade; only portland cement should be used, and the brand selected should conform to the specifications of the American Society for Testing Materials—for these specifications are now accepted as the American standard.

3. Sand.—The sand employed should be free from clay, vegetable loam, sticks, and organic matter and should be of hard, dense, tough material. Siliceous quartz sands are the best, although sands from any durable rock will answer.

Sharp sand was formally a requirement in all important construction, but this property is by no means essential. To be sure, by the use of sharp sand there is a slight tendency toward a concrete of greater crushing strength than when sand of *rounded* grains is employed, but this influence on the result is of less importance than the size of grain, or granulometric composition. Moreover, the sharper the sand employed—the relative sizes of the grains remaining the same—the greater the percentage of voids, and consequently the greater the amount of cement required to produce a given density. (The term *density* is here used to express the ratio of the volume of the solid particles to the total volume of the concrete.) It is now generally conceded that the requirement of sharpness of sand should be omitted from concrete specifications.

Tests of mortar and concrete show that strength and water-tightness increase with density, and so the best sand as to size is one which will produce the smallest volume of mortar of standard consistency when mixed with the given cement in the required proportions. To put it somewhat differently,—the best sand for strength, for water-tightness, and also for economy (as will be seen later) is one which is so graded from fine to coarse that the percentage of voids in the resulting mortar is reduced to a minimum. Such a sand has a very coarse appearance as the amount of fine material required is small.

It has been found that the densest mixture occurs with particles of different sizes and also that the *least* density occurs when the grains are all of the *same* size. Coarse and fine sands are thus inferior to graded sands for concrete, but of the two extremes the coarse sand is preferable. The reason for this is due to the fact that the coarse sand has a less total grain surface in a unit volume, even when the sands considered contain the same pro-

portion of solid matter and voids. Less total grain surface means less cement and water to coat the grains, and less labor required in mixing. The additional amount of cement and water required in the case of the fine sand reduces the density of the resulting mortar and likewise its strength. (The density of neat cement ranges between 0.49 and 0.59, while the density of a sand mortar ranges from 0.60 for a fine sand to 0.75 for a coarse sand or a well-graded sand.)

A fine sand is one containing more than 30 per cent of particles that will pass a No. 40 sieve (diameter of hole = 0.015 in.).

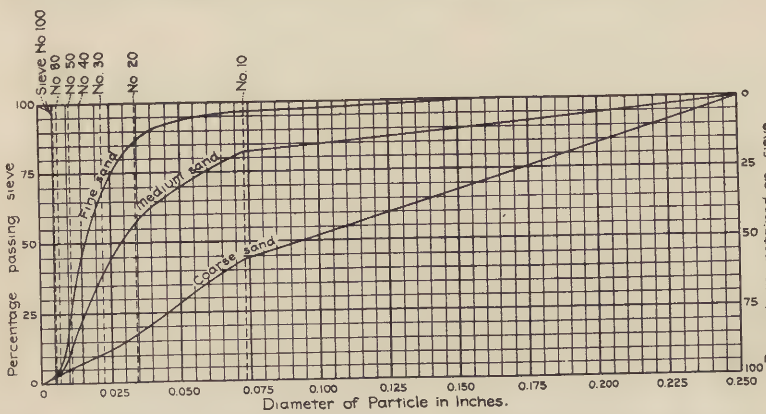

Fig. 1.—Typical mechanical analyses of fine, medium and coarse sands.

The finer the sand, the more nearly uniform the size of the grains, and consequently the greater the proportion of voids. Fine sand is seldom satisfactory and should not be used unless a coarse sand is not available. Even in such cases, tests of strength should be made with the idea of determining what extra cost may be justified in securing a coarser material.

The most accurate method of determining the value of a sand with reference to its size is by means of a sieve analysis. This consists of sifting the sand through several different sieves, and then plotting upon a diagram the percentage by weight which is passed (or retained) by each sieve—abscissæ representing size and ordinates representing percentage. Fig. 1 represents the analyses of three natural sands—a fine, a medium, and a coarse well-graded sand. Uniform grading is indicated by an approach to a straight line. A standard size of sieve is 8 in. in diameter and 2 1/4 in. high. Woven brass wire sieves are employed for

openings less than 1/10 in. in diameter; while for larger openings sheet brass is used, having circular openings drilled to the required dimensions. The woven brass wire sieves are given commercial numbers which approximately coincide with the number of meshes to the linear inch. The actual size of hole, however, varies with the gauge of wire used by different manufacturers and every set of sieves must be calibrated separately. A common defect in sieves is the displacing of the wires so that they are not perpendicular to each other; such sieves should be discarded. Sieves are made to fit together in nests, so that when a sample of sand is placed in the upper (or coarsest) sieve and the nest of sieves is thoroughly shaken, the quantity caught on each sieve can be determined at once. For analyzing sand the following sizes[1] are desirable:

Commercial No.	10	20	30	40	50	80	100	200
Approximate size of hole in inches	0.073	.034	.022	.015	.011	.007	.0055	.0026

A screen with 1/4-in. openings is generally employed for separating out large material from sand.

Specifications should limit the maximum amount of loam or clay to be allowed in any given work. Loam should never be permitted, but clay to the amount of 5 to 10 per cent, if evenly divided, is often beneficial in *lean* mortars. In rich mortars the strength and density is decreased by even slight additions of clay; but in lean mortars the clay helps to fill the voids of the sand, and causes the cementing material to coat the grains better and to bind them together more strongly.

Broken stone screenings have a small percentage of voids and, when free from clay, usually make excellent sand. These screenings ordinarily give a stronger mortar than natural sand but are likely to contain an undue amount of dust, especially when obtained from soft stone; in such a case the mass should be screened before being used in mixing mortar. Gravel screenings also constitute a good material in place of sand. All material passing a 1/4-in. screen is generally considered as sand, or fine aggregate; while all material larger than this size is classed as coarse aggregate.

4. Stone.—For the coarse aggregate, either crushed stone or gravel is generally used. Any stone is suitable which is clean and durable and which has sufficient strength to prevent the strength of the concrete from being limited by the strength of

[1] From American Civil Engineers' Pocket Book, 1st edition, page 415.

the stone. Traps, granites, limestones, and the more compact sandstones are generally employed. Aggregates containing soft, flat, or elongated particles should never be used.

All that has been said concerning voids in sand applies with equal force to the coarse aggregate. Fig. 2 illustrates the analysis of a bank gravel and of a crushed stone. Screens varying by a quarter of an inch from 1/4 in. up are desirable, but a very useful analysis may be made with fewer screens. A uniform size of stone filled with mortar does not make as dense or as

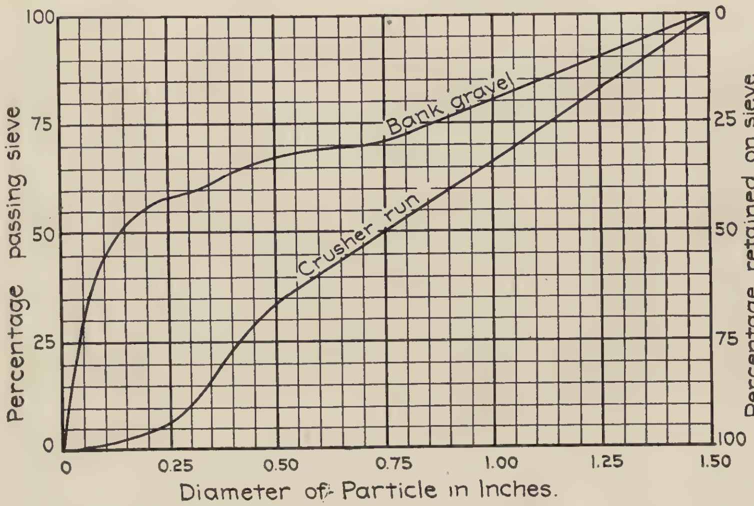

FIG. 2.—Typical mechanical analyses of bank gravel and crushed stone.

strong a concrete as one in which the coarse aggregate is well graded—that is, where the small stones partly fill the larger interstices. A straight line on a mechanical-analysis diagram indicates a uniform grading of size.

Other things being equal, the larger the stone, the stronger and denser the concrete. Experience has shown that for reinforced concrete the maximum size should not be more than about 1 in. to 1 1/2 in., in order that the concrete may fit itself closely around the reinforcing metal. The smaller the stone, the greater the surface to be coated, and the greater the amount of cement required.

Most gravels are sufficiently durable for use in concrete. They should be at least reasonably clean, although a quantity of finely divided clay equal to 5 to 10 per cent of the gravel may add to

the strength of the concrete, if the cement paste does not entirely fill the voids. The presence of clay requires very thorough mixing. When gravel is used, it should be screened to separate the sand and then be remixed in order that the proportions may be definite.

Cinders make good fireproof concrete, but are not recommended by the best authorities for reinforced work. The allowable stress is too low for economical use and, unless great care is taken in having a wet mix and in thorough mixing, there is danger of corrosion of the embedded steel due to porosity. Cinders for use in concrete should not contain many, if any, fine ashes and should consist of hard, clean, vitreous clinker, free from any unburned coal. Concrete containing this aggregate can safely be employed for filling between steel beams and for fireproofing steel or iron columns, and for a concrete fill on top of reinforced concrete floors and roof slabs.

5. Consistency.—Opinion differs as to the quantity of water that should be employed in mixing, but it is safe to say that a somewhat wet or mushy mixture should be used in reinforced concrete construction. Such a mixture flows easily under and around the metal reinforcement and ensures its preservation. It also conforms readily to the molds and gives a smooth surface.

Experiments show that while dry concrete carefully mixed and rammed is stronger at the earlier ages than wet concrete, in six months' time but little difference in strength is found. Moreover, with dry mixtures there is difficulty in obtaining a uniform consistency—occasional batches being too dry.

The water used in mixing concrete should be free from oil, acid, alkalies, or vegetable matter.

6. Unit for Proportioning.—When proportions of the ingredients of a concrete are specified, the specifications should state whether the cement shall be measured loose, or as packed in bags and barrels. The reason for this is clear when it is considered that loose cement occupies about 30 per cent more volume than packed cement. The usual method is to specify the barrel of packed cement as the unit, and to assign it some definite volume—the sand and stone to be measured loose.

A barrel of Portland cement weighs 376 lb., not including the barrel, and a bag of Portland cement weighs 94 lb.; in other words, there are four bags to a barrel. The cement as packed in a barrel occupies, on an average, a volume of about 3.2 cu. ft., but as the unit adopted is an arbitrary one in any case, 3.8 cu. ft.

to the barrel is generally taken as the standard. The value 3.8 has been selected for convenience since 100 lb. of cement can thus be considered as 1 cu. ft.

7. Theory of Proportions.—Two well-established laws govern the theory of proper proportioning, namely:

1. With the same percentage of cement in a unit volume of concrete, the strongest and most impermeable concrete is that which has the greatest density.

2. If the sand and stone remain the same, the strongest and most impermeable concrete is that containing the greatest percentage of cement in a unit volume.

The first law is extremely important. Another way of expressing it is to say that, to obtain the greatest strength and impermeability, the cement should fill the voids of the sand and the resulting mortar should fill the voids of the stone. The second law means that with the same aggregates the strength and water-tightness increases with the amount of cement used—provided, however, that in some cases this amount be not in excess of the voids in the sand, and that the amount of mortar used in each case be the same. If the cement more than fills the voids of the sand, or if the mortar more than fills the voids of the stone, the concrete will be less dense than if the voids were just filled (ordinary concrete has a density between 0.80 and 0.88 and hence is denser than either neat cement or cement mortar); and thus the strength due to increase of cement *may* be offset by the decrease in density.

8. Proportioning by Mechanical Analysis.—Certain standard proportions, such as 1:2:4 and 1:2 1/2:5, are commonly employed in practice; but better results with greater economy can often be secured by the use of mechanical-analysis curves. These curves make it possible to find the best proportions of different aggregates, and they also afford means of finding the best proportions attainable by screening the sand and the stone, and by making artificial combinations of the several portions.

In proportioning by mechanical analysis, the object to be aimed at is to grade the fine and coarse aggregate so that the densest concrete will result from the use of a given amount of cement. This means that the object to be kept in view while grading should be a minimum percentage of voids; however, the use of much very fine material should be avoided for the reason cited in Art. 3. After this grading is accomplished, an amount of

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cement should be used which will give the requisite strength or degree of imperviousness.

The curve of maximum density, or ideal curve, for the combination of sand and crushed stone with a maximum size of 1 in.

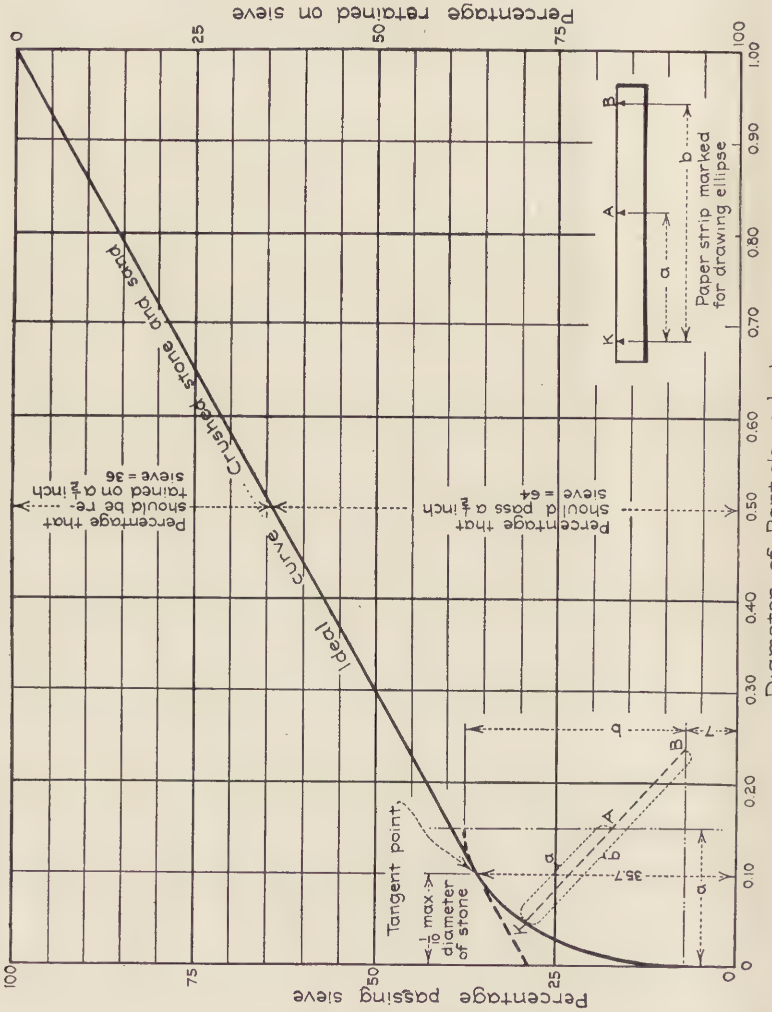

FIG. 3.—Ideal curve for crushed stone and sand.

is shown in Fig. 3. This curve was plotted from a set of rules determined by William B. Fuller and Sanford E. Thompson in an extended series of tests at Jerome Park Reservoir, New York, in 1903 and 1904[1]. These tests, from which the rules

[1] Transactions of the American Society of Civil Engineers, Vol. 59, 1907.

were deduced, comprised the screening of crusher-run stone and bank gravel into twenty-one sizes ranging from 3 in. down to material passing a No. 100 sieve, and then re-combining these sized materials in a predetermined mechanical-analysis curve by weighing out the necessary quantities of each size. Over 400 different mechanical-analysis curves were made. The different mixtures were thoroughly mixed with a given weight of cement to a given consistency; then they were tamped into a strong cylinder, and their volume determined. These tests led to the determination of valuable rules for the plotting of ideal curves for density. Many of the mixtures were also made up into prisms and beams, and the strength of each was determined by breaking tests. These tests substantiated the two laws of the theory of proportions given in the preceding article.

The maximum density curve was found to be of substantially the same form for different materials, whatever the maximum size of stone. The curve in all cases may be taken as a combination of an ellipse and a straight line. First a straight line should be drawn from the point where the largest diameter stone reaches the 100 per cent line, to that point on the vertical ordinate at zero diameter which is given in column (1) in the following table:

DATA FOR PLOTTING CURVES OF MAXIMUM DENSITY

Materials	Intersection of tangent with vertical at zero diameter (1)	Height of tangent point (2)	Axes of ellipse a (3)	Axes of ellipse $(b+7)$ (4)
Crushed stone and sand..........	28.5	35.7	0.150 D	37.4
Gravel and sand..................	26.0	33.4	0.164 D	35.6
Crushed stone and screenings	29.0	36.1	0.147 D	37.8

In this table, D = the maximum diameter of the stone, in inches. Next mark the tangent point on this line—namely, where this line is intersected by the vertical ordinate for one-tenth the maximum stone. This mark should check with the values given in column (2) of the above table. Then plot the location of the axes of the ellipse from the values of a and $(b+7)$ given in columns (3) and (4) respectively in the above table. The major axis of the ellipse should be placed on the 7 per cent line of percentages. (Fig. 3.) Now to plot the ellipse: take a strip of

paper and mark off the lengths of the semi-major and semi-minor axes upon it; each of these lengths should be laid off in the same direction from a common point, which we shall call K; denote the length of the semi-major axis as KA and the length of the semi-minor axis as KB; now swing the strip of paper little by little so that the outline of the curve may be marked off by the point K while the points A and B are kept at all times upon the axes b and a respectively.

The principles by which sand and stone curves are combined into a single curve, with the object in view of approaching closely to the curve of maximum density, may best be explained by some typical problems as developed by Messrs. Taylor and Thompson.[1]

Suppose that we have for concrete the fine sand OA and the crushed stone DE of Fig. 4, and suppose the problem is to find what proportion of each material should be employed. The curves of the two materials are plotted to the same scale and the ideal curve is drawn by the method previously described. Experiments have shown that where the materials to be mixed are represented by only two curves, the best results are obtained when the combined curve intersects the ideal curve approximately on the 40 per cent line at F, and when the finer material is assumed to include the cement. The sand and stone curves in this case do not overlap, and hence for the best proportions, 60 per cent by weight should be stone, and 40 per cent by weight should be sand plus cement. The combined curve is *not* drawn for this simple case. Now the proportion of cement to be used to give the required strength of concrete must always be assumed; and in this case one part by weight of cement to six parts by weight of dry aggregate (measured before the sand and stone are mixed together) will be considered as satisfactory. This will make the cement 1/7, or 14.3 per cent, of the total materials. Deducting this from the percentage of cement plus sand, we have $40\% - 14.3\% = 25.7\%$ sand. The best proportions, then, for a 1:6 mixture by weight are 14.3 parts cement: 25.7 parts sand: 60 parts stone, or a 1:1.8:4.2 concrete. In determining the corresponding proportions by volume, the weights of the sand and stone per cubic foot should be considered.

Consider now the proportioning of the medium sand OB with the stone DE as before. The curve OB may be transformed so that it will pass through F, by changing the distances from the bottom line of the diagram to the curve OB in the proportion

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, pages 784 to 788 inclusive. Copyright, 1905, 1909, by Frederick W. Taylor.

$\frac{HF}{HG}=\frac{40}{93}=43\%$, which means that 43 per cent of the dry materials by weight should be cement plus sand, and 57 per cent stone. About 3 per cent of the stone is overlapped by the sand, but this is so slight it need not be considered here in determining the

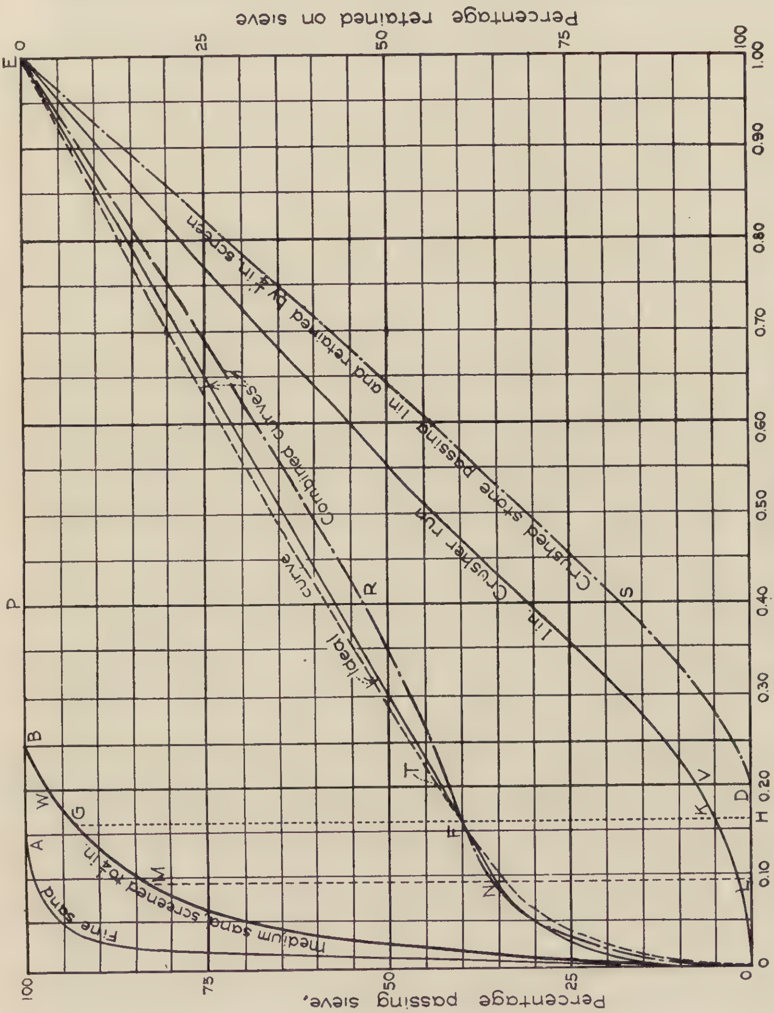

FIG. 4.—Method of proportioning two aggregates.

combined curve. Theoretically, no overlapping in this case should occur, but it is practically impossible in a revolving screen to prevent some fine material being carried over to the openings of larger size. The best proportions of the cement,

sand, and stone should be determined in a similar manner to the previous problem. The combined curve is shown as $ONFRE$. The part of this curve to the left of F very nearly coincides with the ideal curve. For example, 43% of $LM = 0.43 \times 83 = 35.7\%$, or almost exactly LN. From F to the right, the combined curve varies somewhat from the ideal. For example, the percentage of particles larger than 4/10 of an inch $= 57\%$ of $PS = 0.57 \times 82.5 = 47\%$, or PR.

Now suppose that we consider two aggregates, the curves of which overlap—namely, OB and OKE of Fig. 4. The proportion of sand plus cement to use is $\frac{KF}{KG} = \frac{35}{88} = 40\%$, with $\frac{GF}{GK} = \frac{53}{88} = 60\%$ stone. The combination curve, with the exception of the overlap, may be drawn in the same manner as before. The method of finding a point such as T is as follows:

$$DV(0.60) + DW(0.40) = DT = 43\%.$$

The student should understand that the location given for the point F in the above problems can be only approximate. If possible in any given case, it would be advisable to vary the proportions somewhat each way from those obtained in the above manner, and determine the corresponding densities by volumetric tests. It should also be understood in this connection that the curve of maximum density may vary slightly from the so-called *ideal curve;* but the variation is never great, and there is the advantage of being able to plot, by means of simple rules, a curve which will lie at least very close to the maximum density curve for the given aggregates.

It should be clear that plotting mechanical-analysis curves shows approximately not only the best proportions for given materials, but also shows how the materials may be improved by adding or subtracting some particular size. The most valuable use, however, of the method of proportioning by mechanical analysis is in the kind of work which warrants employing several grades or sizes of sand and stone. The process of determining the percentage of each material varies for different cases and is more complicated than where but two aggregates are used. The following problem will give a fair idea of the method of solution for any given case.

The two sizes of crushed stone and the sand shown in Fig. 5 will be considered. The crushed stone curves show the sizes of stone which ordinarily pass through crusher screens of given

diameter of hole, and also illustrate how inefficient the screening process may be. For example, if the sizes of the particles had corresponded exactly to the diameters of the holes and the screening had been more perfectly done, the curves would have

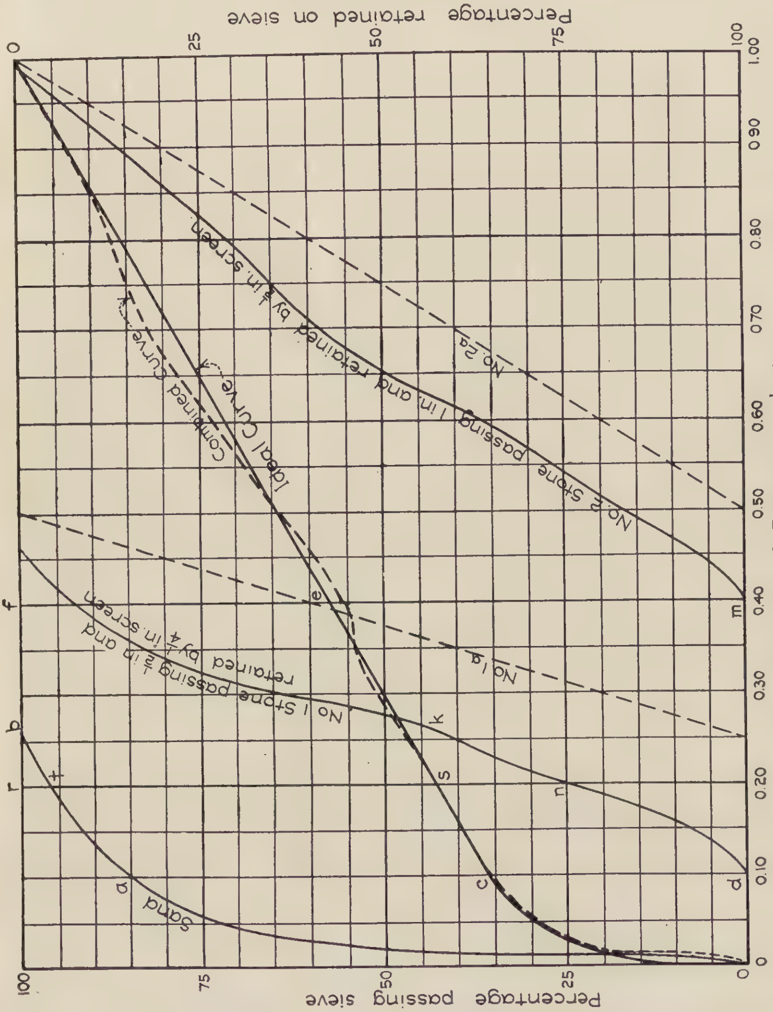

FIG. 5.—Method of proportioning a graded mixture.

had more nearly the direction and location of dotted lines No. 1*a* and No. 2*a*. In combining these curves so as to obtain the strongest and most impermeable concrete, it is quite clear that the grains smaller than 0.10 in. diameter must be supplied wholly from the portion *Oa* of the sand curve; while the larger

grains of the sand, represented by the portion *ab*, are found also in the No. 1 curve. The percentage of sand to use is $\frac{dc}{da}=\frac{36}{85}=$ 42%, since this percentage transforms the sand curve so that it fits very nearly the ideal curve from O to C. The percentage of No. 2 stone required is $\frac{fe}{fm}=\frac{43}{100}=43\%$. This leaves $100-(42+43)=15\%$ to be furnished by the No. 1 stone. Suppose now that a 1:7 concrete is desired. Then, since for more than two aggregates the finer material is *not* assumed to include the cement, there will be $\frac{100}{7}=14.3$ parts cement, and the proportions will be 14:42:15:43 or 1:3.0:1.1:3.1—the parts being by weight. An ordinate such as *rs* may be found as follows:

$$(0.43)(100)+(rn)(0.15)+(rt)(0.42)=57\%$$

With three or more sizes of stone, it is often necessary to assume the required percentages of the intermediate sizes, and make many trial plottings, before we are able to determine the proper proportions to give the best combined curve.

The following statement by William B. Fuller illustrates the possibilities of mechanical analysis:[1] "The ordinary mixture for water-tight concrete is about 1:2:4, which requires 1.57 barrels of cement per cubic yard of concrete. By carefully grading the materials by methods of mechanical analysis, the writer (Fuller) has obtained water-tight work with a mixture of about 1:3:7, thus using only 1.01 barrels of cement per cubic yard of concrete. This saving of 0.56 barrels is equivalent, with Portland cement at $1.60 per barrel, to $0.89 per cubic yard of concrete. The added cost of labor for proportioning and mixing the concrete because of the use of five grades of aggregate instead of two was about $0.15 per cubic yard, thus effecting a net saving of $0.74 per cubic yard. On a piece of work involving, say, 20,000 cu. yd. of concrete such a saving would amount of $14,800.00, an amount well worth considerable study and effort on the part of those in responsible charge."

The above statement does not take into account the cost of the sand and stone which would be needed for the approximate 1:3:7 concrete over and above that required for the 1:2:4 mixture, but this amount would be small compared with the total saving.

9. Quantities Required per Cubic Yard.—The following approxi-

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 183. Copyright, 1905, 1909, by Frederick W. Taylor.

mate rule devised by William B. Fuller gives the quantities of packed cement, loose sand, and loose stone required, to make a cubic yard of concrete. Let c, s, and g be the number of parts by volume of cement, sand, and stone, respectively. Also, let C, S, and G be the required number of barrels of packed cement, the required number of cubic yards of loose sand, and the required number of cubic yards of loose stone, respectively. Then

$$C=\frac{11}{c+s+g}$$

$$S=C\times s\times\frac{3.8}{27}$$

$$G=C\times g\times\frac{3.8}{27}$$

If the stone is of nearly uniform size, about 5 per cent should be added to all quantities computed by the above rule and, on the other hand, if the stone is well-graded, about 5 per cent should be deducted. Although stone is sometimes screened to approximately one size, it is only a waste of labor and material, for the screened stone makes a weaker concrete, and requires more cement.

10. Compressive Strength.—The compressive strength of concrete varies within wide limits, due to the fact that there are so many reasons for variation. The principal factors which affect compressive strength are: (1) the quality of cement used; (2) the quantity of cement in a unit volume of the concrete; (3) the character and size of the aggregates; (4) the density of the concrete; (5) the care taken in mixing; (6) the age of the mixture; and (7) the conditions under which the concrete seasons.

Because of the different conditions met with in practice, it is somewhat misleading to present average values for the compressive strength of concrete. Where possible, the strength of any given concrete should be determined by actual tests and, if the results are too low, the ingredients or proportions should be changed until a satisfactory result is obtained.

The two chief factors which determine the compressive strength of concrete, considering the materials as satisfactory, are age and proportions of ingredients. The relative amount of increase in strength of concrete, from 7 days to 6 months, for two common mixtures, is shown approximately in the table which follows. The table gives average values of the compressive strength in pounds per square inch based on tests made for the Boston

Elevated Railway Company at the Watertown Arsenal in 1899. The test pieces were 12-in. cubes.

Mixture	7 Days	1 Month	3 Months	6 Months
1 : 2 : 4..........	1565	2399	2896	3826
1 : 3 : 6..........	1311	2164	2522	3088

The theoretical angle of rupture in crushing is about 60 degrees with the horizontal, and this theoretical conclusion is borne out by actual tests. For example, cubes of concrete will leave after crushing, pyramids whose surfaces are at an angle of about 60 degrees with the base. Thus, it should be clear that to develop simply the normal compressive strength, the height of a specimen should be at least one and one-half times its least lateral dimension.

Conclusive evidence of the increased strength of cubes as compared with cylinders, due to the reason above given, is shown by the United States Government tests at St. Louis, the results of which are discussed in the U. S. Geological Survey Bulletin No. 344, 1908. Computations from these tests give a ratio of strength of 8 in. $\times$ 16 in. cylinders to 6-in. cubes, at ages of thirteen and twenty-six weeks, as 0.88. This value coincides almost exactly with the empirical formula evolved by Prof. Johnson from results on sandstone and cast-iron prisms. The formula follows:

$$\frac{\text{strength of cylinder}}{\text{strength of cube}} = 0.778 + 0.222 \frac{\text{diameter of cylinder}}{\text{height of cylinder}}$$

A study of a number of tests on concrete tend to show that this formula may be applied with sufficient accuracy, considering the variability of the material, when comparing all sizes of concrete cylinders and cubes.

Tests were formerly made on specimens of cube form, but recently the prismatic or cylindrical form of a height of 2 to 3 diameters has been more commonly employed. Since there is a greater freedom for shearing action in the cylindrical specimen, it is generally used in studying the results of tests on columns. The cube form, however, is useful for comparison with the compressive strength of concrete in a beam.

Tests on cylindrical specimens under different conditions show

a variation in compressive strength at the end of 1 month, for concrete of ordinary proportions, from 1500 to 4000 lb. per square inch. Under reasonably good conditions as to character of material and workmanship, an average strength of 2000 lb. per square inch may be expected of a 1:2:4 concrete at the end of 1 month; and for a 1:3:6 mixture, a strength of 1600 lb. per square inch.

Taylor and Thompson present a practical working formula of sufficient accuracy to compare the compressive strength of mixtures of the *same* materials in *different* proportions. The values in the following table[1] have been obtained from this formula based on cube specimens and medium consistency:

Proportions			Age, one month — Voids in crushed stone or gravel					Age, six months — Voids in crushed stone or gravel				
Cement	Sand	Stone	50 %[2] lb. per sq. in.	45 %[3] lb. per sq. in.	40 %[4] lb. per sq. in.	30 %[5] lb. per sq. in.	20 %[5] lb. per sq. in.	50 %[2] lb. per sq. in.	45 %[3] lb. per sq. in.	40 %[4] lb. per sq. in.	30 %[5] lb. per sq. in.	20 %[5] lb. per sq. in.
1	1½	2	2880	2860	2840	2800	2760	3890	3870	3840	3780	3730
1	1½	3	2780	2750	2720	2670	2610	3750	3710	3680	3600	3530
1	1½	4	2680	2650	2610	2540	2460	3620	3570	3520	3430	3330
1	2	3	2560	2540	2510	2460	2410	3460	3420	3390	3320	3250
1	2	4	2480	2440	2410	2350	2290	3340	3300	3250	3170	3090
1	2	5	2400	2350	2310	2230	2170	3230	3180	3120	3010	2930
1	2	6	2320	2260	2230	2140	2060	3130	3060	3010	2890	2780
1	2½	3	2370	2340	2320	2270	2230	3200	3160	3130	3070	3020
1	2½	4	2290	2260	2230	2180	2110	3090	3050	3010	2940	2850
1	2½	5	2210	2180	2130	2070	2000	2980	2940	2880	2790	2700
1	2½	6	2140	2100	2060	1980	1910	2890	2830	2780	2670	2570
1	3	4	2120	2090	2060	2020	1970	2860	2830	2780	2720	2660
1	3	5	2060	2030	1990	1930	1870	2780	2740	2690	2610	2530
1	3	6	1990	1950	1910	1840	1770	2680	2630	2580	2480	2390
1	3	8	1860	1810	1770	1680	1600	2510	2440	2390	2280	2160

Note.—Proportions are based on a barrel of 3.8 cu. ft. Values are for average ultimate strength, which must be divided by a factor of safety for working loads. Quality of materials and methods of mixing may affect the strength by 25 per cent in either direction, while the relative values for different proportions are not materially changed.

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 360.

[2] Use 50 per cent columns for crushed stone screened to uniform size.

[3] Use 45 per cent columns for average conditions and for crushed stone with dust screened out.

[4] Use 40 per cent columns for gravel or mixed stone and gravel.

[5] Use these columns for graded mixtures.

It should be noticed in the table that the stone with the smaller percentage of voids gives the lower strength. This *seeming* irregularity is due to the fact that this stone measured loose has more solid material per cubic foot than the stone with the higher percentage of voids, and hence with the same proportions by volume, this stone gives a greater bulk of concrete, and less cement per unit volume. A smaller amount of cement shows here to have more influence in decreasing the strength than the greater density has in increasing it. It must not be inferred from this that the aggregate with the largest percentage of voids is best to use. Such an aggregate requires more cement to a given volume of concrete, and although the aggregate with fewer voids is sometimes slightly inferior in strength, the latter is the more economical.

All the preceding results for the crushing strength refer to a compressive force applied over the entire upper surface of the specimen. Experiments show that if the load is applied upon only the central portion of the upper surface, the specimen will bear a greater unit load, due to the fact that the outer portions will aid the interior portion in resisting stress. In connection with the designing of concrete footings for the Boston Elevated Railway, thirty-six 12-in. cubes of 1:0:2 and 1:2:4 concrete were crushed at different ages by applying the load over the entire upper surface of the cube; then the same number of similar cubes were crushed by applying the stress over an area of 10 by 10 in. and then a third set by applying the stress over an area of 8 by 8 1/4 in. The third series gave a strength 128 per cent of the first, and the second series gave 112 per cent. Age of the concrete and proportions of ingredients did not seem to have any influence on the above results.

11. Tensile Strength.—Concrete is frequently subjected to tension in reinforced concrete construction; not directly, however, but as the result of bending as in beams, girders, and slabs. The value of this transverse strength is of little importance; because, on account of the brittleness of concrete in tension and its liability to crack from shrinkage or from the shock of some of the applied loads, it is unsafe to depend upon the tensile strength of concrete in reinforced concrete design. In certain computations, however, the tensile strength must be considered.

The character of the sand and aggregate, and poor workmanship, have a greater influence on the tensile strength than upon

the compressive. Tests seem to show that reasonable values for ultimate strength in direct tension at the end of one month are about as follows:

1:2:4 mixture. . . . 160–200 lb. per square inch
1:3:6 mixture. . . . 100–125 lb. per square inch

The tensile strength of concrete at the place of greatest stress, that is, at the fiber most remote from the neutral axis, limits the strength of unreinforced concrete beams. Transverse tests of plain concrete should therefore show about the same relative results as tensile tests, and, in fact, they are quite as significant in this connection. The value of the ratio of modulus of rupture to tensile strength will ordinarily range from 1.8 to 2.0.

12. Shearing Strength.—A true shearing failure can only occur when the external forces producing the failure act in opposite directions and close together. The actual strength of concrete in direct shear is greater than was formerly supposed because, in the earlier experiments, shear was confused with the diagonal tension which occurs in the web of a beam. Diagonal tension is explained at length in Chapter IV. At the present time it is sufficient for the student to realize that, in this article, shear is considered to be the strength of the material against a sliding failure, when tested as a rivet or bolt would be tested for shear.

Very few tests have been made to determine the shearing strength of concrete since it is difficult to arrange an experiment to determine this strength without involving some bending stress. Perhaps the best set of tests was made at the Massachusetts Institute of Technology in 1904 and 1905 under the direction of Prof. Charles M. Spofford. Three grades of concrete were used —1:2:4, 1:3:5, and 1:3:6—and the specimens were stored in both air and water from 24 to 32 days. The specimens tested were cylinders 5 in. in diameter and 18 in. long, and the end thirds were held rigidly by cast-iron yokes. The pressure was applied through a cast-iron, half-cylinder bearing which fitted between the two yokes and caused the shearing of the concrete across two planes. Six extra cylinders of the same concrete were tested at the same time for compression. The tests gave a shearing strength ranging in general from 60 to 80 per cent of the compressive strength of the concrete.

13. Contraction and Expansion.—The contraction and expan-

sion of concrete occurs from two causes: (1) temperature, and (2) hardening action. Like most building material, concrete contracts as the temperature falls, and expands as it rises. Its volume is also affected when it hardens, a shrinking action taking place when hardened in air and some expansion when hardened in water.

Experiments show an average value of the coefficient of expansion of concrete, due to temperature changes, to be about 0.000006 per degree Fahr. which, it should be noted, is very nearly that of steel. Thus, when steel is embedded in concrete, the two materials will be but slightly stressed because of any difference in their rates of expansion.

The amount of change in volume of concrete due to hardening increases with the proportion of cement used in the mixture. When hardened in air, concrete contracts about 0.0005 of its length; while the expansion, when hardening takes place under water, is only about 0.0001 of the length. The whole change is usually not accomplished for some months, but about half the whole is effected in about a week.

14. Fireproofing Qualities.—The value of concrete as a material to be used in the construction of the walls, columns, and floors of buildings is largely dependent upon its fireproofing qualities.

Concrete has a low conductivity of heat due to two causes: (1) the presence of combined water, and (2) porosity. The water of crystallization, being chemically combined, is not given off at the boiling-point. A part of this water goes off at about 500° Fahr., but the dehydration is not complete until 900° Fahr. is reached. This vaporization of the water at the surface absorbs heat and protects the interior. The layer of changed material is then a better nonconductor than before, so the process goes on very slowly. The porosity of concrete also offers considerable resistance to the passage of heat since it is well known that air spaces are a most efficient protection against conduction.

Steel is said to lose 10 per cent of its strength at 600° Fahr. and 50 per cent at about 750° Fahr. A fire in a building may reach a temperature of 2000° to 2300° Fahr., and the importance of sufficiently protecting the reinforcing steel from loss of strength due to overheating is therefore evident. The surface of a mass of concrete exposed to the action of flames for some time may be

ruined and may be flaked off by the application of a stream of water from a hose; but fortunately, the depth to which the heat penetrates is very limited. The conclusions to be drawn from experimental tests, and also from observations of the results of large fires, are that sharp corners of beams and columns are more susceptible to attack than wide, flat surfaces, such as slabs. Sufficient protection seems to be afforded the reinforcing metal by a 1 1/2 to 2 1/2 in. covering in beams and columns, and a 3/4 to 1 in. covering in slabs. Little difference was observed between stone and cinder concrete—the burning of the bits of coal in poor cinder concrete being often balanced by the splitting of the stones in the stone concrete. It seems probable, from the composition of the rock, that hard trap or gravel may be preferable to hard limestone, slate, or conglomerate, as fire-resisting material, although further experiments are needed to determine their relative durability in this respect. Soft limestone as the aggregate should never be employed, as under the action of fire and water it disintegrates.

Prof. C. L. Norton, in his report on the Baltimore fire to the Insurance Engineering Experiment Station, says: "Where concrete floor arches and concrete-steel construction receive the full force of the fire, it appears to have stood well, distinctly better than the terra-cotta." The reason for this he considers to be due to the fact that terra-cotta expands about twice as much as steel, while concrete expands about the same.

15. Waterproofing Qualities of Concrete.—Most of the tests and observations on the waterproofing qualities of concrete have been made with the idea of determining whether or not embedded steel is subject to corrosion from moisture and other causes. In general, the rusting of iron occurs only under the combined action of moisture and carbon dioxide. A coating of Portland cement has long been known to not only exclude moisture and carbon dioxide, but in hardening to *absorb* any carbon dioxide which may be present. Concrete as actually deposited around reinforcing steel cannot be said to be as effective as an unbroken coating of cement, but experiments show that this is practically accomplished if the concrete is mixed quite wet, so as to furnish a thin coating on the steel, and if it is free from voids and cracks. The preserving quality of concrete is thus dependent upon its consistency and the proportions employed, but for ordinary reinforced work it may be concluded from the results of tests

and observations that, when well placed, the concrete affords complete protection of the steel against corrosion.

16. Modulus of Elasticity.—To gain clearness in this study, modulus of elasticity will be treated in Art. 25.

17. Weight of Concrete.—A dense, well made, 1 : 2 : 4 concrete, when dry, will weigh approximately 155, 152, 148, and 143 lb. per cubic foot, according as the aggregate is trap rock, gravel, limestone, or sandstone. These figures represent average values although the weight is affected but little by any ordinary variation of proportions. It is found that a wet concrete when dried out will weigh less than a well compacted concrete containing originally less water. The addition of reinforcing steel in the usual proportions will add from 3 to 5 lb. Cinder concrete has an average weight of 112 lb. per cubic foot.

The assumed weight of reinforced concrete is usually 150 lb. per cubic foot.

CHAPTER II

STEEL

18. General Requirements.—The reinforcing steel in reinforced concrete construction is mostly in the form of rods, or bars, of round or square cross-section. These vary in size from about 1/4 to 3/8 in. for light floor slabs, up to 1 1/2 in. as a maximum size for heavy beams. Somewhat larger sizes are sometimes used for columns. Rods can be procured varying by 1/16 in. increments from 1/4 in. to 1 in., and then by 1/8 in. increments to 2 in. Above 2 in., rods to the nearest 1/4 in. should be selected. Stock lists of steel companies should be consulted as these show the normal stock in tons of the different sizes. Various bars of deformed cross-section are widely advertised, some of the more common types being shown in Fig. 6. Woven wire and thin punched plates are sometimes used in thin slabs with good effect.

Authorities differ as to the quality of steel to be used for reinforcement—medium and high carbon steel being used by different engineers. Medium steel is better fitted for most classes of structures than high steel, while in some structures high steel will answer just as well although probably with very little gain in economy. No matter what kind of steel is used in beams, low unit stresses are much to be preferred on account of the large distortions involved.

Brittleness is to be feared in high steel, although this quality is not so dangerous when the metal is used in heavy reinforced concrete members—for example, in heavy beams or slabs—

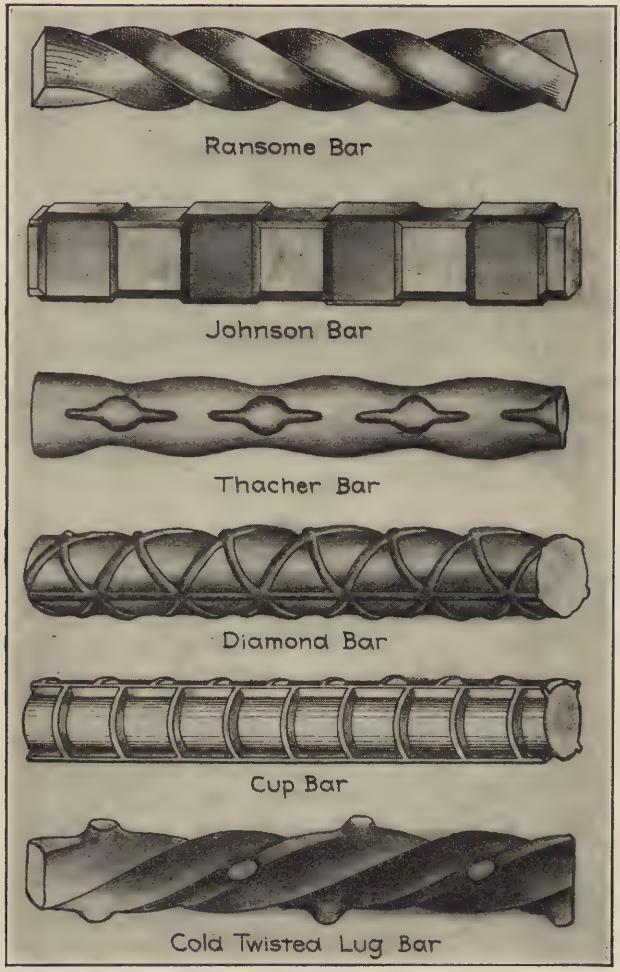

FIG. 6.—Deformed bars.

as the concrete to a large extent absorbs the shocks and protects the steel. Medium steel is manufactured and sold under standard conditions in the open market. This steel is of excellent quality, and an engineer can feel sure of the safety of his structure without the expense of exhaustive tests. As a rule, a satis-

factory high elastic limit steel cannot be obtained in this way. To prevent brittleness a careful inspection of high steel should be made during manufacture. The expense of doing this may so increase the cost of the material that little or no economy will result from the use of high steel, even though it be considered permissible to employ a greater working stress than in the case of medium steel. Open hearth steel is preferable to Bessemer steel, as it is more uniform in quality and does not possess the brittleness sometimes met with in Bessemer steel.

An important test in the specifications for reinforcing steel is the bending test, and no steel which fails to pass this test should be used under any circumstances. In case a lot of steel has been delivered without previous test by the purchaser, one bar selected at random in every 100 should be subjected to this test and, if it fails to pass, the portion from which it is taken should be rejected.

19. Tensile Strength.—The elastic limit and ultimate strength of the two general grades of steel above mentioned will range about as follows:

	Medium	High
Elastic limit, lb. per sq. in....	35 to 40,000	50 to 60,000
Ultimate strength, lb. per sq. in.	60 to 70,000	80 to 100,000

20. Coefficient of Expansion.—The coefficient of expansion of steel may be taken at 0.0000065 per degree Fahr.

21. Modulus of Elasticity.—The modulus of elasticity of all grades of steel is very nearly the same and is usually taken at 30,000,000. (Art. 25.) If steel could be made with a high modulus of elasticity, it would be especially fitted for reinforced concrete work, because the higher the modulus the less the deformation under any given loading. Unfortunately, however, the modulus of elasticity is a constant for all the different kinds of steel.

22. Bending Test for Steel.—*Medium Steel:* Full-sized material 1 in. thick, and over, should bend cold 180 degrees around a pin, the diameter of which is equal to twice the thickness of the bar, without fracture on the outside of bend. *High Elastic Limit Steel:* Specimens for bending should bend cold under the following conditions without fracture on the outside of the bent portion:

Around twice their own diameter	Around their own diameter
1 in. diameter, 80 degrees.	$\frac{1}{4}$ in. diameter, 130 degrees.
$\frac{3}{4}$ in. diameter, 90 degrees.	$\frac{3}{16}$ in. diameter, 140 degrees.
$\frac{1}{2}$ in. diameter, 110 degrees.	$\frac{1}{8}$ in. diameter or less, 180 degrees.

Test pieces of steel wire used for reinforcement should bend 180 degrees around their own diameter without fracture.

PROBLEMS

1. A concrete as proportioned by mechanical analysis is a 1 : 2.1 : 4.9 mixture by weight. If both the sand and stone weigh 100 lb. per cubic foot, what are the corresponding proportions by volume considering 100 lb. of cement as 1 cu. ft.?
2. What would be the correct proportions by volume of the concrete of Problem 1, if the sand weighed 95 lb. per cubic foot and the stone 90 lb. per cubic foot?
3. Consider a 1 : 2 : 4 concrete, and let a cubic yard be composed of 1.57 bbl. of cement, 0.44 cu. yd. of sand, and 0.88 cu. yd. of stone. The cement costs $1.50 per barrel and the stone $1.20 per cubic yard. Two sands are available, one at 50¢ and the other at $1.00 per cubic yard, which give a corresponding compressive strength of concrete of 2000 and 2200 lb. per square inch respectively. Which sand is the more economical if the concrete is to be used in members sustaining purely compressive stresses?
4. By carefully grading the materials in Problem 3 using the 50¢ sand, it was found that practically a 1 : 3 : 6 mixture having 1.11 bbl. cement, 0.47 cu. yd. sand, and 0.94 cu. yd. stone, per cubic yard of concrete, could be used to give the same strength that the $1.00 sand gave in Problem 3. The added cost of labor for proportioning and mixing the concrete, due to the use of graded materials, was 10¢ per cubic yard. What would be the saving on a piece of work involving 10,000 cu. yd. of concrete?
5. The sieve analysis of a bank gravel is as follows:

Sieve number	Size of mesh in inches	Percentage passing
1-in.	1.00	100.0
3/4-in.	0.75	92.5
1/2-in.	0.50	80.0
1/4-in.	0.25	71.0
.15-in.	0.15	64.4
10	0.073	50.0
20	0.034	34.0
30	0.022	25.7
40	0.015	20.8
50	0.011	17.5
80	0.007	11.0
100	0.0055	9.5

Show by mechanical analysis curves how you would screen the gravel into two sizes and combine to secure a concrete of greater density and strength.

CHAPTER III

CONCRETE AND STEEL IN COMBINATION

23. Advantages of the Combination.—The highest success in the use of concrete and steel in combination is attained only when maximum strength is secured at minimum cost. This is accomplished when the steel and concrete are placed in such a manner as to derive their greatest strength, and when economical proportions of these materials are employed.

Steel can be put into a form to resist a given tensile stress much more cheaply than it can be put into a form to resist a corresponding compressive stress. This should be clear, when it is considered that the solid bar is well adapted to take tensile stresses, while for compressive stresses the steel must be made into forms of more extended cross-section, in order to provide the necessary lateral rigidity. There is a serious objection to the use of steel in many locations due to its lack of durability and its failure to stand up under a high heat. Steel is also a relatively expensive building material.

Concrete, on the other hand, cannot be used in tension except to a very limited extent, but its compressive strength is fairly high. It is also a good fireproof material and has great durability. In addition, concrete is a comparatively cheap material; is readily available in almost any locality, and tests and the results of observations show that it thoroughly protects embedded steel from corrosion.

From the above considerations, it follows that reinforced concrete construction is advantageous to varying degrees in different types of structures. In structural forms subjected to both tension and compression, such as beams, the proper combination of the two materials meets with the best success. Steel rods embedded in the lower side of the beam carry the tensile stresses while the compressive stresses are carried by the concrete. Here, then, the steel is used in its cheapest form and the whole structure may be made strong, economical, and very durable. For columns, also, a combination of the two materials is quite advantageous although to a varying degree and, in any case, not to such a large extent as in beams. In this type of

structure, steel may be used with concrete in the form of small rods to reinforce the concrete; or it may consist of structural steel shapes in the form of a core, simply surrounded and held rigidly in place by the concrete—most of the load being carried by the steel; or finally, a steel column may be used and the concrete employed merely for the purpose of adding symmetry and providing fire protection. This last case does not come under the head of reinforced concrete and will not be considered in this course. The use of a moderate amount of steel with concrete so as to produce a material with a reliable tensile and bending resistance has opened the way for the use of this combination in a great variety of structures and in special individual forms.

24. Bond Between Concrete and Steel.—Usually the entire stress which is carried by the steel of a reinforced concrete beam is transmitted to the reinforcing bars by the bond or adhesion between the concrete and the steel. Stress is also conveyed to the steel of a reinforced concrete column in like manner. Experience has shown the bond to be reliable and permanent, and that plain bars may be used in most structures with success. Deformed and twisted rods are used to a large extent in structures where the stress between the steel and concrete exceeds the safe working adhesion of the plain rod. The indented surfaces of deformed bars produce a mechanical bond in addition to the adhesion already mentioned.

The values derived from different experiments for the adhesion in pounds per square inch of contact surface vary quite widely. With plain rods, results vary from about 200 to 750 lb. per square inch. The quality of the concrete and the manner of making the tests are important factors.

Tests to determine the strength of bond are usually made either by pulling out a rod that has been embedded in a block of concrete or by forcing the steel out by compression. In either case the concrete is in compression, and there is the tendency to increase the friction by adding to the pressure at the surfaces in contact. In an important series of tests made at The University of Wisconsin, test beams were arranged as shown in Fig. 7—the reinforcing bars being embedded only a short distance from each end, leaving the middle portion exposed. The stress in the rods was computed from the observed deformations. The conditions were quite similar to those which exist in an ordinary beam, but the beam was prevented from failing in the early

stages of the tests, by an upper set of auxiliary rods. Failure finally occurred by the pulling out of the lower rods, as intended. Tests indicate that the bond per square inch of surface is not affected by the size of rod. A study of the results of many experiments leads to the conclusion that for ordinary round or square bars, not too smooth, the adhesive strength may be taken at from 200 to 300 lb. per square inch. After bond is once broken, plain rods give a frictional resistance of about two-thirds of the original bond resistance; in other words, the reinforcement may be moved slightly with reference to the concrete, as by a sharp blow, and still leave about two-thirds of the bond strength effective.

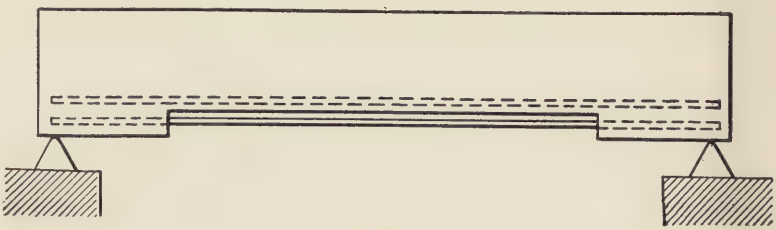

Fig. 7.

A rough surface gives a higher adhesive value than a smooth surface, consequently a thin film of rust on the reinforcing metal should not cause its rejection. Loose or scaly rust should not be allowed, however. Bars in this state of corrosion may be used, if they are first cleansed with a stiff wire brush or given a pickling bath of sulphuric acid solution (consisting of 1 part acid to 6 parts of water), and then dipped in clean water. Oiling and painting of reinforcing steel should not be permitted, as the adhesion is greatly reduced thereby. Round bars show the greatest adhesion—flat bars the least.

Let f_s be the working tensile strength of the steel, a_s the area of bar, o the circumference of bar, d the diameter or thickness of bar, u the working unit bond strength, and x the required length of embedment (or grip) for the above values of f_s and u. Then, to develop the strength of the steel, using either round or square bars,

$$xou = a_s f_s = \frac{\pi d^2}{4} \cdot f_s$$

or

$$x = \frac{f_s d}{4u}$$

The initial movement in the case of indented bars is probably due to a slight crushing of the concrete under the projections. The strength mechanically obtained from the indented surfaces, however, depends upon the shearing strength of the concrete. The bars cannot be pulled through the concrete without shearing off an area equal to the total area of the indented portion and, in addition, overcoming adhesion along the remaining surface. In tests of such bars, failures have usually occurred by the splitting of the specimen or by the breaking of the bar, but the results indicate that at least 500 to 600 lb. per square inch in bond may be expected.

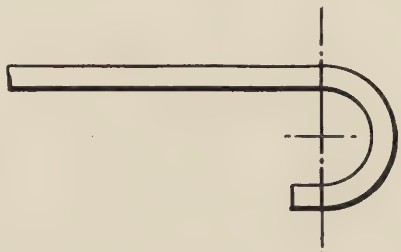

Fig. 8.

It is a common practice to bend the ends of reinforcing rods into hooks consisting of either curved or right angle bends. Experiments on the efficiency of such hooks have shown that the ultimate bond strength was greatly increased. It was found that with an embedment in concrete in all directions equal to 8 diameters of the bar, a hook of 5 diameters may be assumed to develop the elastic limit of the steel. When curved ends are used, they should consist of bends through 180 degrees with a short length of straight rod beyond the bend, as shown in Fig. 8. Short square hooks upon the ends of bars are not of great value.

25. Ratio of the Moduli of Elasticity.—In order to treat the subject clearly, it will be necessary to give a somewhat extended discussion of the term *modulus of elasticity*. The modulus of elasticity of a material such as steel is the ratio of the unit stress to the corresponding unit deformation, provided the elastic limit of the material is not exceeded. This ratio is usually denoted by E, or

$$E = \frac{f}{d}$$

in which f denotes the stress in pounds per square inch and d the corresponding deformation per unit of length.

Thus, if an external load of P lb. be applied to a steel bar whose length is l in. and whose cross-section is A sq. in., and if the compression under this load be d in., we should have

$$E=\frac{f}{d}=\frac{P/A}{d/l}=\frac{Pl}{Ad}$$

Suppose an external force of 30,000 lb. be applied to a bar whose length is 10 in. and whose cross-section is 1 sq. in., and suppose the extension under this force is 0.01 in. We should have

$$E=\frac{Pl}{Ad}=\frac{(30{,}000)(10)}{(1)(0.01)}=30{,}000{,}000$$

If the external force, or load, in pounds per square inch be represented by ordinates and the corresponding elongations or

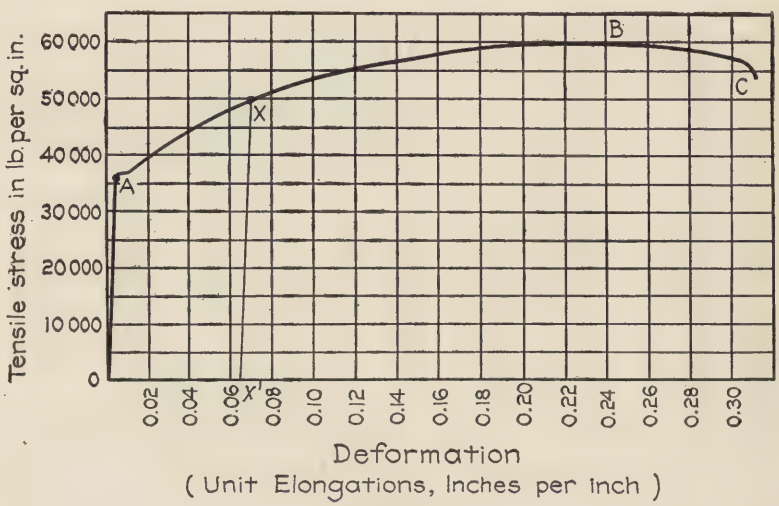

FIG. 9.

compressions be represented by abscissæ, then the action of the specimen under test may be indicated by what is known as a stress-deformation diagram. In Fig. 9 a typical stress-deformation diagram is shown for medium steel in tension. There are three significant points which need to be noted. These are: the *elastic limit*, the *ultimate strength*, and the *breaking-point.* These three points are indicated by the letters A, B, and C, respectively.

Steel has a well-defined elastic limit in tension, as is readily seen by the diagram, and which is at least one-half the ultimate strength. It is represented by the ordinate of the point of tangency between the straight line and the rest of the curve. For medium steel in compression, the transition from the straight line to the curve is more gradual, and the point of tangency is more difficult to locate. If loads are applied which cause stress less than the elastic limit, no appreciable permanent deformation will exist when the loads are removed. Loads causing stress in the steel above this limit will, however, leave a permanent set on their removal. For example, suppose a steel bar is stretched with a force of 50,000 lb. per square inch of cross-section. The point x will be reached in Fig. 9. The elongation of the bar will be about 0.07 in. per inch of length. If the load is removed, the diagram will go back along some line such as xx'. The permanent set is ox'. If the bar is stretched so that the resulting stress per square inch is less than the elastic limit, then the diagram will go back along the line OA, and no deformation results. Since the line OA is straight, it is clear that below the elastic limit, stress is proportional to deformation. The above illustration could be applied equally well to steel in compression.

Above the elastic limit the elongation of steel increases at a rate which becomes more and more rapid until finally the condition of perfect plasticity is reached, and the body elongates under a constant force, while the lateral dimensions reduce more and more rapidly. The point B is reached when this action occurs. The piece at this point draws out rapidly, the breaking point C is soon reached, and the piece breaks under a reduced load. For a piece tested in compression, the points B and C are almost identical.

The modulus of elasticity may be figured theoretically by any stress and its corresponding deformation from O to A, but is generally figured near the elastic limit stress since practically a more accurate result can be thus obtained. The modulus of elasticity for steel in both tension and compression is about 30,000,000. This value may be taken the same for all grades of steel, the variation being so slight. For medium steel, the elastic limit is about 36,000 lb. per square inch for both tension and compression.

In the design of reinforced concrete structures, it is necessary to know the relative stresses in the concrete and steel of com-

pression members under like distortions; and in beams, it is necessary to find the position of the neutral axis and the required percentage of steel. These may easily be determined if the modulus of elasticity is known for both the concrete and the steel. The modulus of elasticity for steel we already know. Let us now discuss the method of determining the modulus for concrete.[1]

With concrete we have an entirely different material to deal with as regards elasticity. Fig. 10 shows a typical stress-deformation diagram for concrete in compression. As you can see, the compression curve is slightly curved almost from the beginning, the curvature gradually increasing toward the end in approximately the shape of a parabola. A release of load at even moderate stresses will show some permanent set, as OY, indicating imperfect elasticity. After a few repetitions of such loads, however, there will be no further set, and the stress deformation diagram line will become straight up to the load applied, as YA, for example. There is a limit of stress beyond which repeated applications of the same load will continue to add to the permanent deformation, and the specimen will ultimately fail. This limit is found to be from one-half to two-thirds the ultimate strength and is called the elastic limit. Point S is the breaking-point and ultimate strength.

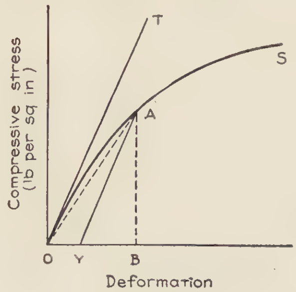

Fig. 10.

For very low stresses, up to perhaps 300 to 400 lb. per square inch, the variation of the curve from a straight line is so small that it may be considered straight, and an average straight line may be drawn, as OT, and its slope taken as the modulus of elasticity. For higher stresses, the total deformation is OB and the modulus elasticity becomes

$$E=\frac{AB}{OB}$$

This ratio will be less than the slope of the line OT.

Later in the course when referring to these two methods of calculating the modulus, the slope of the tangent OT will be called the *initial modulus* and the slope of the line OA, the *secant modulus.*

[1] Discussion as given in Turneaure and Maurer's "Principles of Reinforced Concrete Construction," 2nd edition, page 20.

Suppose a load passes frequently over a reinforced concrete structure, and let us consider a certain member in compression. The first application, let us say, causes d deformation with a permanent set d'. Then it should be clear, that the second application causes an additional deformation of only $d-d'$, which is the same as saying that both the concrete and the steel did not return to their original position before the second application. There must have been some compression still left in the steel, with an equal amount of tension in the concrete, when the second load was applied; in other words, the total deformation fixes the stress in the steel. Thus, for the determination of the relative stresses in the two materials for a working stress in the concrete, the modulus for the concrete should be expressed by $\frac{AB}{OB}$ (Fig. 10) and not by the ratio $\frac{AB}{YB}$.

Let us take the case of a beam. The compressive stresses in the concrete at any section will vary from zero at the neutral axis to the value AB, for example, at the extreme fiber. At intermediate points, the stresses and corresponding deformations follow approximately the law of the curve OA. In this case the slope of the chord OA does not exactly represent the facts, but for working loads the error is small.

Tests on prisms show that the modulus of elasticity as designated above for ordinary concrete, 30 days old and under working loads, ranges from 2,500,000 to 3,500,000 lb. per square inch. As a rule, the denser and older the concrete, the higher the modulus.

Now that the meaning of the term modulus of elasticity has been made clear with reference to both steel and concrete, the reason for the occurrence of a constant ratio of the moduli in specifications and building codes will be discussed.

Consider the stresses in the steel and concrete of a reinforced concrete column, reinforced with longitudinal rods only. Let

f_s = unit stress in steel.
f_c = unit stress in concrete.
E_s = modulus of elasticity of steel.
E_c = modulus of elasticity of concrete.

Since the modulus of elasticity of a material is the ratio of stress to deformation, it follows that, for equal deformations, the

stresses in the steel and concrete of a reinforced concrete column will be as their moduli of elasticity. Thus,

$$\frac{f_s}{f_c}=\frac{E_s}{E_c}$$

This ratio of the moduli is generally denoted by the letter n, or

$$f_s=nf_c$$

The term n, then, is the subject of this discussion and is the value referred to in the various specifications and building codes.

The equation just given shows that if the stress in either the steel or concrete of a concrete column is known, the stress in the other material can be found, and this relation is made use of in the derivation of column formulas. Fig. 10 shows that the modulus of elasticity of concrete in compression is less for the greater loads, and hence the value of n is greater. Thus, it is plain that with increasing loads in concrete columns the steel receives a greater proportionate stress, the variation in the amount carried by the steel depending on the variation in the value of n. In order to take account of the fact that under increasing loads the steel receives an increasing proportion, it is desirable to use a value of n in the computations for design somewhat larger than that which is obtained by taking a value of E_c corresponding to working loads on small prisms (about 10). A value of 15 for n may well be used for all ordinary mixtures and for all types of columns. The value of 12 sometimes specified is now considered unduly low.

In concrete beams, experiments show that the tension which remains in the concrete just below the neutral axis, and properly not allowed for in the derivation of the beam formulas, has its effect in the position of the neutral axis and the strength of the beam. It is found that a value of 15 for n—the same as is used for columns—is not too large for calculations of strength of beams under the usual assumptions, although great accuracy in this respect is not necessary. This value of 15 for n is the one most generally used, but a value of 12 is also frequently employed. The value of 15 corresponds to a value of E_c of 2,000,000 which is somewhat low as determined by compressive tests.

Comparatively few tests have been made on the elasticity of concrete in tension, but these seem to indicate that for small stresses, it is practically the same as in compression, although probably slightly less.

26. Behavior of Reinforced Concrete Under Tension.—The behavior of reinforced concrete under tension, and especially when constituting the tensile side of a beam, has been the result of much study by many experimenters. Early tests indicated that the ultimate strength of reinforced concrete is as much as ten times that of plain concrete, but such results were due to the fact that it was found extremely difficult to determine just when the concrete begins to crack. Cracks do not become noticeable, even on very close examination, until a stretching occurs corresponding to a tensile stress much beyond the ultimate strength of the concrete. The steel forces the concrete to elongate uniformly throughout, so that a crack will open up very slowly and will remain invisible for some time.

A method of detecting minute cracks was accidentally discovered in 1901–2 in some experiments made at the University of Wisconsin. It was found that when beams were hardened in water and only partially dried before testing, very fine hair-cracks became noticeable at a moderate load. Before these cracks occurred, however, dark wet lines appeared across the beam, and it was observed that each of these lines was later followed by a very fine crack. That these watermarks were incipient cracks was determined by sawing out a strip of concrete along the outer part of the beam. Careful measurements of extension showed that these streaks, or watermarks, occurred at practically the same deformation at which the concrete ruptured when not reinforced. This same phenomenon has since been observed by many careful experimenters, and the fact is now generally established that concrete, reinforced with steel, does not elongate under tensile stress to any greater extent before cracking than plain concrete.

A reinforced concrete beam for working loads is usually more heavily stressed on the tension side than the ultimate tensile strength of plain concrete—enough steel being usually embedded near the lower face to permit the full allowable compressive strength of the concrete to be utilized. The presence, then, of the cracks above referred to, occurring long before a reinforced concrete beam has obtained its working load, must seriously affect the tensile strength of the concrete. The formulas now in most general use for the design of reinforced concrete beams neglect entirely the tensile strength of the concrete.

The important question which arises is how far concrete may

be cracked without exposing the steel to corrosion. Concrete, fortunately, contains a large proportion of lime which readily absorbs carbon dioxide. For this reason, steel is effectually protected by concrete if it is covered with even a thin film. Experiments tend to show that concrete when well placed and mixed wet, completely protects the steel in the tensile side of a beam from corrosion, even when the unit stress in the steel approaches the elastic limit.

27. Shrinkage and Temperature Stresses.—In reinforced concrete structures which are free to contract and expand, the stresses occurring from temperature changes and from shrinkage in hardening are due wholly to the mutual action of the steel and concrete. Of the stresses produced from these two causes, those which result from hardening are the greater, but experiments show that even these are not sufficient to be of practical importance. In regard to the temperature stresses, they are negligible by reason of the nearly equal rates of expansion of the two materials.

On the other hand, if reinforced concrete structures are restrained by outside forces, or if they are of such dimensions that they cannot be considered as sufficiently well bonded to act as a unit—such as long retaining walls—then the stresses resulting are much greater, and the tensile strength of the concrete will be reached (this will occur with a drop in temperature somewhere between 10 and 20° Fahr.), thus producing cracks, called contraction cracks. To prevent plainly noticeable cracks due to shrinkage and lowering of the temperature, steel should be inserted—the amount used varying in practice from 0.2 of 1 per cent to 0.4 of 1 per cent, based on the cross-section of the concrete. This is less than the amount required theoretically, but experience shows this amount to give very satisfactory results where the foundations are stable. If the structure is fixed in two directions, the reinforcement must be placed accordingly.

No amount of reinforcement can entirely prevent contraction cracks. The steel can, however, if of small diameter and placed close to the surface, force the cracks to take place at such frequent intervals that the required deformation occurs without any one crack becoming large. No cracks will open up to be plainly noticeable until the steel is stressed beyond its elastic limit. The amount of steel should be such, then, that without being stressed beyond its elastic limit, it will withstand the tensile stress resulting from the maximum fall of temperature (usually

considered to be 50°) in the steel itself plus the tensile stress necessary to crack the concrete. A high elastic-limit steel is thus advantageous.

The size and spacing of the cracks will also depend upon the bond strength of the reinforcing rods. The distance between cracks in any given case will be the length required to develop a bond strength equal to the tensile strength of the concrete. Thus, bars with irregular surfaces which provide a mechanical bond with the concrete are in general more effective than smooth bars.

28. Repetition of Stress.—Experiments on cubes of neat cement and concrete for repeated loads have shown that the limit of

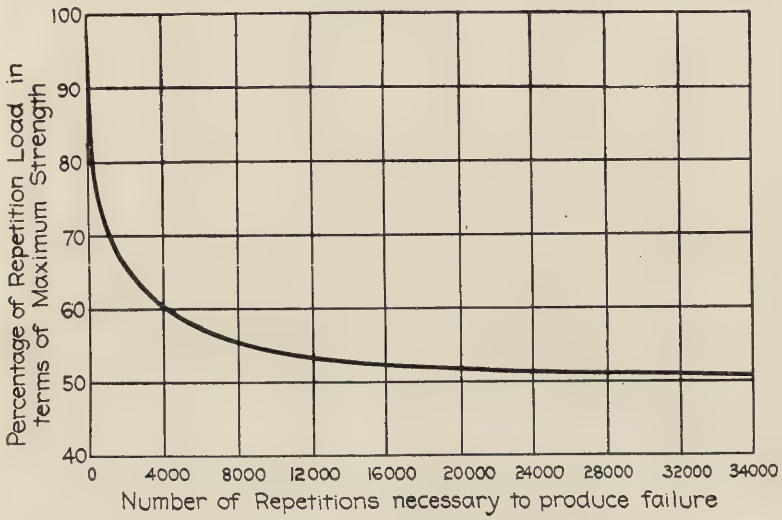

Fig. 11.—General curve of the fatigue of concrete.

permanent elasticity for such loads is from 50 to 60 per cent of the ultimate strength. Experiments on reinforced beams also indicate similar results. In the case of beams, the failure under repeated loads appears to be largely a gradual fracture in diagonal tension, ending with a crushing of the concrete at the upper surface of the beam.

An account of experiments to determine the effect of repeated stress on concrete is given in Trans. Am. Soc. C. E., Vol. LVIII. The summary is shown in Fig. 11, and it is seen that the number of repetitions required to produce failure varies with the load applied. For example, if the load applied is 60 per cent of the ultimate, 4000 applications cause failure; while if the load applied is 70 per cent, about 1000 applications are required.

Problems

6. A steel bar of 20-in. length and a cross-section of 2 1/2 sq. in. is put in tension. The force applied is 40,000 lb. and the extension under this load is 0.011 in. Determine the modulus of elasticity of this steel in pounds per square inch.
7. If steel costs 4¢ per pound with a working compressive strength of 16,000 lb. per square inch, and concrete costs 30¢ per cubic foot with a working compressive strength of 450 lb. per square inch, what will be the relative cost of the two materials in members sustaining purely compressive stresses?
8. A plain concrete retaining wall has a height of 20 ft. and a width of 8 ft. at the base. The front face has a batter of an inch to the foot or 1 : 12. (a) Compute the number of cubic yards in 1 ft. length of wall. (b) Compute the approximate amount of cement, sand, and broken stone required—the mixture being 1 : 2 1/2 : 5. (c) What is the approximate weight of wall?
9. What change of length will tend to take place in a long plain concrete wall by a drop of temperature of 50° Fahr.? How do you think the disfiguration of such a wall by irregular cracks may be prevented?
10. The allowable tensile stress in a 1-in. plain round steel rod is 16,000 lb. per square inch and the working bond strength is 80 lb. per square inch. What length of embedment is necessary to develop the allowable stress in the steel? What would be the length of embedment if a 1/2-in. plain round rod had been employed?
11. Explain how it is that a constant value of n, equal to 15, may be used in the computations for the design of beams and columns.
12. At a given point in the tensile part of a reinforced concrete beam, the ultimate tensile strength in the concrete of 200 lb. per square inch is reached. Approximately what is the corresponding tensile stress in steel located at about this point?
13. A load is a lowed to pass over a reinforced concrete beam which is in amount about 65 per cent of the ultimate load. About how many repetitions of this load will cause failure?
14. A long reinforced concrete wall is subjected to a drop of temperature of 50° Fahr. Consider 160 lb. per square inch the ultimate strength of the concrete and 40,000 lb. per square inch the elastic limit of the steel. (a) What stress is caused in the steel by the drop of temperature in the steel itself considering the ends of the steel as fixed? (b) How much more stress can be put into the steel before stressing it beyond its elastic limit? (c) If the steel is to be stressed just to the elastic limit, what percentage of steel is necessary to prevent large contraction cracks in the concrete?
15. Assuming the steel rods in Problem 14 to be 1/4 in. in diameter with an ultimate bond strength of 400 lb. per square inch, how far apart would the minute cracks occur?

PART II

THE THEORY AND DESIGN OF SLABS, BEAMS, AND COLUMNS.

CHAPTER IV

RECTANGULAR BEAMS

[Reinforced concrete is not used for plain tension members so that the stresses to be resisted in this type of construction come from simple compression, simple bending, or combined compression and bending. The simple beam supported at its two ends will be studied first and the stresses which must be provided for will be analyzed. The student should know how to determine the outer forces (loads and reactions) which are applied to a beam. Before taking up reinforced concrete beams it will be desirable to consider the nature of the stresses in a plain concrete or homogeneous beam of any material.]

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29. Inner Forces in a Homogeneous Beam.—The inner forces in a beam are tension, compression, and shear. The following principles concerning homogeneous beams should be familiar to the student:

1. At any cross-section the internal forces, or stresses, may be resolved into normal and tangential components. The components normal to the section are stresses of tension and compression, while the tangential components add together and form a stress known as the resisting shear.

2. The shear at any cross-section is borne by the tangential stresses in that section. The moment at any section is borne by the component stresses normal to that section.

3. The neutral axis passes through the center of gravity of the cross-section.

4. The intensity of stress normal to the section increases directly with the distance from the neutral axis and is a maximum at the extreme fiber. The intensity of this stress at any given

point in the cross-section is given by the following formula:

$$f=\frac{My}{I}$$

in which f = fiber stress at distance y from neutral axis.
M = external bending moment at section in inch-pounds.
y = distance in inches from neutral axis to any fiber.
I = moment of inertia of the cross-section about the neutral axis.

5. The general formula which gives the longitudinal shear per square inch (v) at any desired point in the cross-section is

$$v=\frac{VQ}{Ib'}$$

in which V = total shear at the section in pounds.
Q = statical moment about the neutral axis of that portion of the cross-section lying either above or below (depending upon whether the point in question is above or below the neutral axis) an axis drawn through the point in question parallel to the neutral axis.
I = as before.
b' = width of beam at the given point.

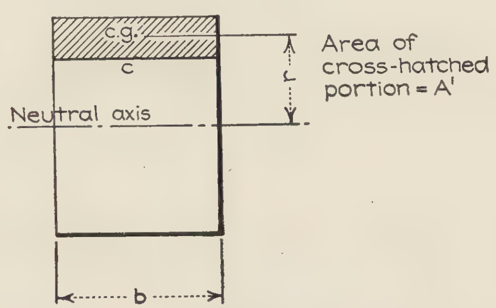


FIG. 12.

In the above formula, by the term *statical moment* is meant the product of the area mentioned by the distance between its center of gravity and the neutral axis. For example, the longitudinal shearing intensity at a point c in a rectangular beam, Fig. 12, may be expressed as follows:

$$v=\frac{VA'r}{Ib}$$

For rectangular beams and all beams of uniform width, the largest value of v for any given section will occur at the neutral

axis since the statical moment Q has its maximum value for a point on this axis, and b is constant.

6. If a beam is of constant cross-section throughout, the maximum values of f and v will occur at the section where M and V respectively have maximum values.

The case of longitudinal shear is shown in Fig. 13. Let the fiber stresses at section m be represented by f_1 and those at section n by f_2, while the longitudinal variations of the fiber stresses between these two sections are indicated at section n by the cross-shaded areas. This increase of horizontal stress from one section to another (which we know to be true since the bend-

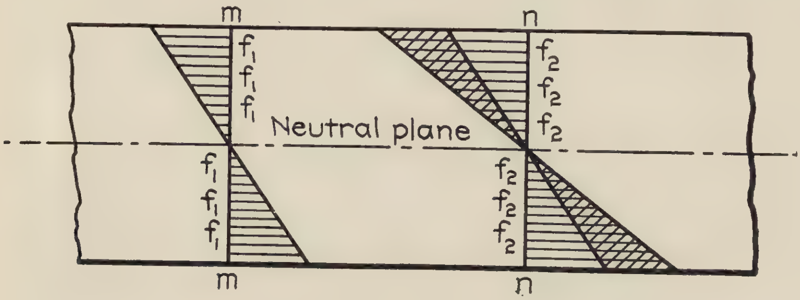


FIG. 13.

ing moment M increases from the ends toward the center of span and with it the intensity of the horizontal stresses) induces a force at every longitudinal layer tending to slide the upper portion past the lower; and this sliding or shearing force, which increases at every layer, attains its maximum intensity at the neutral plane.

In addition to the longitudinal or horizontal shear at any point, as explained above, there co-exists a vertical shear and the intensity of this vertical shear is equal to the intensity of the horizontal shear. This may be proved as follows:

Fig. 14 represents an infinitely small portion of the side of a beam at any given point. The sides of the element, as far as any difference in the result is concerned, may each be represented by h, and the breadth of the beam at this point will be denoted by b. Now there are two sets of shearing forces acting upon it, one vertical and the other horizontal; and these shears form two pairs of couples, acting as indicated by the vertical and horizontal arrows. For an infinitesimal distance (h) the horizontal fiber

stresses balance each other and need not be considered. If the intensity of the horizontal shear at this point be represented by s, and that of the vertical shear by v, then in order that the total horizontal and vertical shears acting on the particle may be in equilibrium, the moments of these two shears must be equal, thus:

$$(sbh)h = (vbh)h$$
$$\therefore s = v$$

Since the intensities of the horizontal and vertical shears are equal, they will both be represented by the common symbol v.

Using our general formula, it should be clear that the intensity of the shear at the top and bottom of a beam is zero and, by substituting the proper values of the separate terms for a rectangular cross-section, the student will find that the intensity of shear (horizontal and vertical) along a vertical cross-section for a rectangular beam varies as the ordinates to a parabola, as shown graphically in Fig. 15. The maximum value occurs at the neutral axis and is 3/2 the average intensity, or $\frac{3}{2} \cdot \frac{V}{bd}$.

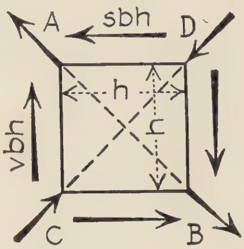


Fig. 14.

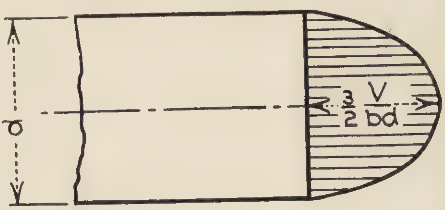


Fig. 15.

We now know that shear on a vertical cross-section is greatest at the neutral axis and at this axis the horizontal fiber stress is zero; in other words, there are no other forces on the web but those of shear. Consider again the infinitely small prism shown in Fig. 14, but this time assume it to lie at the neutral plane. The shearing forces acting on this prism will develop inclined stresses of tension and compression. The value of these inclined stresses which are indicated by the diagonal arrows, Fig. 14, may be found by resolving the shearing forces into components parallel to AB and CD. The value of each component is found to be $\frac{vbh}{\sqrt{2}}$. The effect of these components is to produce on CD a total

tension of $\frac{2vbh}{\sqrt{2}}$ and on AB a total compression of the same amount. Since the length of the diagonals is $h\sqrt{2}$, the intensities of these inclined stresses are equal and with a magnitude of $\frac{2vbh}{(\sqrt{2})(h\sqrt{2})(b)}=v$. It, therefore, follows that at the neutral plane there exists a tension and compression at angles of 45 degrees to the horizontal, and that the intensity of these forces is equal to that of the shear.

Above and below the neutral plane the direction and magnitude of the inclined stresses are not as above found, due to the fact that the final value of the tension or compression at any point would have to be obtained by combining the horizontal fiber stresses due to bending with the inclined stresses due to shear. At the end of a beam, however, where the shear is a maximum and the bending moment a minimum, these stresses lie practically at 45 degrees to the horizontal throughout the entire depth of beam. Also, at the section of maximum moment the shear is zero and the stresses become horizontal.

It is proved in treatises on mechanics that if f represents the intensity of horizontal fiber stress and v the intensity of vertical or horizontal shearing stress at any point in a beam, the intensity of the inclined stress will be given by the formula

$$t=1/2f \pm\sqrt{1/4f^2+v^2}$$

and the direction of this stress by the formula

$$\tan 2K=\frac{2v}{f}$$

where K is the angle of the stress with the horizontal.

The two formulas given above are general formulas and apply when f is either tension or compression. By their use the lines of maximum stress may be traced throughout the beam. With the $\left\{\begin{matrix}\text{plus}\\\text{minus}\end{matrix}\right\}$ sign before the radical in the first formula, the resulting value of t is the maximum tension $\left\{\begin{matrix}\text{below}\\\text{above}\end{matrix}\right\}$ or the maximum compression $\left\{\begin{matrix}\text{above}\\\text{below}\end{matrix}\right\}$ the neutral plane. The formula for K shows that two values of K, differing by 90 degrees, will satisfy the equation. At any given point, then, maximum compressive stress and maximum tensile stress make an angle of 90 degrees with each other. Fig. 16 shows approx-

imately the directions of the maximum stresses for a beam uniformly loaded. It should be noted that the common theory of flexure gives the unit stress correctly at the important section of maximum moment and also for the extreme fibers in other sections, since at these points the shear is zero. Where the

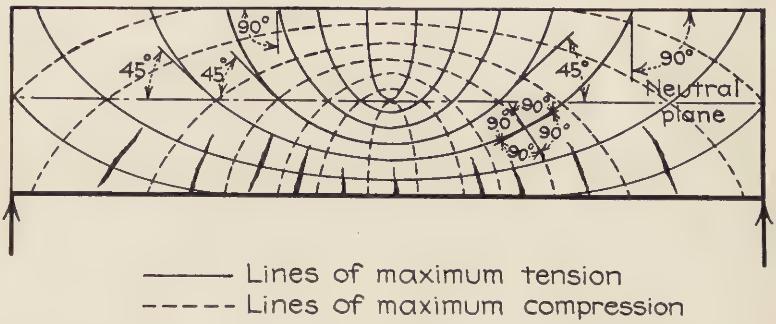


FIG. 16.

shear is not zero an inclined stress is the result and our flexure formula gives only the horizontal component of this stress—namely, the *fiber* stress.

By the use of the above formulas the value of the maximum tensile (or inclined) stress at any point may be determined,

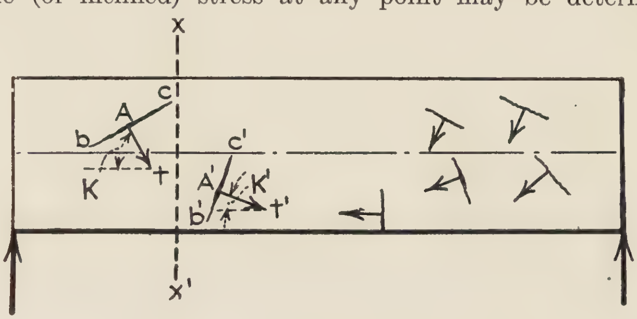


FIG. 17.

also the angle K it makes with the horizontal. The stress t, Fig. 17, produces the greatest intensity upon a plane perpendicular or normal to its line of action—namely, upon the plane bc. In the same manner considering point A', it is a comparatively simple matter to derive the magnitude of t' and the angle K' it makes with the horizontal. The stress t' causes the greatest stress on the inclined plane $b'c'$.

Theoretically, a plain concrete beam will fail by cracks opening up along the zigzag lines which are shown in Fig. 16. This is found to be true in tests on this type of beam. A plain concrete beam will always fail in tension due to the low ultimate strength of concrete in tension as compared to its strength in compression.

The exact direction of the maximum tensile stress at any point in a beam depends only upon the relation between shear and bending moment. Hence by means of the formulas given above, the student should be able to trace the general direction of the stresses in a beam with the loading other than uniform. The introduction of steel into a concrete beam, however, changes the direction of the stress lines somewhat. The exact effect of adding steel will be taken up later.

The student should have some idea of the variation of the

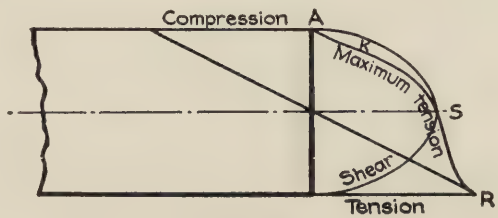


Fig. 18.

maximum tensile stress throughout a vertical cross-section. Of course, in studying this variation of stress, it should be remembered that the direction of the stress is not normal to the vertical section at the different points. Maximum tensile stresses, as we have already shown, are normal to a vertical section only when the section is taken where the shear is zero. In other sections these stresses are normal only along the lower fibers of the beam. Fig. 18[1] shows the variation in the maximum tensile stress on a section such as XX', Fig. 17. The variation of the normal and shearing stresses is also shown.

It should now be clear that the steel reinforcement in a concrete beam for uniform loading should have the general directions shown in Fig. 19 in order to take the tension in the beam and prevent the cracks starting along the lines indicated. Fig. 20 is the simplest method of reinforcement and quite often used for light loads. In beams highly stressed, the student can appreciate the reason for curved reinforcement in addition to the horizontal rods. The most common method is to use several rods for the horizontal reinforcement and then to bend

[1] From Turneaure and Maurer's "Principles of Reinforced Concrete Construction," 2nd edition, page 50.

a part of these upward as they approach the end, where they are not needed to resist bending stresses. This matter will be taken up in detail later. The concrete is depended upon for the compressive and shearing stresses, its resistance to such stresses being large.

It cannot truly be said that the above remarks refer directly to homogeneous beams of concrete. As we have already seen,

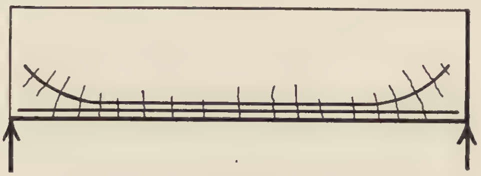


Fig. 19.

the elastic property of concrete is very much inferior to that of steel, consequently the formula $f = \frac{My}{I}$, does not exactly apply except for very low stresses. In fact, in the previous investigation we have erred slightly by representing the direction of the maximum internal stresses in a concrete beam to be identical with the direction of the maximum stresses in a steel beam. The difference, however, is inappreciable, especially for low stresses. The student should appreciate the fact that the

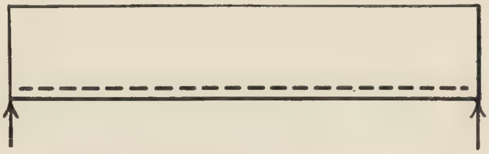


Fig. 20.

directions of the internal stresses at the principal points in concrete beams are the same as in steel beams—namely, at the upper and lower fibers and at the neutral plane. This statement should be understood when it is considered that the direction at these points, as determined by the formula $\tan 2K = \frac{2v}{f}$, depends only upon a zero value for either v or f. For example, at the lower and upper fibers, the value of v becomes zero and the magnitude of f does not control. Likewise, at the neutral plane, f becomes zero and the value of the angle K is not affected by the magnitude of v.

30. Assumptions in Common Theory of Beams.—In order to derive a formula by which we can design concrete beams reinforced with steel, it will first be necessary to overhaul the assumptions on which the common theory of flexure is founded and see if we can derive a formula, or formulas, by which we can determine stresses in both the steel and concrete of such beams.

The two main assumptions in the common theory of beams may be stated as follows:

1. If, when a beam is not loaded, a plane cross-section be made, this cross-section will still be a plane after the load is put on and bending takes place. (Navier's hypothesis.)

2. The stress is proportional to the deformation—namely, to the elongation or compression per unit of length. (Hooke's Law.)

From the first assumption the following principle is obtained: the unit deformations of the fibers at any section of a beam are

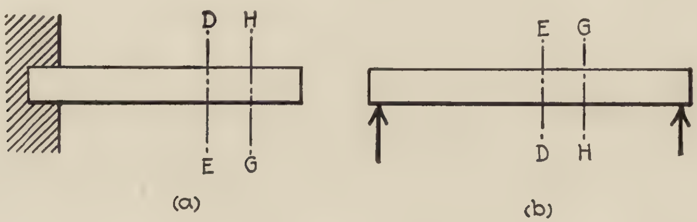


Fig. 21.

proportional to their distances from the neutral axis. From the second assumption: the unit stresses in the fibers at any section of a beam are also proportional to the distances of the fibers from the neutral axis.

It may be well to explain more at length what is meant by the two assumptions and the results derived from them.

Assumption 1.—Imagine two originally parallel cross-sections, ED and GH, Fig. 21a or Fig. 21b, so near to each other that the curve taken after bending by that part of the neutral plane between these sections may, without appreciable error, be accounted circular. Let ED and GH, Fig. 22a or 22b, be the lines in the loaded beam in which these cross-sections cut the plane of the paper, and let O be the point of intersection of the lines ED and GH. Let $OF = r$, $FL = y$, $FK = l$, $LM = l + al$, in which a is the elongation per unit of length of a fiber at a distance y from the neutral axis, y being variable; then, because FK and LM are concentric arcs

subtending the same angle at the center, we shall have the proportion

$$\frac{l+al}{l}=\frac{r+y}{r} \text{ or } 1+a= 1+\frac{y}{r}$$

$$a=\frac{y}{r} \text{ or } a=\left(\frac{1}{r}\right)y$$

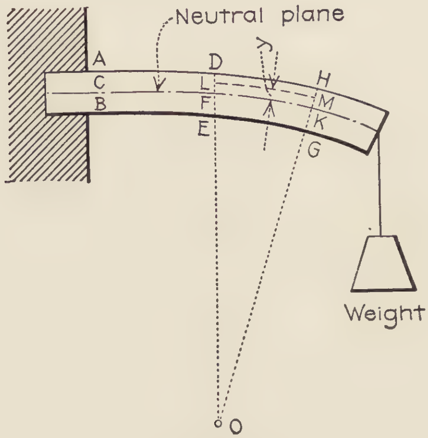


Fig. 22a.

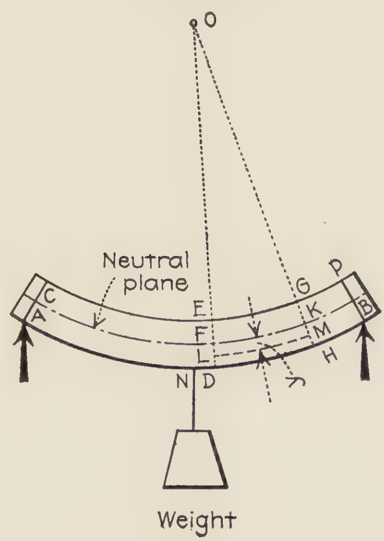


Fig. 22b.

but, as y varies for different points in any given cross-section while r remains the same for the same section, it follows that if a certain cross-section be assumed, the deformation of any fiber

at the point where it cuts this cross-section is proportional directly to the distance of this fiber from the neutral axis of the cross-section.

Assumption 2.—As to the evidence in favor of this law, experiment shows that as long as a material such as steel is not strained beyond safe limits, this law holds. However, wrought iron and steel are the only important structural materials which closely follow this law, and they only within their elastic limits. But under working conditions, these materials are not stressed beyond their elastic limit and so the formulas ordinarily hold. Timber, stone, and cast iron can hardly be said to obey Hooke's law, yet for working conditions the common flexure formulas for these materials are roughly correct and they are in general use.

The important question to be decided is—does concrete follow the laws stated above? In regard to the first assumption, it can be said that careful measurements show some deviation from a plane, but in general this assumption seems to be warranted by the results of observed deformations under working loads. Concerning the second law, as regards the concrete in a reinforced concrete beam, it should be clear that the assumption will not strictly apply except for low stresses.

31. Plain Concrete Beams.—*OS* in Fig. 23 is the stress-deformation diagram for concrete in compression, with which the student is already familiar. The curve shown here is identical with Fig. 10, the only difference being that the deformations are represented vertically instead of horizontally. This change has been made in order that we may apply the curve directly to the cross-section of a beam. The curve *OT* is the stress-deformation diagram for concrete in tension and is of use only in the analysis of plain concrete beams.

We have just seen that *Assumption* 1 may be applied to beams of concrete. This assumption leads us to the conclusion that deformations of the fibers are proportional to the distances of the fibers from the neutral axis. Figs. 24 and 25 show the curves applied to the vertical cross-section AX of a beam. For example, suppose the deformation at point a, Fig. 25, is the same as represented by Oa in Fig. 23. The corresponding stress given by the compression curve in Fig. 23 is aa'. Lay off the distance aa' in Fig. 25 to represent that stress. Proceeding similarly for all points and connecting, we have the stress curve $A''OX''$, which is nothing more than a portion of the stress-deformation diagrams in Fig. 23 plotted to a different scale.

Turning our attention to the principles of mechanics, in order to determine which ones may be used to find the resisting moment in a plain concrete beam, we find we have three, as follows:

1. For beams rectangular in section, the average unit tensile and compressive fiber stresses on any cross-section are represented

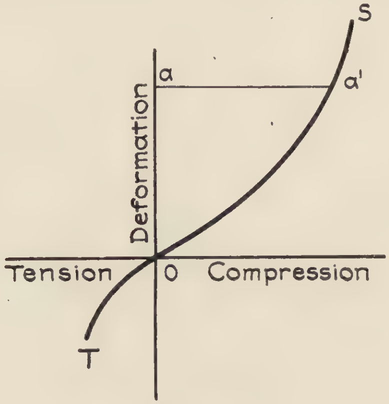


FIG. 23.

by the average abscissæ in the tensile and compressive parts of the combined stress-deformation diagram respectively. Also, the total tension T, Fig. 26, and the total compression C on a cross-section are proportional respectively to the areas XOX''

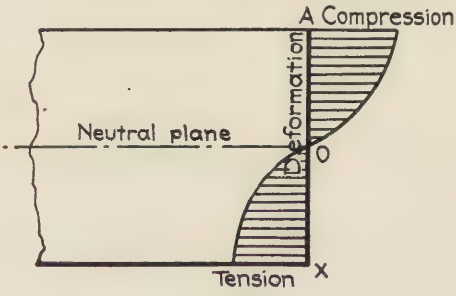


FIG. 24.

and AOA''; hence, according to some scale, the areas represent T and C respectively.

2. The resultant tension T and resultant compression C act through the centers of gravity of the tensile and compressive areas in the combined stress-deformation diagram.

3. When all the forces (loads and reactions) applied to the beam act at right angles to it, then the resultant tension T equals

the resultant compression C; hence, the two stresses constitute a couple which is the resisting couple. (This principle comes from one of the three conditions of equilibrium; namely, $\Sigma H = O$.)

In order to introduce important ideas concerning concrete beams, the perfectly general method of figuring the ultimate resisting moment at any cross-section will be explained. Fig.

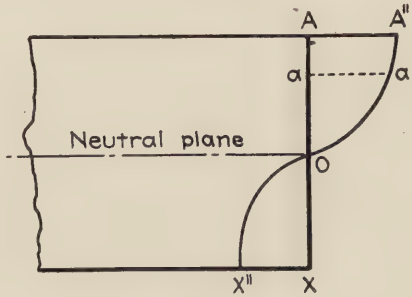


Fig. 25.

27 shows the combined stress-deformation diagram for a given concrete. A plain concrete beam will break in tension because of the low strength of concrete in tension as compared to its strength in compression. BB' then gives this breaking tensile stress and T is represented by the area BOB'. Likewise, C

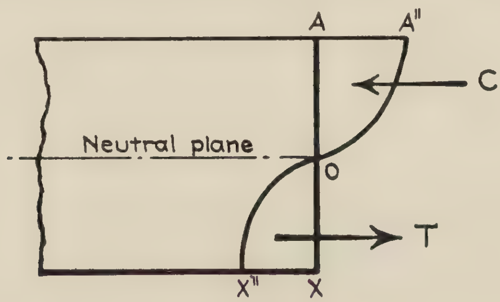


Fig. 26.

is represented by the area AOA'—this area being determined from the principle that BOB' and AOA' are equal. The next step is to locate the center of gravity of each area and scale the vertical distance between them. The resisting couple is T or C multiplied by this scaled distance.

Partly to test the correctness of the theory of flexure of concrete beams, Professor Mörsch[1] made three beams and several tension and

[1] Of the Zurich Polytechnic, Zurich, Switzerland.

compression specimens of the same mix of concrete.[1] From tests on the specimens, he obtained a combined stress-deformation diagram from which he computed the probable resisting moment of the beams. The average of the actual resisting moments of the beams and the moment computed by the theory of flexure agreed very closely. This general theory of the flexure of concrete beams may thus be considered highly satisfactory.

It should be clear to the student by this time that a great deal of compressive strength cannot be made use of in a plain concrete beam. If concrete were only stronger in tension, then the plain concrete beam might be of some structural value. In order to offset this disadvantage of plain concrete, steel is used. In the discussions which follow it will be assumed that the concrete and steel adhere perfectly and therefore deform equally. Many tests show that, under proper design, this is true for all practical purposes.

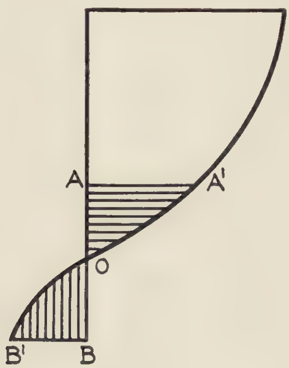


Fig. 27.

32. Flexure Formulas for Reinforced Concrete Beams.—A great many formulas have been proposed from time to time to be used in the design of reinforced concrete beams. Of course, the object has been in each case to obtain a formula whereby the resisting moment may be obtained at any cross-section in a sufficiently accurate manner, but at the same time with a view to the ease with which it may be used in practice. As might be expected, many of the earlier formulas considered the concrete to carry its share of the tension which we know now cannot be done with absolute safety. By means of some formulas the ultimate resisting moment of a beam may be computed and the ultimate loads at once determined which will produce this moment. A factor of safety is then applied and the safe load obtained. In other formulas the working loads are found directly. When the ultimate load is obtained from a formula, the utimate strength of the concrete and the elastic limit of the steel (the student will find later that the elastic limit of the steel may determine ultimate strength) must be used. Likewise, when deriving safe loads, the working strengths should be substituted.

Unlike steel beams, reinforced concrete beams require a prelim-

[1] From Turneaure and Maurer's "Principles of Reinforced Concrete Construction," 2nd edition, page 55.

inary formula to be solved before the formula for resisting moment may be employed. Solving this preliminary formula locates the position of the neutral axis which is in the same position only for beams of a given concrete and of a given percentage of steel reinforcement.

The two most general varieties of flexure formulas in practical use will be taken up in Arts. 33 and 34. In each of these two classes of formulas, tension in the concrete is neglected. For purposes of discussion, the subject of beams will first be treated with reference only to the horizontal reinforcement. The inclined tensile stresses will be considered separately. Analysis and results of experiments discussed in succeeding assignments prove beyond any doubt that the rational formulas which we will now develop may be used safely and economically in designing. No empirical formula is needed.

The student should realize that the following assumptions must be made in order to derive working formulas: (1) the union between the steel and the concrete is sufficient to cause the two materials to act as one material; (2) no initial stresses are considered in either the concrete or the steel due to temperature or shrinkage; (3) the applied forces are parallel to each other and perpendicular to the neutral surface of the beam before bending; (4) sectional planes before bending remain plane surfaces after bending within the elastic limit of the steel.

33. Flexure Formulas for Working Loads Based on Rectilinear Variation of Stress in Concrete.—The loads being working loads, the unit stress in the steel is within the elastic limit, and the unit stresses in the concrete may be considered without material error to vary as the ordinates to a straight line. The following notation will be employed referring to Fig. 28.

Let f_c = maximum intensity of compressive stress in the concrete under a given load. It is represented by the distance AA''.

f_s = maximum intensity of tensile stress in the metal under the same load (the area of reinforcement is assumed to be so small with reference to the total area of cross-section of the beam that the stress in the metal is practically uniform).

AA' represent the deformation, or deformation per unit length, in the concrete which is stressed to the amount f_c.

$C'C$ represent the unit deformation, or deformation per unit length, in the metal which is stressed to the amount f_s.

C = total compression in concrete at a section of the beam.

T = total tension in steel at a section of the beam.

E_c represent the modulus of elasticity of concrete in compression.

E_s represent the modulus of elasticity of steel in tension.

n = ratio $\frac{E_s}{E_c}$.

d = distance from compression surface to axis of reinforcement.

k = proportionate depth of neutral axis from below the compression surface.

a_s = area of cross-section of steel.

b = breadth of a rectangular beam.

p = "steel ratio" = the ratio of the area of steel to area of concrete $=\frac{a_s}{bd}$.

M_c = resisting moment as determined by concrete.

M_s = resisting moment as determined by steel.

M = bending moment or resisting moment in general.

Now the total compressive resistance is equal to the area of the triangular figure AOA'' multiplied by b, the breadth of the beam. But the area $AOA''=1/2\,(AA'')kd=1/2\,f_ckd$. Hence, the total compressive resistance C is equal to $1/2\,f_ckbd$.

The total tensile resistance T is evidently the cross-sectional area of the metal multiplied by the uniform intensity of stress thereon $=a_sf_s$.

Since the total compressive resistance above the neutral axis must be equal to the total tensile resistance below the same, we have

$$1/2\,f_ckbd=a_sf_s \tag{1}$$

From the assumption that deformations vary as the distances of the fibers from the neutral axis,

$$\frac{AA'}{OA}=\frac{CC'}{OC} \text{ or } \frac{AA'}{kd}=\frac{CC'}{d(1-k)}$$

But by definition of the values represented by AA' and CC', we have

$$AA'=\frac{f_c}{E_c} \text{ and } CC'=\frac{f_s}{E_s}$$

Substituting

$$\frac{f_c}{E_c kd} = \frac{f_s}{E_s d\,(1-k)}$$

which reduces to

$$f_s = f_c n \,\frac{1-k}{k} \qquad (2)$$

The stress-deformation diagram of the concrete being straight, the center of action of the compressive stresses is at a point 2/3 of the height of OA from O. The lever arm for the resisting

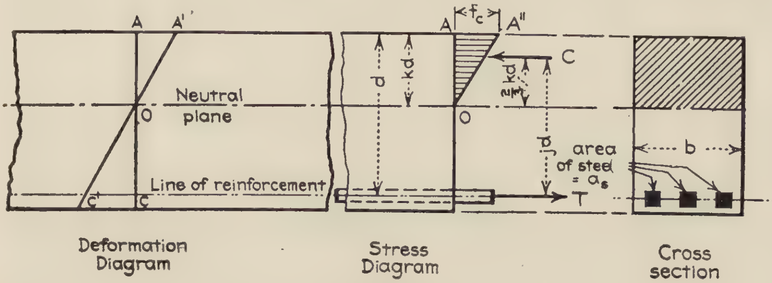


FIG. 28.

moment of the summation of the compressive stresses with respect to the neutral axis, is therefore represented by $2/3kd$.

The center of action of the tensile stresses is at a point distant OC from O. The lever arm for the resisting moment of the summation of the tensile stresses with respect to the neutral axis, is therefore represented by $d(1-k)$.

The total resisting moment of the beam is the sum of the moments of the total compressive stresses and of the total tensile stresses about the neutral axis. Therefore, we have

$$\begin{aligned} M &= 2/3\; kd(1/2\, f_c kbd) + d(1-k)a_s f_s \\ &= 1/3\, f_c k^2 bd^2 + a_s f_s d\,(1-k) \qquad (3) \end{aligned}$$

Eliminating k between equations (1) and (2), the following formula for steel ratio results

$$p = \frac{1/2}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{nf_c}+1\right)} \qquad (4)$$

This formula shows that for a given concrete and ratio of working stresses, p has the same value for all sizes of beams.

Introducing the value of f_s from equation (2) into equation (1), we have

$$1/2k^2bd - a_s n(1-k) = 0$$

or

$$1/2k^2b - pbn(1-k) = 0$$

5

from which

$$k=\sqrt{2pn+(pn)^2}-pn \tag{5}$$

Sustituting the value of a_sf_s from (1) into (3), we get

$$M_c=1/2f_ck(1-1/3k)bd^2 \tag{6}$$

Substituting the value of f_c from (1) into (3) and remembering that $a_s=pbd$,

$$M_s=pf_s(1-1/3k)bd^2 \tag{7}$$

Solving equation (1) for f_c,

$$f_c=\frac{2f_sp}{k} \tag{8}$$

It will be noted that equation (6) gives the resisting moment when the maximum allowable value of f_c is introduced as the limiting factor, and that equation (7) gives the resisting moment when the maximum allowable value of f_s is the limiting factor. The lesser of these two resisting moments, when proper working values are assigned to f_c and f_s, is the allowable resisting moment of the beam in question.

The distance from C to T is in most formulas denoted by jd; namely, j denotes the ratio of this distance to the total depth of the beam. Since $jd=d-1/3kd$, then $j=1-1/3k$ and the above formulas may be simplified by substituting j for the quantity $(1-1/3k.)$

The formulas which we shall need to use in the design of reinforced concrete beams are collected for convenience as follows:

$$p=\frac{a_s}{bd}$$

$$p=\frac{1/2}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)} \tag{4}$$

$$k=\sqrt{2pn+(pn)^2}-pn \tag{5}$$

$$M_c=1/2f_ckj(bd^2) \quad \text{or} \quad bd^2=\frac{M}{1/2f_ckj} \tag{6}$$

$$M_s=pf_sj(bd^2) \quad \text{or} \quad bd^2=\frac{M}{pf_sj} \tag{7}$$

$$f_c=\frac{2f_sp}{k} \tag{8}$$

Illustrative Problem.—What will be the resisting moment (M) for a beam whose breadth (b) is 8 in. with a distance from the center of the reinforcement to the compression surface (d) of 12 in., the area of steel section

being 0.96 sq. in.? $E_s = 30{,}000{,}000$. $E_c = 2{,}500{,}000$. $f_c = 500$ lb. per square inch. $f_s = 16{,}000$ lb. per square inch.

$$n = \frac{E_s}{E_c} = \frac{30{,}000{,}000}{2{,}500{,}000} = 12. \quad p = \frac{a_s}{bd} = \frac{0.96}{(8)\ (12)} = 0.01$$

From equation (5)

$$k = \sqrt{(2)(0.01)(12) + (0.01)^2(12)^2} - (0.01)(12)$$
$$= 0.384$$

From equation (6)

$$M_c = 1/2(500)(0.384)(0.872)(8)(12)^2$$
$$= 96{,}500 \text{ in.-lb.}$$

From equation (7)

$$M_s = (0.01)(16{,}000)(0.872)(8)(12)^2$$
$$= 160{,}700 \text{ in.-lb.}$$

M_c is the lesser of the two resisting moments and hence controls in the design $= 96{,}500$ in.-lb.

Illustrative Problem.—Suppose that the beam of the preceding problem is 14 in. deep and is subjected to a bending moment of 130,000 in.-lb. Compute the greatest unit stresses in the steel and concrete.

$$p = \frac{a_s}{bd} = \frac{0.96}{(8)(14)} = 0.0086$$

From equation (5)

$$k = \sqrt{(2)\ (0.0086)\ (12) + (0.0086)^2\ (12)^2} - (0.0086)(12)$$
$$= 0.363$$

From equation (6)

$$130{,}000 = \left(\frac{f_c}{2}\right)(0.363)(0.879)(8)(14)^2$$
$$f_c = 520 \text{ lb. per square inch}$$

From equation (7)

$$130{,}000 = (0.0086)(f_s)(0.879)(8)(14)^2$$
$$f_s = 11{,}000 \text{ lb. per square inch}$$

Illustrative Problem.—A beam is to be figured to withstand a bending moment of 300,000 in.-lb. A 1 : 2 : 4 concrete will be used with $E_c = 2{,}000{,}000$ and $f_c = 600$ lb. per square inch. The pull in the steel is to be limited to 14,000 lb. per square inch. Its modulus of elasticity E_s is 30,000,000.

$$\frac{E_s}{E_c} = 15$$

From equation (4)

$$p = 0.0084$$

With this value of p, *Eq.* (5) gives

$$k = 0.391$$
$$j = 0.870$$

Either equation (6) or (7) may now be used in determining b and d since

the amount of steel to be employed will cause simultaneous maximum working stresses.

From equation (7)

$$bd^2=\frac{300{,}000}{(0.0084)(14{,}000)(0.870)}=2930$$

Many different values of b and d will satisfy the last equation. If b is taken as 10 in., then

$$d^2=\frac{2930}{10}=293, \text{ or } d=17\ 1/4 \text{ in.}$$

Finally

$$a_s=(0.0084)(10\times 17.25)=1.45 \text{ sq. in.}$$

If 1 3/4 in. is allowed between the tension surface of the concrete and the center of the steel, the entire depth of the beam should be 19 in.

PROBLEMS

16. What will be the resisting moment for a beam with $b=12$ in. and $d=16$ in., the area of steel section being 3.5 sq. in.? $E_s=30{,}000{,}000$. $E_c=3{,}000{,}000$. $f_c=600$ lb. per square inch. $f_s=16{,}000$ lb. per square inch.
17. Suppose the beam of the preceding problem is 20 in. deep (d) and is subjected to a bending moment of 550,000 in.-lb. Compute the maximum unit stresses in the concrete and steel.
18. Design a beam of 12.5 ft. span to carry a load of 1000 lb. per linear foot (including weight of beam), using a 1 : 2 : 4 Portland cement concrete having an allowable working strength of 650 lb. per square inch. A mild steel, with an allowable working stress of 14,000 lb. per square inch is to be used. $E_s=30{,}000{,}000$. $E_c=2{,}500{,}000$.
19. By means of the formulas $t=1/2f\pm\sqrt{1/4f^2+v^2}$ and $\tan 2K=\frac{2v}{f}$, prove the following statements concerning homogeneous beams:

    (a) Wherever shearing stress exists, the maximum stress will not be on a vertical section, but on an inclined one.

    (b) At all points in a beam where the shear is zero, the direction of the maximum tension is horizontal, as at points of maximum bending moment and along the outer fibers of the beam.

    (c) Wherever the horizontal fiber stress is zero (at the neutral plane and at all sections of zero bending moment), the direction of the maximum tension is inclined 45 degrees to the horizontal, and its intensity is equal to the vertical shearing stress at the same place.

---

34. Flexure Formulas for Ultimate Loads, Based on Parabolic Variation of Stress in Concrete.—The stress-deformation curve for concrete in compression agrees very closely with the parabola; in fact, so nearly so that the parabola is often used in theoretical analysis to represent the distribution of compressive stress in a

reinforced concrete beam. The form of parabola used has its axis vertical in the stress-deformation diagram, Fig. 10, and its vertex at the point S of the curve representing the ultimate strength.

The assumption is made in deriving formulas for ultimate loads, that the amount of reinforcement is sufficient to develop the full compressive strength of the concrete without stressing the steel beyond its yield point. Failure under such conditions will occur by crushing the concrete, with the yield point of the steel not exceeded. Then, the parabola representing the variation of compression is a *full* parabola, the upper end being the vertex and the axis horizontal. (Fig. 29.)

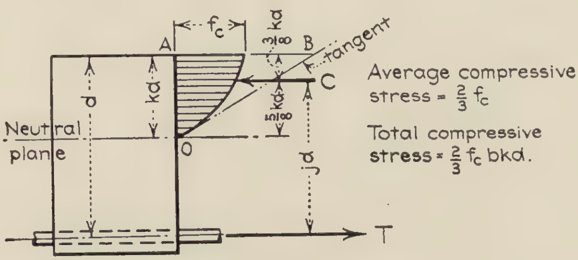


FIG. 29.

When ultimate loads are considered, the *secant modulus* for the concrete should not be used. The *initial modulus* should be employed and will be denoted in this article by E_c. Considering Fig. 29 as drawn to scale,

$$E_c = \frac{AB}{AO}$$

It is a well-known property of the parabola that

$$AB = 2f_c.$$

Since AO represents the deformation AA' (Fig. 28).

$$E_c = \frac{2f_c}{AA'}, \quad or \ AA' = \frac{2f_c}{E_c}$$

In the present connection, the two following properties of a parabola (Fig. 29) are useful: (1) the average abscissa of the parabolic arc equals two-thirds the greatest (f_c); (2) the distance from the center of gravity of the parabolic area to its top equals three-eighths the total height kd.

Following the same procedure as employed in deriving formulas for a rectangular variation of stress in the concrete and using

the value of AA' given above, the following equations may be obtained:

$$p=\frac{a_s}{bd}$$

$$p=\frac{2/3}{\frac{f_s}{f_c}\left(\frac{f_s}{2nf_c}+1\right)}$$

$$k=\sqrt{3pn+\left(\frac{3}{2}pn\right)^2}-\frac{3}{2}pn.$$

$$j=1-3/8k$$

$$M_c=2/3f_ckj(bd^2)\quad\text{or } bd^2=\frac{M}{2/3f_ckj}$$

$$M_s=pf_sj(bd^2)\quad\text{or } bd^2=\frac{M}{pf_sj}$$

$$f_c=\frac{3f_sp}{2k}$$

In the above formulas, f_s = elastic limit of the steel, and f_c = ultimate compressive strength of concrete.

When using the above formulas, it should be remembered that it was assumed at the outset that the amount of steel in the beam is sufficient to cause the ultimate resisting moment to be due to the concrete. Thus, the resisting moment of the beam may be figured by using the formula for M_c. If an amount of steel is used such that the ultimate strength of the concrete and the elastic limit of the steel would be reached simultaneously, either M_c or M_s may be used to determine the ultimate resisting moment. If a less amount of steel is used than the amount just mentioned, the conditions of the assumption do not hold, and the formulas given above cannot be used. When this happens the ultimate moment may be figured by means of formulas based on a parabolic variation of compression in the concrete and applicable for *any* load up to the ultimate. The parabola for such a case is not a full one and the formulas are cumbersome to use and not at all fitted for practical use.

The formulas for ultimate loads, however, can easily be employed in designing. The method is to find the amount of steel to give equal strength in tension and compression. Then either

$$bd^2=\frac{M}{2/3f_ckj}$$

or

$$bd^2 = \frac{M}{pf_s j}$$

may be used to determine the size of beam necessary.

Illustrative Problem.—A beam is to be figured to safely withstand a bending moment of 200,000 in.-lb., the ultimate compressive strength of the concrete being taken at 2000 lb. per square inch and the elastic limit of the steel at 40,000 lb. per square inch. $n=15$.

$$p = \frac{2/3}{20\left(\frac{20}{30}+1\right)} = 0.02$$

$$k = \sqrt{3(0.02)\ (15) + \left(\frac{3}{2}\right)^2 (0.02)^2 (15)^2} - \frac{3}{2}\,(0.02)\ (15)$$

$$= 0.598$$

$$j = 1 - 3/8k = 0.775.$$

With a factor of safety of 3, the ultimate bending moment is 600,000 in.-lb. and

$$bd^2 = \frac{600{,}000}{(2/3)\ (2{,}000)\ (0.598)\ (0.775)} = 972.$$

With $b = 8$ in., then

$$d^2 = \frac{972}{8} = 121.5, \text{ or } d = 11 \text{ in.}$$

Also,

$$a_s = (0.02)(8)(11) = 1.76 \text{ sq. in.}$$

Some designers consider the stress-deformation curve to be *a full* parabola even at working loads and use working formulas similar to those derived for a rectilinear variation of stress. The results obtained in designing under such an assumption are not as much on the side of safety as those obtained by the straight line formulas. This method of figuring beams is more likely to be found in practice where the allowable unit stress on the concrete as specified by a building code is considered lower than conservative practice requires.

Formulas for ultimate loads are open to the objection that when a factor of safety is applied which will bring the stress in the concrete to about a good working stress, the stress in the steel becomes unduly low from a standpoint of economy. A factor of safety of 3 or 4 as is usually taken leaves a high stress in the concrete with the stress in the steel far below what is

usually considered a safe stress. Beams designed by the ultimate load formulas will generally be of smaller cross-sectional dimensions than when the straight-line formulas are employed, but, on the other hand, a larger amount of steel is required. Experienced designers, however, will arrive at satisfactory results by either the "factor of safety" (referred to ultimate strengths) or the "working stress" methods, but there seems to be no good reason why the simple formulas based on the straight line stress variation should not be used for purposes of design,—safe working stresses being employed.

It may be well to state here, that the straight line theory of stress distribution will be assumed in all the discussions which follow.

Problems

20. Solve Problem 16 for the ultimate resisting moment, assuming a 2000-lb. concrete and assuming the elastic limit of the steel equal to 40,000 lb. per square inch. Consider the given value of E_c to be the initial modulus.

21 Solve Problem 18 by the ultimate load formula, assuming a 2400 lb. concrete and assuming the elastic limit of the steel equal to 35,000 lb. per square inch. Use a factor of safety of 4. Consider the given value of E_c to be the initial modulus.

[In the preceding paragraphs have been shown the two usual methods of calculating the maximum fiber stresses in the concrete and steel of a reinforced concrete beam. The method of procedure is to determine the vertical section of the beam where the moment is a maximum and apply the formulas at that section. The formula for p, containing the values of f_c and f_s, determines the amount of steel reinforcement which is needed to cause the beam to be of equal strength in tension and compression. The formulas for resisting moment determine the bending moment which a beam will safely withstand (for an existing structure) or the size of the beam needed to resist a given bending moment (for a proposed structure).

In *steel* I-beams the above mentioned calculations, which for convenience we shall call "moment calculations," are the only ones needed except in the case of short beams heavily loaded, when the matter of diagonal compression must be investigated. Steel beams are strong in tension but the thinness of the web

makes some investigation necessary in the design of short deep beams to prevent crippling of the web due to column action.

In *reinforced concrete* beams, on the other hand, the concrete is strong in compression but exceedingly weak in tension and usually the diagonal tensile stresses become fully as important as the maximum fiber stresses. An investigation will now be made of the distribution of tension and shear in the concrete to determine what effect such stresses have on the diagonal tensile stresses, and then we can decide on the method of providing for these stresses. The variation of the bond stress between the concrete and steel will also be considered.]

35. Shearing Stresses.—As previously mentioned, the introduction of steel into a concrete beam affects the direction of the diagonal tension lines to a certain extent, by reason of the large shearing stresses which are brought into existence immediately above the steel. First of all, however, let us see the method employed in determining this shear.

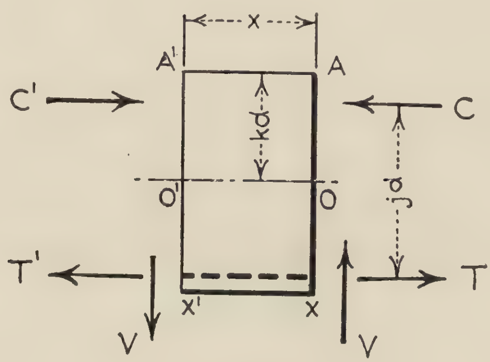


FIG. 30.

In Fig. 30 is represented a short portion of a beam which we will assume so short that no part of the load need be directly considered. The total vertical shear will be denoted by V. The student has already learned that the horizontal and vertical shearing stresses per unit area at any point in a beam are the same. Let v = intensity of either stress at the neutral axis. The tension area of the concrete may be neglected. Then, $C' = T'$ and $C = T$. The total shearing stress on any horizontal plane between the steel and the neutral plane will be equal to

$T' - T$, and, if b is used to denote the breadth of beam, the stress per unit area

$$v = \frac{T' - T}{bx} \tag{1}$$

The forces acting on this portion of the beam must be in equilibrium, hence the couples must balance. Thus,

$$Vx = (T' - T)jd$$

or

$$(T' - T) = \frac{Vx}{jd}$$

Substituting in equation (1), we have

$$v = \frac{V}{bjd} \tag{2}$$

The value of j for working loads varies within narrow limits and v will change but slightly if the different values of j are

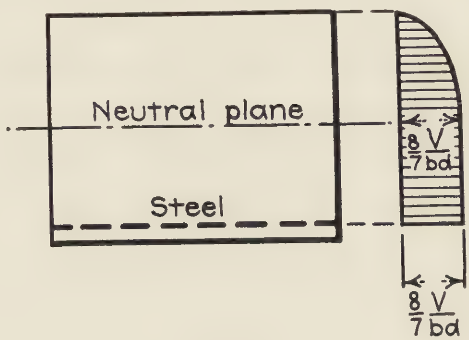


FIG. 31.

inserted in equation (2). The average value of j for beams in ordinary construction is 7/8. Using this value, equation (2) reduces to

$$v = \frac{8}{7} \cdot \frac{V}{bd} \tag{3}$$

It is clear that the shearing stress is the same at all points between the neutral axis and the steel; above the neutral axis the shear follows the parabolic law as in the plain concrete beam. Fig. 31 represents the distribution of shearing stress (both horizontally and vertically) on a vertical cross-section where the bending moment is not a maximum.

It is quite as satisfactory for purposes of comparison to use the average value of the shearing stress, or

$$v_o = \frac{V}{bd} \tag{4}$$

It should be remembered that the *true* maximum intensity of shear will generally be from 10 to 15 per cent higher than the value thus determined.

The longitudinal tension in the concrete near the end of beam modifies the distribution of the shear, increasing the shearing stress somewhat at the neutral axis and decreasing it at the level of the reinforcement. Equation (3), however, gives results which are sufficiently accurate and are derived for beams having the horizontal bars straight throughout. When any web reinforcement is used, the distribution and the amount of the shearing stresses at the end of a simply supported beam are materially different from the foregoing. The analysis of the stresses become more complex and a determination of their value impracticable. Even here, however, the above formula serves a useful purpose. It is found that shear is the chief factor in the failure of a beam by diagonal tension and either formula (3) or formula (4) may be used in design if properly controlled by the results of experiments.

36. Inclined Tensile Stresses.—To determine the approximate amount and direction of the diagonal tensile stresses in the concrete of reinforced concrete beams, the two equations given in Art. 29 will apply. The equations there given are as follows:

$$t=1/2f\pm\sqrt{1/4f^2+v^2} \tag{1}$$

$$\tan 2K=\frac{2v}{f} \tag{2}$$

Although we properly neglected any tension which may exist in the concrete, when making the "moment calculations" and when deriving the formula for horizontal and vertical shear, still when it comes to the consideration of diagonal tensile stresses this tension must be taken into account. As already explained, when the maximum fiber stresses in a beam at the section of maximum bending moment have reached their allowable working values, the cracks near this section, across the bottom of the beam, show that tension in the concrete no longer exists. This is the reason why the concrete is not considered to take any tension in the moment calculations. But it must be understood that cracks due to the same cause do not exist near the ends of the beam where the bending moment is small and where the diagonal tension cracks are likely to occur. Also, maximum vertical and maximum horizontal shear will be found at the ends of a beam where the tension in the concrete exists,

but in the preceding equations for shear we have neglected the tension existing in the concrete, because the effect of such tension on the distribution of the shear is very small and need not be taken into account. When it comes to the matter of diagonal tensile stresses, however, the tension in the concrete must be considered.

As already explained, it is impossible at any point to determine how much tension still remains in the concrete and, because of this, our formulas for direction and magnitude of the diagonal tensile stresses cannot be used to give us accurate results nor can they be used in design. By means of them, however, we can arrive at some important conclusions.

Suppose, for example, at some point between the middle and end of a beam, the stress in the steel is 3000 lb., due to a smaller bending moment than the maximum. Consider $E_c = 2{,}000{,}000$ and $E_s = 30{,}000{,}000$. The steel and concrete at the bottom of the beam will have the same deformation due to plane sections remaining plane. We know, also, by definition that

$$\text{Deformation} = \frac{\text{tensile stress in concrete}}{E_c} = \frac{f_s}{E_s}$$

or tensile stress in concrete $= \dfrac{f_s E_c}{E_s} = 200$ lb. per square inch.

This is about the ultimate tensile strength of a good concrete and so between this point and the end of the beam, tensile stress will exist in the concrete, even along the bottom of the beam. Between this point and the center, tension will also exist in the concrete but the lower limit of the stress will gradually approach the neutral plane until the amount is approximately zero near the section of maximum moment. The resulting diagonal tension at the point of the beam referred to above, assuming a reasonable shearing stress in the lower part of the beam, say 80 lb. per square inch, will be by equation (1)

$$t = 1/2(200) + \sqrt{1/4(200)^2 + 80^2} = 228 \text{ lb. per square inch.}$$

and it will have a direction inclined about 19 1/4 degrees from the horizontal. This stress may exceed the ultimate strength of the concrete and the result will be an inclined crack.

The above discussion shows us that the maximum tensile stresses become considerably inclined immediately above the line of the steel. From equation (2) it is plain that this inclination is greater, the greater the shear, and the less the horizontal

tension. It will, therefore, increase toward the end of the beam. At points nearer the neutral plane, the horizontal tensile stresses become less and the inclined tension approaches the value of the shearing stress, while its inclination approaches 45 degrees. The student through all this discussion should keep in mind that these diagonal tensile stresses can only occur where the concrete still takes its proportion of the tensile stress. Fig. 32 is an attempt to represent roughly the general direction of the inclined tensile stresses in a uniformly loaded beam with horizontal reinforcement.

From equation (1) it is evident that the intensity of the diagonal tensile stress at any point depends upon the shear and horizontal tension in the concrete. The percentage of reinforcement is also a factor to be considered, since a large percentage reduces the

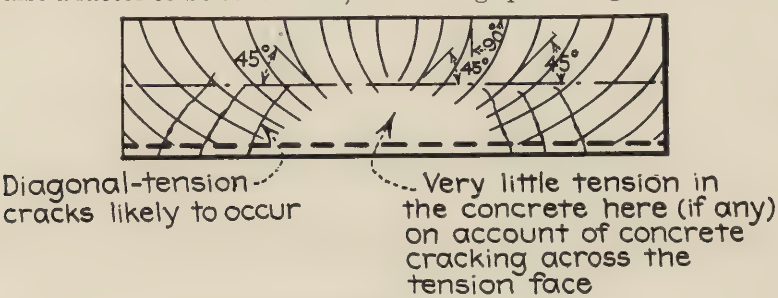


Fig. 32.

horizontal deformation and consequently the tension in the concrete, and tends to strengthen the beam as regards failure in diagonal tension. Remembering that for a given percentage of reinforcement, the horizontal tension in the concrete depends entirely upon the bending moment, we may say that the strength of a beam as regards diagonal tension failure depends upon the relation between shear and bending moment and upon the amount of reinforcement; shear, however, is the chief factor.

From the preceding considerations the student should now see clearly that the character of the loading influences the strength of a beam as regards diagonal tension, the amount of reinforcement remaining the same. For example, Fig. 33 represents the variation in moment and shear in a beam with a concentrated load at the center; Fig. 34 represents the variation of these functions in a beam loaded at the third points; while Fig. 35 shows similar curves for a uniformly loaded beam. In the first

and second cases maximum shear occurs where maximum moment exists, while in the latter case it occurs at the point of zero moment. Conditions are thus seen to be somewhat more favorable in the continuously loaded beam.

The next question to be decided is this: How may we prevent diagonal tensile stresses which are excessive? Obviously, they will be reduced by keeping the horizontal tension small through the use of considerable horizontal steel at points of heavy shear, by avoiding heavy shearing stresses, and by providing some type

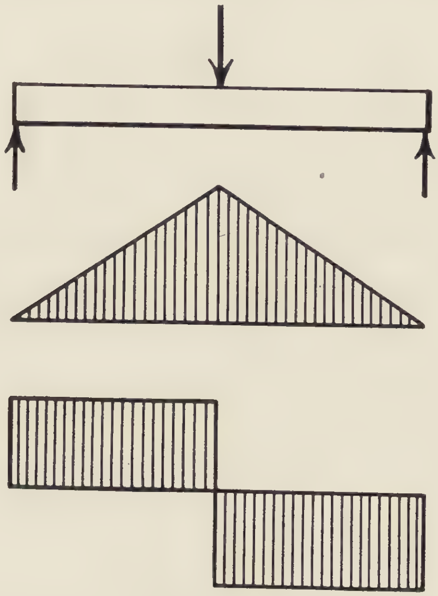


FIG. 33.

of web reinforcement. The different methods of reinforcing for diagonal tension failure will now be taken up. From the preceding discussion it is sufficient to bear in mind that the shear is the principal factor to be considered in the matter of diagonal tension and is a convenient measure of such stress. Also, that the shearing strength of the steel cannot be considered to aid the strength of a beam with respect to diagonal tension.

37. Methods of Web Reinforcement.—There are in use many methods of placing steel in the web of a reinforced concrete beam in order to reinforce it against diagonal tension failure. The various methods may, for convenience, be divided into three

groups: (1) reinforcing metal placed at an inclination; (2) reinforcing metal placed vertically; (3) miscellaneous methods.

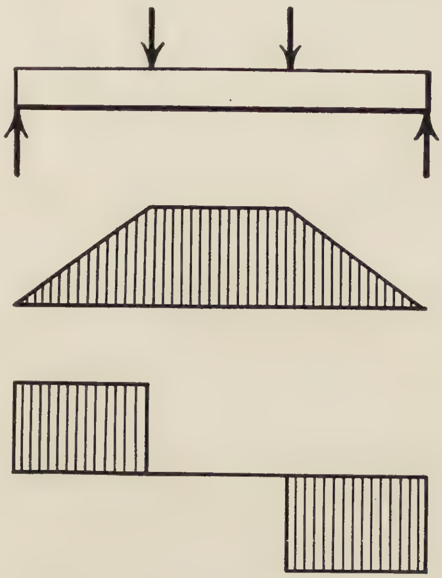


Fig. 34.

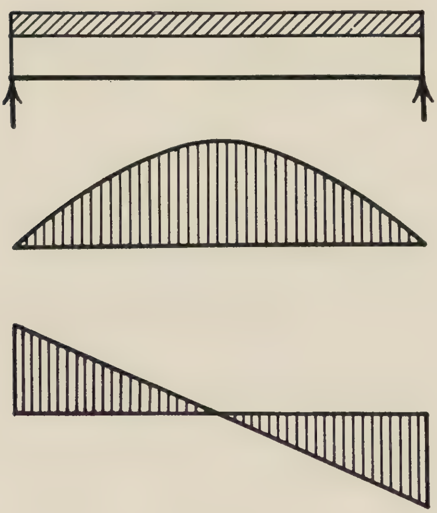


Fig. 35.

Referring to Fig. 32, it is evident that the ideal web reinforcement for uniform loading would be a system of rods arranged

approximately as shown in Fig. 36. In any given case the *exact* arrangement must depend upon the nature of the loading, concentrated loads tending to extend the region of large shear to greater distances from the supports. Some horizontal rods should be carried to the end of the beam in order to keep the tension in the concrete low and reduce the tendency to the formation of inclined cracks. Also, if some of the horizontal rods are bent up to take the place of inclined stirrups, the bends should be made somewhat beyond the theoretical points required for bending moment, so that the actual working stresses in the horizontal steel near the end of the beam will be low.

The method of reinforcement indicated in Fig. 36 cannot, however, be conveniently used in practice. Often the horizontal

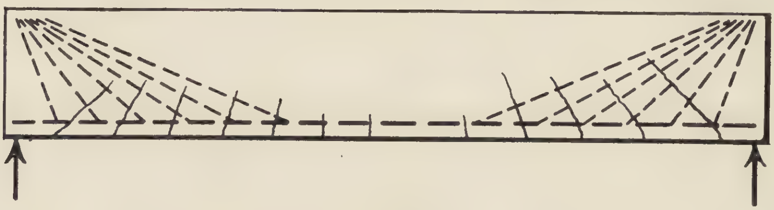
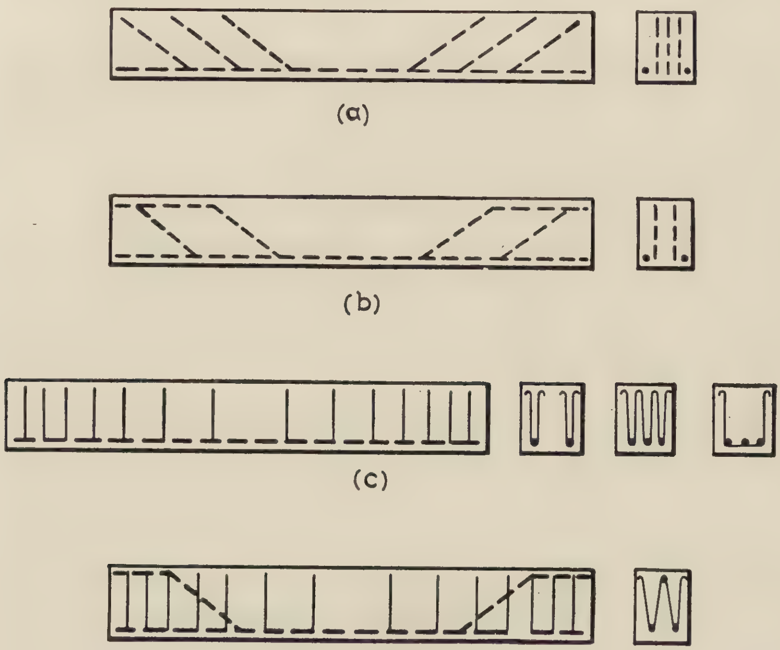


FIG. 36.

rods are too few in number to bend up at the required number of points for thorough web reinforcement, and besides it is not convenient to handle rods with various inclinations at their ends. The common practice is to use bent rods (all bent at the same angle) combined with vertical stirrups. It should be clear that rods bent at a moderate angle are well suited for sections toward the center of beam, and vertical stirrups for sections near the end where the steel must be spaced closer together and at greater inclinations. Sometimes separate inclined reinforcement (called inclined stirrups) is used and in such a case there is danger of its slipping along the horizontal rods if the inclination is too great. If attached to the horizontal rods, however, such reinforcement is very effective. Special forms of bars may be used, such as the well-known Kahn bar, in which strips are sheared from the main bar and bent up. Fig. 37 will give the student an idea of some of the more common forms of beam reinforcement.

A discussion of this kind would not be complete without some comparison being made between vertical stirrups and inclined rods in regard to their effectiveness in preventing inclined tension failure. To assist the student in seeing clearly the relation

between the action of vertical stirrups and inclined rods, Figs. 38 and 39 have been prepared, based on the diagrams of Messrs. Taylor and Thompson.[1] In these figures the beams are considered broken as shown. The load to the left of the break being heavier will tend to drop, and this downward force, combined with the pull in the horizontal steel, may be resisted either by the vertical rod AB, or by the inclined rod CD. The inclined rod, except close to the end of beam, is in a better position to take stress immediately and, of course, must be more effective in preventing initial rupture. Practically, however, there is no



(a)

(b)

(c)

(d)

FIG. 37.

great difference in the effectiveness of the two forms of reinforcement if closely spaced so as to prevent excessive deformation along the lower portion of the beam. The calculation of the necessary web reinforcement will be taken up later after some definite results of experiments have been discussed.

38. Bond Stress.—Slipping of the bars in a reinforced beam may cause failure, but it can readily be prevented by proper construction. The tension in the horizontal steel near the

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 445.

lower surface of a reinforced concrete beam is a maximum near the center of beam and decreases each way toward the end. The difference in the tension between any two points is transmitted to the concrete by the bond between the steel and the concrete. This increment (or decrement) of the tension in the steel is finally transferred to the compression area of the

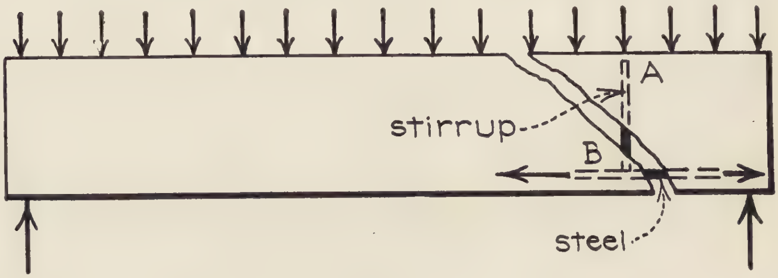


FIG. 38.

concrete, and becomes an increment (or decrement) to the compressive stress in the concrete above the neutral axis. The formula for bond will now be derived for beams in which the reinforcement is horizontal or straight throughout.

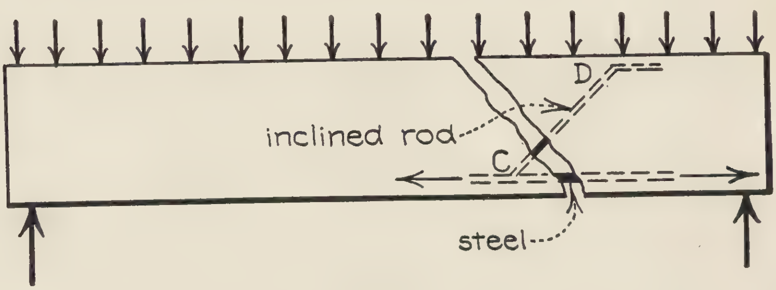


FIG. 39.

The total shearing stress per linear inch between the steel and the concrete, considering a length of beam equal to x, is

$$\frac{T'-T}{x}$$

From Fig. 30

$$Vx=(T'-T)\ jd,$$

or

$$\frac{T'-T}{x}=\frac{V}{jd}\text{(bond stress per linear inch)}$$

and the bond stress per square inch of the surface of the steel

bars is $\frac{V}{jd}$ divided by the sum in inches of the circumference of the bars at the given vertical cross-section. *If* u=unit bond stress, and Σo the total circumference of all bars in a beam at the given section, then

$$u=\frac{V}{\Sigma ojd}$$

The above formula shows that the bond stress is a simple function of the shear and varies with the shear. Thus, shear diagrams may be used to represent the variation of bond stress along a beam. In most cases, plain horizontal rods are sufficient but, under unusual circumstances, deformed bars are employed to obtain a mechanical bond in addition to the strength resulting from adhesion of concrete to steel. When using the above formula, the average value of $j=7/8$ may be taken.

If we consider simply supported beams, experiments show that when stirrups are used the beam is stiffened and the bond stress along the horizontal rods near the end of beam is somewhat reduced. A reason for this may be shown in the fact that, after the concrete begins to crack from diagonal tension (Fig. 38), the stirrups aid in carrying part of the tensile stress which results from the bending moment then existing at the line of the diagonal crack; the stress in the horizontal rods at the end of beam is thus reduced and likewise the liability of failure through bond. A greater reduction of the bond stress has been found to exist when the web reinforcement is provided by means of bent rods. The reduction becomes considerable when about one-half of all the rods are bent up provided, however, that a sufficient number of rods be thus employed. Results seem to indicate that no reduction should be considered in design unless the number of rods bent be greater than two and that the bends be made at at least two points at each end of beam. Tests show that for conditions especially favorable, an average of 50 per cent more bond stress may safely be allowed on the horizontal rods at the end of beam than would be considered safe by the above formula. No allowance should be made when only stirrups are employed for the web reinforcement. The bond stress in continuous beams will receive attention in Art. 63.

The bond strength of vertical (or inclined) stirrups may be insufficient to develop the required strength of the stirrups with respect to tension. This possibility must also be investigated in

the design of beams having web reinforcement in the form of bent rods. It is evident, however, that the above formula for bond stress along horizontal rods can in no way apply to the bond stress along vertical or inclined stirrups. The method of obtaining the necessary diameter of stirrups, and the spacing of stirrups and bent rods will be explained later.

39. Tests.—It would be poor judgment to base design of reinforced concrete beams on theory alone. Seeing is believing and if tests could not be reconciled with theory, there are few engineers who would have much dependence upon theoretical formulas. The reason for making tests, in the first place, is to see if the theoretical formulas deduced from the principles of mechanics give results closely agreeing with the actual conditions in practice.

A reinforced concrete beam will usually fail in one of three ways:

(a) By the yielding of the steel at, or near, the section of maximum bending moment.

(b) By the crushing of the concrete at the same place.

(c) By a diagonal-tension failure of the concrete at a place where the shear is large.

Methods (a) and (b) are called *moment* failures. Method (c) is sometimes improperly called a *shear* failure. To be sure, shear is the chief factor in the cause of such a failure, but the true reason for the destruction of the beam in such a case is the cracking of the concrete due to inclined tensile stresses.

We have already spoken of the fact that the bond stress between concrete and steel may be exceeded and that the beam may fail by the slipping of the bars. This, however, is not possible if the beam is designed properly and if a fairly wet mixture of concrete is employed and carefully placed around the steel. In an important series of tests on beams by Bach in which the primary cause of failure was the slipping of the bars, the bond stress along the horizontal bars at the end of beam was calculated by the preceding formula. The average maximum result for the rectangular beams with straight rods only was 291 lb. per square inch, and with stirrups 330 lb. per square inch. With a number of bent rods, the average value was 493 lb. per square inch.

Failure by the shearing of the concrete near the support is possible where the load is very close thereto, but as the shearing strength of concrete is at least one-half the crushing strength,

such failures are exceedingly unlikely and need rarely be considered. The usual so-called shear failures are in reality diagonal tension failures.

When a beam is to be tested to destruction in order to investigate any particular type of failure, the steel and concrete used are tested separately. In this way, the properties of the materials are known when separately subjected to stress.

When a beam begins to fail by the yielding of the steel, any further load rapidly increases the deformation, large cracks open up in the concrete on the tension side, the neutral axis rises on this account, and the ultimate failure soon occurs by the crushing of the concrete. A steel tension failure is found to occur when the amount of steel used is less than the amount determined by theoretical formulas which makes the beam of equal strength in tension and compression. This result agrees, then, with what is expected. Likewise it is found that with a larger amount of steel than is theoretically required, the yield point of steel is not reached and the beam fails directly by crushing of the concrete. Beams with no web reinforcement and with the existence of large shearing and moment stresses, fail by inclined cracks opening up in the concrete, thus substantiating to a considerable degree the theoretical deductions regarding the internal stresses in beams.

The results of breaking tests on reinforced beams with different percentages of steel reinforcement compare well with the results derived from theoretical formulas. Considering the nature of the material, the calculations by the two assumptions of stress variation are found to agree sufficiently with the experimental results to justify their use in problems of design.

Another method of testing reinforced concrete beams is by the use of extensometers to measure distortions, so that the deformation of the steel and of the extreme fiber of the concrete may be calculated and the neutral axis determined. In making beam tests it is customary to place equal loads at points dividing the length into three equal parts. (See illustration of beam marked *C*1.) The advantage of this arrangement lies in the fact that the bending moment is practically uniform between the loads and, if measuring devices are attached, the deformations of the fibers at the top and at the bottom may be easily determined. If in Fig. 40 the deformations be aa' and bb', the neutral axis O is located by connecting a' and b' with a

straight line, intersecting ab at O. In our moment calculations, the position of the neutral axis was found to be of prime importance and we can readily see that, once this is known, the actual strength may be determined with little uncertainty. In determining the position of the neutral axis, the formula for k shows it to depend only upon the amount of steel used and upon the ratio $\frac{E_s}{E_c}$ or n. The only element of uncertainty is the value of E_c. It might be well to take the value as determined by the ordinary compression test for use in our theoretical formulas, but closer results can be obtained from these formulas if the value of n is taken so that for average conditions the neutral axis is in as nearly as possible the same position theoretically and experimentally. The reason that E_c, as thus determined, will give better results at working loads, is due to the effect of the remaining tension in the concrete below the neutral

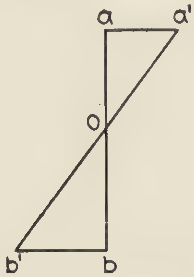


Fig. 40.

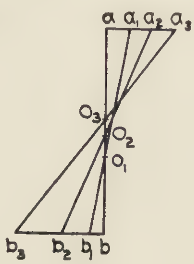


FIG. 41.

axis—a stress which is properly not allowed for in the resisting moment. From the results of experiments by extensometers for determining the position of the neutral axis, it appears that a value of 15 for n is not too large for calculations of strength of beams under the usual assumptions, although great accuracy in this respect is found unnecessary.

In making the experiments above described, it was observed that the neutral axis raised as the loading increased, k being approximately 3/8 at working loads. It was also noted that for the lower loads the neutral axis as determined from the theoretical formulas is more uncertain and generally lower in the beam than for the higher loads. This is undoubtedly due to the relatively large influence of the tensile strength of the concrete in such cases. This rise of the neutral axis as the load increases is shown in Fig. 41. Consider a_1b_1 to be the plane ab after a bending

takes place just sufficient to bring the maximum tensile stress in the concrete to its ultimate value. When loads are applied which cause a greater bending moment, the concrete in tension becomes broken by fine cracks, and the steel takes a greater part of the tensile stress. The elongation at b now increases faster than at a, and the neutral axis rises rapidly. When working loads are reached, the position of the neutral axis moves but little, and the steel takes all the tension.

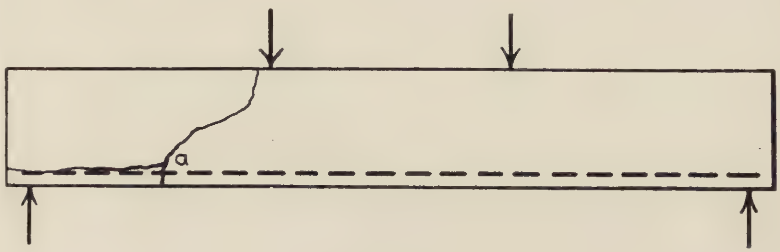


Fig. 42.

Fig. 42 illustrates a typical diagonal tension failure where only horizontal bars are used. The initial crack forms at a and branches in a diagonal direction, running in a direction away from the support. A little later the concrete begins to fail in a horizontal tension crack just above the rods, progressing from a toward the end of the beam. This horizontal crack is brought about by the new conditions which exist after the concrete has become cracked along the diagonal line and the normal diagonal tension has thus ceased to act. Sometimes this horizontal crack does not extend to the end of the beam—the final failure occurring either by the diagonal crack extending to the top of the beam or the horizontal rods pulling out. Thus final failure often occurs from stresses which are developed after initial failure has occurred. However, the initial failure and its cause is what is of importance in design.

In regard to tests of beams having web reinforcement, it is found that vertical stirrups spaced a distance apart equal to, or greater than, the depth of the beam, will give little aid in the prevention of diagonal cracks between successive stirrups, although they may prevent final failure by preventing the extension of a crack horizontally along the reinforcing rods.

Using stirrups with part of the horizontal rods bent up, the diagonal tension cracks appear with an inclination about the

same as those which cause diagonal tension failure in a beam without web reinforcement. The beam, however, is found to withstand a great deal more shear, confirming the theoretical deductions.

Tests by Talbot in which curved and inclined rods were used, but in which no rods continued straight for the entire length of the beam, showed results very little better than for straight rods.

Vertical stirrups and bent rods combined are found by tests to give the very best results. Tests also seem to indicate that too much reliance should not be placed upon one or two bent rods. For this reason, even if one or two rods are bent up properly to take the diagonal tension, it would be good design to consider this rod as not taking any diagonal tensile stress and to provide a thorough web reinforcement by means of stirrups.

Fig. 43 represents the conditions which developed in the test of a beam. The cracks are numbered in the order of their

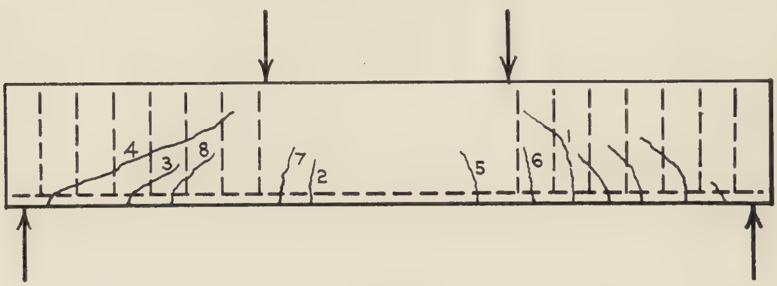


Fig. 43.

appearance, final failure occurring at crack No. 4 and being due to inadequate web reinforcement. The stirrups were stressed beyond their yield point.

It appears from tests of beams in which bent rods were employed with a good anchorage at their ends, that considerable arch action is developed, and that the anchorage is quite advantageous in increasing web resistance. This form of construction is also found to be an insurance against failure at low loads through defective concrete or insufficient bond.

The results of experiments show that the ultimate compressive strength of concrete in a beam is at least equal to its crushing strength as determined by tests on cubes hardened under similar conditions; also, that the yield-point of the steel should be regarded as ultimate strength as far as reinforced beams are concerned. When the steel reaches its yield point, the beam de-

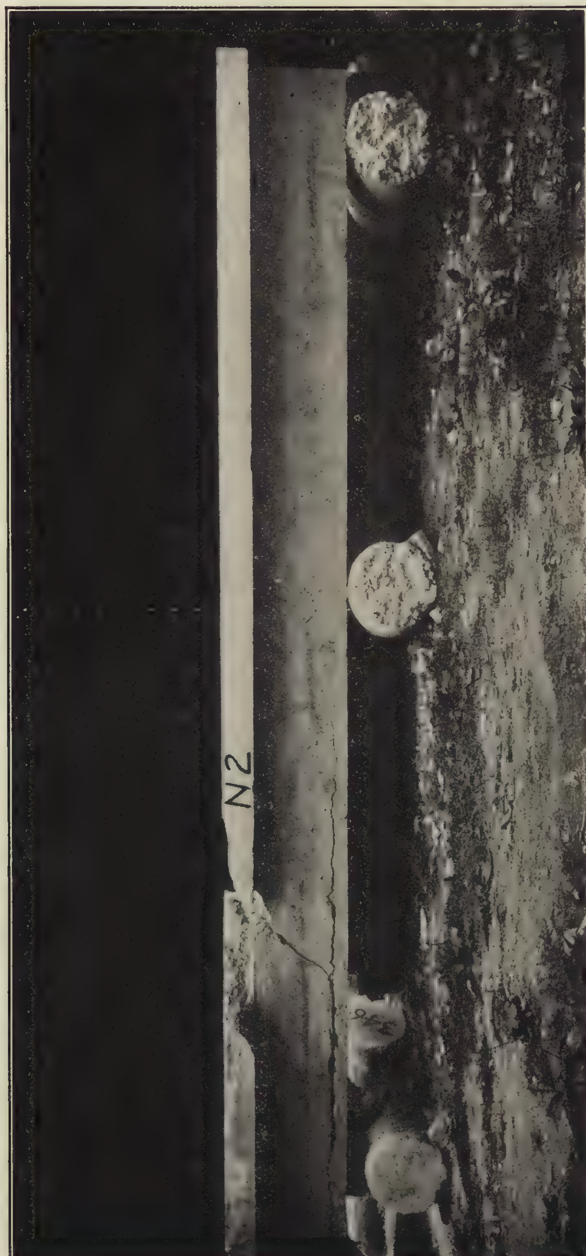
flects, and failure soon occurs by the crushing of the concrete.

The ultimate shearing strength of a beam having no web reinforcement may be taken at 100 to 150 lb. per square inch, for a 1:2:4 concrete, calculated as average shearing stress on the cross-section. The stresses here considered relate to shearing stresses involving large diagonal tensile stresses. Where such tensile stresses are not developed to any extent, as in *punching* shear, a value of at least one-half the compressive strength (as previously mentioned) may be employed. However, it is almost impossible in practice to avoid altogether such tensile stresses, and it is not advisable to raise the first value given any considerable amount. The student must continually bear in mind that the kind of failure denoted as a *shear* failure is so called for convenience; they are diagonal tension failures brought about by large shearing stresses and hence may be measured by the shearing forces present. The average shearing stress on a vertical section at failure is the value always given. While the maximum shearing stress is 12 to 15 per cent greater than this, the average stress is practically as good a standard of measure and is much more readily calculated.

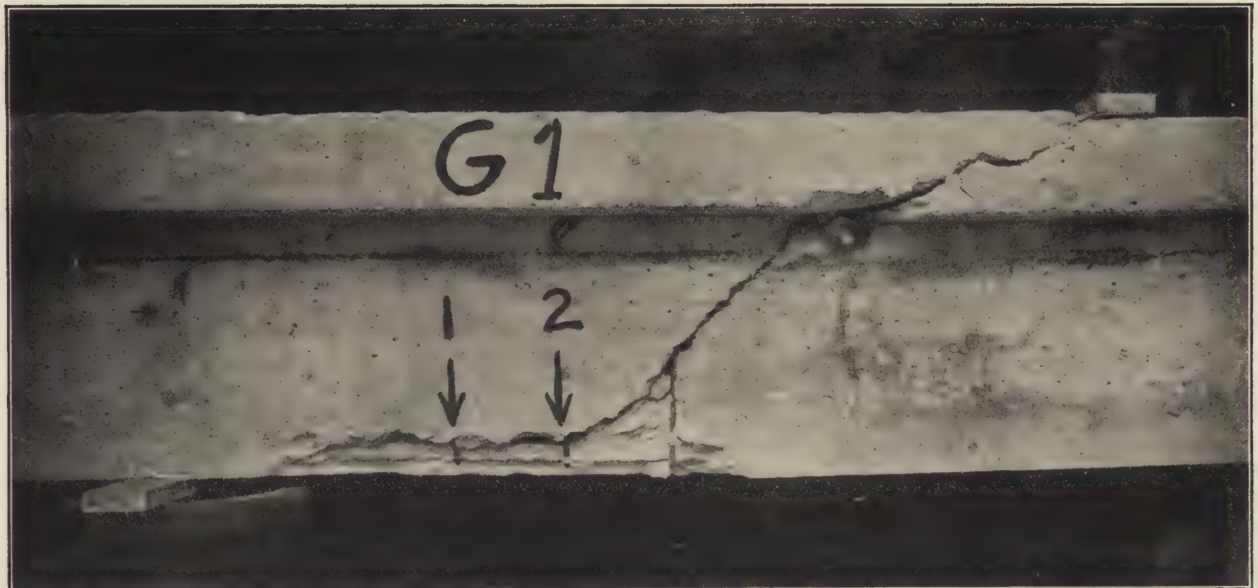
The following is taken from "Principles of Reinforced Concrete Construction" by Turneaure and Maurer: "As already stated the amount of horizontal steel has a direct bearing on shear failures for the reason that large areas of steel with low unit stresses permit less extension of the concrete than small areas with high working stresses. This effect is shown in a marked manner in a series of tests made at The University of Wisconsin on small mortar beams of 1:3 mixture. The beams were 3 in.$\times$4 1/2 in. in cross-section and 4 ft. span length. Loads were applied at two points a varying distance apart. Only straight reinforcement was used, amounting to 1.41 per cent. The tensile strength of the material was high, being 490 lb. per square inch. The results were as follows:

Distance apart of loads Inches	Average shearing stress Lb. per sq. in.
Center load	177
12	200
24	220
32	316
36	512
40	850
44	1035

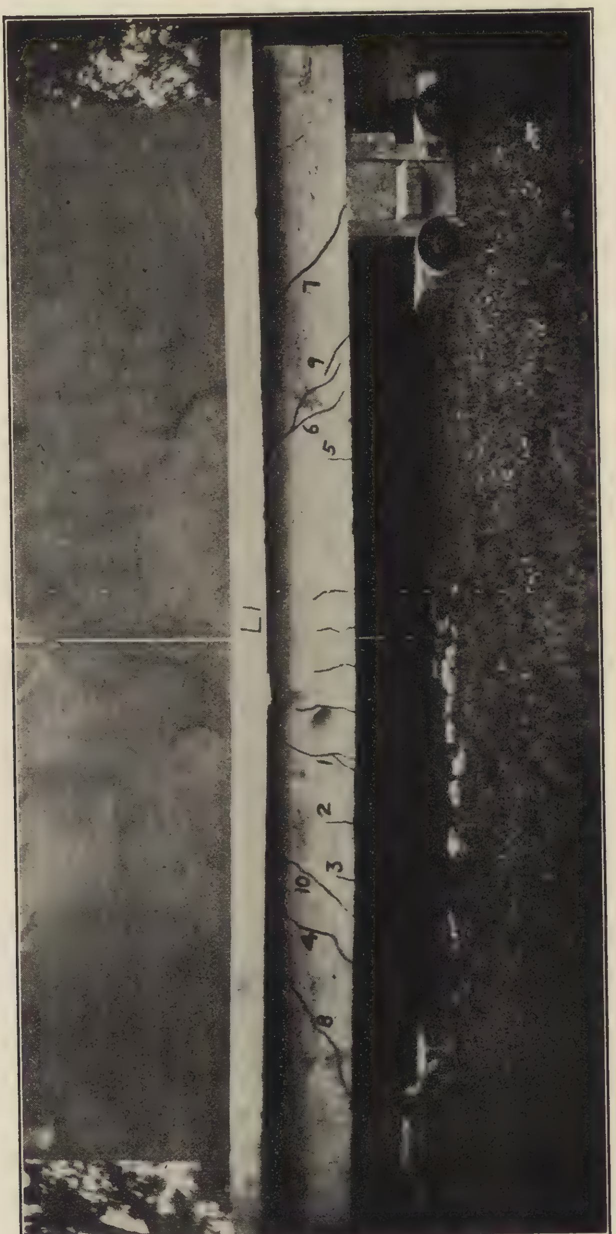
The beams and columns illustrated in this text were all tested in the laboratory for testing materials at the University of Wisconsin. The T-shaped beam C1 shown above had thorough web reinforcement, including both stirrups and inclined rods, so that failure came through the weakening of the tension side of the beam when the horizontal steel passed the yield point. The increased stretch in the steel caused a large amount of compressive stress to be thrown upon a small area at the top and this part of the beam finally crushed. Upon removing the concrete from around the rods at the point of failure (marked 2), the metal was found to have scaled, showing conclusively that it had passed the yield point. The failure of the beam developed slowly.



The beam N2 illustrates the typical diagonal tension failure where only horizontal bars are used.



The beam G1 shown was reinforced by 1/4-in. vertical stirrups and failed through diagonal tension owing to insufficient web reinforcement. The two stirrups near the failure crack were broken, indicating that the increased stress brought upon them by the failure of the concrete loaded them beyond their capacity. The beam failed suddenly, accompanied by a sharp snap, due to the breaking of the stirrups. The broken stirrups may be seen just below the arrow tips.



The beam L1 was so well designed as to the strength of its component parts that it had about equal strength in tension, compression, and shear.

The increase in strength as the loads approach the supports must be due largely to the decrease in moment stress and consequent distortion which is essentially what occurs when large areas of steel and low working stresses are used."

Tests on beams with web reinforcement show that the ultimate average shearing strength may reach 300 to 400 lb. per square inch. The latter figure may, from our present knowledge, be taken as about the maximum value with ordinary, closely spaced, web reinforcement.

Diagonal tension failure in a beam occurs suddenly. It is the failure to be most feared and therefore should be most carefully guarded against.

40. Working Stresses.—The following working stresses were recommended by the Joint Committee on Concrete and Reinforced Concrete in its first Progress Report presented early in 1909. This Joint Committee is composed of members selected from the American Society of Civil Engineers, the American Society for Testing Materials, the American Railway Engineering and Maintenance of Way Association, and the Association of American Portland Cement Manufacturers, and therefore represents the highest authority in the United States. The report on working stresses will be given in full.[1] The student need not concern himself with the stresses in columns for the present.

> "*General Assumptions.*—The following working stresses are recommended for static loads. Proper allowances for vibration and impact are to be added to live loads where necessary to produce an equivalent static load before applying the unit stresses in proportioning parts.
>
> "In selecting the permissible working stress to be allowed on concrete, we should be guided by the working stresses usually allowed for other materials of construction, so that all structures of the same class but composed of different materials may have approximately the same degree of safety.
>
> "The stresses for concrete are proposed for concrete composed of one part Portland cement and six parts aggregate, capable of developing an average compressive strength of 2000 lb. per square inch at 28 days when tested in cylinders

[1] The form given corresponds essentially with the 1909 Report of the Reinforced Concrete Committee of the American Society of Civil Engineers.

8 in. in diameter and 16 in. long, under laboratory conditions of manufacture and storage, using the same consistency as is used in the field. In considering the factors recommended with relation to this strength, it is to be borne in mind that the strength at 28 days is by no means the ultimate which will be developed at a longer period, and therefore, they do not correspond with the real factors of safety. On concrete in which the material of the aggregate is inferior, all stresses should be proportionally reduced, and similar reductions should be made when leaner mixtures are to be employed. On the other hand, if, with the best quality of aggregates, the richness is increased, an increase may be made in all working stresses proportional to the increase in compressive strength at 28 days, but this increase shall not exceed 25 per cent.

"*Bearing.*[1]—When compression is applied to a surface of concrete larger than the loaded area, a stress of 32.5 per cent of the compressive strength at 28 days, or 650 lb. per square inch on the above-described concrete may be allowed. This pressure is probably unnecessarily low when the ratio of the stressed area to the whole area of the concrete is much below unity, but is recommended for general use rather than a variable unit based upon this ratio.

"*Axial Compression.*—(a) For concentric compression on a plain concrete column or pier, the length of which does not exceed 12 diameters, 22.5 per cent of the compressive strength at 28 days, or 450 lb. per square inch on 2000 lb. concrete, may be allowed.

"(b) Columns with longitudinal reinforcement only, the same unit stress as recommended for (a). (It is recommended that the ratio of the unsupported length of a column to its least width should be limited to 15 in any type of reinforced concrete column.)

"(c) Columns with reinforcement of bands or hoops,[2] stresses 20 per cent higher than given for (b), or 540 lb. per square inch on 2000 lb. concrete.

"(d) Columns reinforced with not less than 1 per cent

[1] For beams built into pockets in concrete walls, the lower compressive stress of 450 lb. per square inch should not be exceeded.

[2] Where bands or hoops are used, the total amount of such reinforcement shall not be less than 1 per cent of the volume of the column enclosed. The clear spacing of such bands or hoops shall not be greater than one-fourth the diameter of the enclosed column.

and not more than 4 per cent of longitudinal bars and with bands or hoops, stresses 45 per cent higher than given for (b), or 650 lb. per square inch on 2000 lb. concrete.

"(e) Columns reinforced with structural steel column units which thoroughly encase the concrete core, stresses 45 per cent higher than given for (b), or 650 lb. per square inch on 2000 lb. concrete.

"*Compression on Extreme Fiber.*—The extreme fiber stress of a beam, calculated on the assumption of a constant modulus of elasticity for concrete under working stresses, may be allowed to reach 32.5 per cent of the compressive strength at 28 days, or 650 lb. per square inch for 2000 lb. concrete. Adjacent to the support of continuous beams, stresses 15 per cent higher may be used.

"*Shear and Diagonal Tension.*—Where pure shearing stress occurs, that is, uncombined with compression normal to the shearing surface, and with all tension normal to the shearing plane provided for by reinforcement, a shearing stress of 6 per cent of the compressive strength at 28 days, or 120 lb. per square inch on 2000 lb. concrete, may be allowed. Where the shear is combined with an equal compression, as on a section of a column at 45 degrees with the axis, the stress may equal one-half the compressive stress allowed. For ratios of compressive stress to shear intermediate between 0 and 1, proportionate shearing stresses shall be used.

"In calculations on beams in which diagonal tension is considered to be taken by the concrete, the vertical shearing stresses should not exceed 2 per cent of the compressive strength at 28 days, or 40 lb. per square inch for 2000 lb. concrete.

"*Bond.*—The bonding stress between concrete and plain reinforcing bars may be assumed at 4 per cent of the compressive strength at 28 days, or 80 lb. per square inch for 2000 lb. concrete; in the case of drawn wire, 2 per cent or 40 lb. on 2000 lb. concrete.

"*Reinforcement.*—The tensile stress in steel should not exceed 16,000 lb. per square inch. The compressive stress in reinforcing steel should not exceed 16,000 lb. per square inch, or 15 times the working compressive stress in the concrete.

"*Modulus of Elasticity.*—The value of the modulus of

elasticity of concrete has a wide range, depending upon the materials used, the age, the range of stresses between which it is considered, as well as other conditions. It is recommended that in all computations it be assumed as one-fifteenth that of steel, as, while not rigorously accurate, this assumption will give safe results."

The yield-point of ordinary mild steel purchased in the open market cannot safely be fixed at a higher value than from 30,000 to 32,000 lb. per square inch, although frequently, and in fact in the majority of cases, a value of at least 35,000 to 40,000 lb. per square inch will be found. Since the yield-point of steel is regarded as its ultimate strength when used in reinforced concrete beams, the maximum working stress of 16,000 lb. per square inch recommended by the Joint Committee, gives, it would seem, a factor of safety of at least about 2 with respect to a steel-tension failure. In well-designed beams, however, where the concrete used is of high grade, the steel stress at failure will considerably exceed its elastic limit, the high crushing strength of the concrete enabling the steel to elongate very considerably before final failure occurs through the crushing of the concrete. For the above reason, the working stress of 16,000 will be found to give a much greater factor of safety than 2—generally somewhere between 2 1/2 and 3.

The elastic limit of good concrete is between one-half and two-thirds its ultimate strength. Considering 2000 lb. per square inch as the ultimate strength in concrete, the elastic limit will be about 1000 to 1300 lb. per square inch. The working stress of 650 lb. per square inch gives the beam, then, a factor of safety as regards elastic limit of concrete of about 2, and as regards the ultimate strength of concrete, from 4 to 5 (this value is greater than $\frac{2000}{650}$, since above the elastic limit the stress in concrete does not vary even approximately in the same ratio as the increase in the loading).

Thus, the elastic limit of a reinforced concrete beam is determined by the concrete (minimum factor of safety about 2) and its point of failure by the steel (minimum factor of safety between 2 1/2 and 3), which may be regarded as satisfactory conditions. The greater uniformity and reliability of the steel, as compared to the concrete, should be noted in this connection.

It must be plain to the student that with the values recommended by the Joint Committee for the compression in concrete and the tension in steel, that a beam will reach its elastic limit (determined by the concrete) before it fails (determined by the steel). Of course, the preceding statement applies only when the beam is adequately reinforced against diagonal tension and bond stress. In choosing the above factors, the margin of safety between the elastic limit and ultimate strength has received consideration.

The student should note that a working tensile stress of 16,000 lb. per square inch is recommended by the Joint Committee for all grades of steel. High working stresses in the steel involve large distortions in the concrete, not only at the center of beam but also diagonally near the end of beam. Low unit stresses in the steel are greatly to be preferred on this account. It can also be shown that very little is to be gained in economy by using high stresses such as is often done with high elastic limit steel.

In determining the relative working stresses in steel and concrete some attention should be given to the question of repeated loads. If the live load is a large percentage of the total load and subject to frequent repetitions, a relatively low working stress in the concrete may well be employed in order to maintain elastic conditions. As regards the steel, more perfect elasticity exists up to a definite point, and hence repetition of load need not be considered in the selection of its working stresses. Low unit stresses in the steel are to be preferred, in order to prevent excessive deformations and reduce the liability of diagonal tension cracks. The working stress of 16,000 should be the maximum for medium steel.

The "General Assumptions" of the Joint Committee's report should be studied until the student feels that he can use his judgment correctly in the selection of working stresses for any given concrete and steel. The table of crushing strengths of concrete given in Art. 10 should be employed in this connection. A 1:2:4 concrete is considered to have an average ultimate strength of 2000 lb. per square inch, and the strengths of other concretes should be proportional to this. If high carbon steel is used a stress of 20,000 lb. is frequently permitted.

Age of concrete is another point frequently overlooked when choosing working stresses. The stresses recommended by the Joint Committee refer to the strength at the end of one month,

so that the factors of safety already mentioned with regard to concrete are by no means the ultimate.

The standard compression specimen adopted by the Joint Committee is a cylinder 8 in. in diameter by 16 in. long. Most of the important tests of concrete up to the present time have been made on either 6 by 6 in. or 12 by 12 in. cubes. It is often important to compare the strength of cylinders and cubes and this can be done by means of the formula given in Art. 10.

Problems

22. A simply-supported reinforced concrete beam having $b=8$ in. and $d=12$ in. has a span of 15 ft. and sustains a uniform load (live plus dead) of 350 lb. per foot. The beam contains four 1/2-in. square rods straight throughout and properly spaced. What is the maximum bond stress on these rods in pounds per square inch?
23. Determine whether or not the beam of Problem 22 is of equal strength in tension and compression, assuming the allowable working stresses to be $f_s=16{,}000$ lb. per square inch and $f_c=650$ lb. per square inch. Take $n=15$.
24. (a) What would be the maximum bond stress in Problem 22 if two 3/4-in. square rods were employed? (b) If one 1-in. square rod was used? (c) What fact is brought out by (a) and (b)?
25. What is the maximum shearing stress in the beam of Problem 22? Where does this maximum occur?
26. Is web reinforcement needed in the beam of Problem 22 according to the recommendations of the Joint Committee?
27. If two of the rods in Problem 22 were bent up near the end of beam, approximately what would be the maximum bond stress along the two remaining rods?
28. If you were designing a beam under the conditions of Problem 22 and the depth (d) was limited to 12 in., what breadth of beam and amount of steel would you consider satisfactory?
29. A reinforced concrete structure is subjected to a constant repetition of a live load which is large in comparison with the dead load. Explain what you know pertaining to the proper working stresses for the same.
30. A 1 : 2 1/2 : 5 concrete (see table on page 17) is to be used under average conditions in a reinforced concrete beam. Medium steel. Determine the working stresses to use.
31. A 12-in. by 12-in. cube of a concrete, which is to be used in a reinforced beam, has an ultimate strength of 3000 lb. per square inch. High carbon steel. Average conditions. What working stresses would you employ in design?
32. A simply-supported beam of 10-ft. span and 30-in. depth is loaded with a heavy concentrated load at a point one-fourth the span length from the left support. Considering horizontal steel, show the general directions of the diagonal tensile stresses throughout the beam.

41. Vertical and Inclined Reinforcement.—Thus far in the course the student has been brought to realize the effectiveness of inclined and vertical bars in preventing diagonal tensile cracks in reinforced concrete beams. This type of failure we have seen is the one to be most feared. The inclined reinforcement may be separate members firmly connected with the horizontal reinforcement to prevent slipping, or some of the horizontal bars may be bent up near the ends of the beam where they are not needed to resist bending. The vertical reinforcement may be used separately or in combination with inclined reinforcement, depending upon the preference of the designer and upon the amount of diagonal tension to be provided for. When vertical stirrups are used, they should be looped around the horizontal bars so as to be firmly anchored at their lower ends where the stress is a maximum. The value of a stirrup unless looped or hooked at the top is limited by its strength of bond and, as its length is not great, this point may need consideration.

The proportioning of web reinforcement cannot be done with any degree of exactness since very little experimental work has been performed along this line. However, rough determinations of what is required may be obtained on rational grounds. The only information given us by tests, is the value of the shearing stress which measures diagonal tension failure—(1) for beams with horizontal bars only, and (2) for beams having an effective system of web reinforcement. Also, tests on beams, with and without web reinforcement, show that when reinforcement is provided for diagonal tension, the concrete may be assumed to carry its full value of the shear and the steel the remainder. This action of the concrete and web reinforcement is found to exist at ultimate loads—namely, when the beam has failed by the slipping of the bars. Since this is found true at ultimate loads, it undoubtedly would be even more certain at working loads when the concrete at the most is only slightly cracked.

The Joint Committee limits the shearing unit stress in a beam with an effective web reinforcement to 120 lb. per square inch for a 2000 lb. concrete—namely, three times the allowable shearing stress in a 2000 lb. concrete without reinforcement. In the discussion which follows, we shall assume the concrete to take one-third of the total shear and the stirrups the remaining two-thirds. The point in the beam beyond which web reinforcement is unnecessary will, however, be determined by the maximum allowable shear in the concrete.

When the unit shear, which equals $\frac{V}{bjd}$, is found to only slightly exceed 40 lb. per square inch, it would seem that the beam designed according to the above assumption would be unnecessarily strong as regards web reinforcement. This is true, but on account of the small amount of reinforcement which is usually needed for such cases and the importance of such reinforcement in reinforced concrete beams, the value $\frac{2}{3}$ is proposed for all ordinary cases. The student should always remember this element of safety when using the following formulas in design. If desired, the coefficient $\frac{2}{3}$ in the formulas may be changed as required by the problem under consideration.

Using Taylor and Thompson's method of analysis,[1] let

V = total shear.
s = horizontal spacing of stirrups.
a_s = area of section of stirrup. (In a U-shaped stirrup, a_s is the sum of the areas of the two legs.)
f_s = unit stress in steel.

The maximum intensity of vertical shear v at a vertical section AA, Fig. 44, is $\frac{V_1}{bjd}$. The maximum intensity of horizontal shear

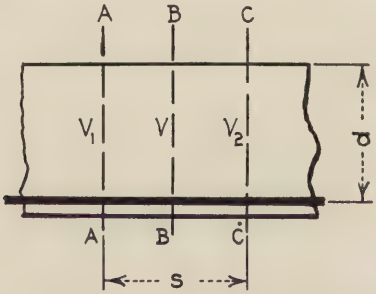


Fig. 44.

at a section AA is equal to the maximum intensity of vertical shear v at the same section. Multiplying this shear by b, the breadth of the beam, gives the maximum horizontal shear per unit of length of the beam $= \frac{V_1}{jd}$. Consider now a horizontal plane of length s at, or below, the neutral plane. The intensity

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 448.

of the horizontal unit shear on this plane varies proportionally with the total shear, and is equal to $\frac{V_1}{jd}$ at section AA and to $\frac{V_2}{jd}$ at section CC. Hence, the total amount of shear on this plane, considering uniform variation of shear, is equal to $\frac{V_1+V_2}{2jd}(s)$, or, when V at BB is the average shear, to $\frac{V}{jd}s$. This is the vertical component of the diagonal tensile stress in a length of beam s.

When a stirrup is placed at B, Fig. 44, and the distance between stirrups is s, then the vertical component of the diagonal tension to be taken by this stirrup is $\frac{2}{3}\cdot\frac{Vs}{jd}$. This stress causes tension in the stirrup, the corresponding horizontal component of the diagonal tension being carried by the horizontal rods.

The stress in a vertical stirrup is $a_s f_s$, hence the cross-sectional area of vertical steel required in a length of s may be taken approximately as

$$a_s=\frac{2}{3}\cdot\frac{Vs}{f_s jd} \quad \text{(Vertical stirrups)} \qquad (1)$$

For stirrups or bars inclined at 45 degrees, the lines on a beam representing the direction in which the diagonal tensile cracks are likely to occur, are crossed more times per unit of length for a given horizontal spacing than would be the case if vertical stirrups were employed; that is, a given amount of inclined steel is much more effective in taking diagonal tension than the same amount of vertical steel. It may be assumed, then, that the stress in the inclined bars is approximately $\frac{a_s f_s}{\sin 45°}$ and the required area of steel, assuming the steel to take two-thirds of the shear, is

$$a_s=\frac{2}{3}\cdot\frac{0.7(Vs)}{f_s jd} \quad \text{(Inclined bars 45 degree angle)} \qquad (2)$$

For other angles of inclination K, it may be assumed as approximately correct to use the formula

$$a_s=\frac{2}{3}\cdot\frac{\sin K\ (Vs)}{f_s jd} \quad \text{(Inclined bars any angle)} \qquad (3)$$

The recommendations of the Joint Committee for the allowable shearing stresses in beams, both with and without effective web

reinforcement, apply to the maximum unit shear at any point of a vertical section, which we have previously found to be $\frac{V}{bjd}$. This maximum extends from the horizontal steel up to the neutral axis. The stress in the stirrups will consequently be the greatest along a plane located between the neutral plane and the horizontal rods.

Assuming that the distance between centers of compression and tension (jd) is approximately $7/8d$, it is clear that stirrups are required with horizontal bars, only when $\frac{V}{bd}$ is greater than $7/8(40) = 35$ lb. per square inch. Also, $\frac{V}{bd}$ should not be greater than $7/8(120) = 105$ lb. per square inch with the most effective web reinforcement. $\frac{V}{bd}$ has been previously denoted by v_o, and we shall represent it in this manner hereafter.

The manner of providing for diagonal tension in beams cannot be impressed too strongly upon the student. First of all, notice the most unfavorable part of the given beam as regards diagonal tensile stresses. This will be at points of excessive shear combined with considerable bending moment. Also, be careful to extend sufficient horizontal steel to the ends of the beam to provide for bending with low unit stresses in the steel. A great many times it will be impracticable to provide as much reinforcement as desired by means of bent-up rods, and some vertical stirrups will be needed. In small beams, vertical stirrups only are often employed throughout, but in large beams under heavy shearing stresses, both should be used. In detail, stirrups may be made in various forms, as indicated in Fig. 45. Special attention is called to the many excellent features of the continuous stirrup. Woven wire bent around the rods is a satisfactory and very effective reinforcement. The Joint Committee recommends that the longitudinal spacing of stirrups or bent rods should not exceed three-fourths the depth of the beam.

42. Vertical Stirrups.—We have seen that the required area of cross-section of a vertical stirrup may be determined by the formula

$$a_s = \frac{2}{3} \cdot \frac{Vs}{f_s jd}$$

With a given stirrup, this formula may be solved to give the spacing required, or

$$s=\frac{3}{2}\cdot\frac{a_s f_s jd}{V}$$

The value of V should be taken at the section where the spacing is desired.

Tests have shown that little or no value is derived from stirrups spaced a distance apart about equal to d. Thus, the spacing of

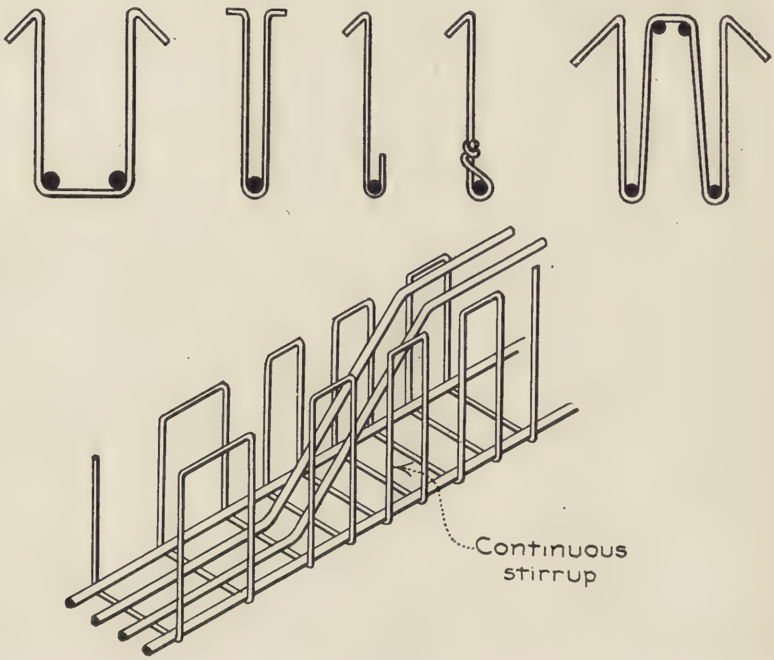


FIG. 45.

stirrups must not exceed the depth of the beam, and, as we have seen, the practical limit suggested by the Joint Committee is three-fourths the depth. Many constructors advise the insertion of occasional stirrups throughout the entire length of the beam, especially T-shaped beams, even if they are not theoretically necessary.

Where only a small amount of web reinforcement is needed, stirrups are occasionally spaced uniformly for convenience. When this is done, only the minimum value of s needs to be figured, and this can be obtained by substituting for V in the above equation, the total shear at the end of beam.

The distance from the support to the point where no stirrups are required is helpful in design. This distance for uniform loading may easily be expressed by means of a formula.

Let v = unit shear.
v' = unit working shear. (V' = corresponding total shear).
x_1 = distance in feet from left support to point beyond which stirrups are unnecessary.
l = span of beam in feet.
w = uniform load in pounds per foot.

Then

$$v = \frac{V}{bjd}$$

Stirrups become unnecessary at section where $v = v'$. Thus, at the required section

$$v' = \frac{V'}{bjd}$$

But

$$V' = \frac{wl}{2} - wx_1$$

Substituting and solving for x_1, we have

$$x_1 = \frac{l}{2} - \frac{v'bjd}{w}$$

Suppose a 10-ft. beam ($b = 10$ in. and $d = 20$ in.) is uniformly loaded with 2900 lb. per foot, and assume $v' = 40$ lb. per square inch according to recommendation of Joint Committee for 2000 lb. concrete. Also assume $jd = 7/8d$. Then,

$$x_1 = \frac{10}{2} - \frac{(40)(10)(17.5)}{2900}$$
$$= 2.59 \text{ ft.}$$

The following graphical method may be employed to determine the spacing of stirrups in large and important beams:

Lay off one-half the span to any convenient scale as shown in Fig. 46. Compute the values of s at a number of points (1, 2 and 3, for example) and lay off these values on the perpendiculars erected at the respective points to the same scale as the span. Draw a smooth curve through the upper ends of the perpendiculars. From point a on the curve directly above the point where the first stirrup will be placed, draw a line at 45 degrees to intersect with the horizontal line and erect at the point of intersection

B a perpendicular to cut the curve in point b. A line drawn from b at 45 degrees will intersect the horizontal in point C, where the above process is repeated. The points A, B, C, D, E, thus obtained, are the points at which stirrups are required.

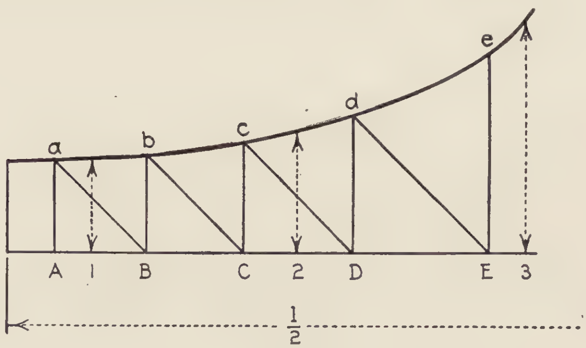


FIG. 46.

In the case of uniformly loaded beams, it is only necessary to compute the minimum spacing of stirrups—that is, at the support. The spacing at two other points may be obtained from the fact that the spacing for a distance from the support $\frac{l}{8}$ is

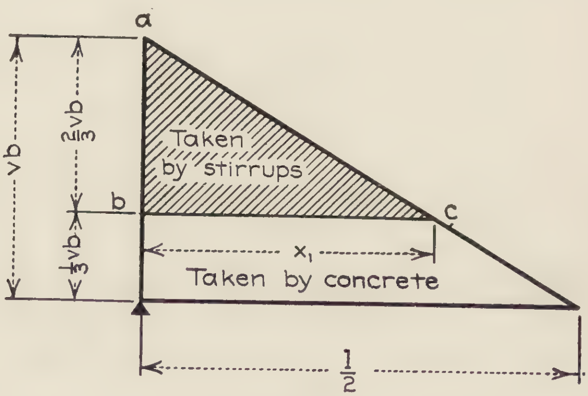


FIG. 47.

four-thirds the minimum, and for a distance $\frac{l}{4}$ is twice the minimum. At the center, the spacing is infinity.

The diagram of maximum shear intensity along a uniformly loaded beam is shown in Fig. 47. If the shear at the end of beam

is the maximum allowable, the triangle *abc* represents the total stress to be taken by the stirrups at each end of beam. This may be proved as follows: Formula (1), Art. 41, shows the total stress for a distance s along the beam (where V is the average shear) to be $a_s f_s = \frac{2}{3} \cdot \frac{Vs}{jd}$; in the case at hand $s = x_1$ and, if v is considered the unit shear at the end of beam $= \frac{2V}{bjd}$, the total stress for the distance x_1 is $\frac{2}{3} \cdot \frac{Vx_1}{jd} = \frac{1}{3} \cdot vbx_1$, which is the area of triangle *abc*. The ordinate *ab* represents two-thirds of the shear at the support per 1-in. length of beam.

Some attention must be paid to the diameter of stirrup which it will be possible to employ in any given case. Of course, the diameter should not be so small that the stirrups will be placed too close together for convenience in construction, nor yet so far apart that the limiting value $3/4d$ is exceeded. But, in addition to such consideration, the bond strength of the stirrup must be carefully investigated since the danger of slipping determines the maximum diameter which may be employed.

Let us derive a formula for the maximum diameter to be used in any given case. We shall consider straight stirrups only.

Let i = diameter of stirrup.
a_s = area of stirrup.
o = circumference of stirrup.
u = allowable bond stress per unit of surface of bar.

The distribution of bond stresses developed on the surface of the stirrups is indeterminate. Evidently it must not be expected that tension will be transferred to the concrete until the compression area of the beam is reached, or until a point but little below is reached. Experiments show that it is safe to assume the grip of a stirrup to be 0.6 the depth of beam.

$$f_s a_s = 0.6\ dou$$

or

$$\frac{a_s}{o} = 0.6 \frac{u}{f_s} d$$

But, for round or square stirrups,

$$\frac{a_s}{o} = \frac{\frac{i^2}{4}\pi}{\pi i} = 1/4\ i$$

Then

$$i = (2.4 \frac{u}{f_s})\ d$$

The table below gives the values of $2.4\frac{u}{f_s}$ for different working values of tension and bond, as developed by Messrs. Taylor and Thompson in "Concrete, Plain and Reinforced."[1]

Allowable unit bond stress in pounds per square inch (u)	Vertical bars Allowable unit tension in stirrups in pounds per square inch (f_s)				
	12,000	14,000	15,000	16,000	20,000
80	0.016	0.014	0.013	0.012	0.010
100	0.020	0.017	0.016	0.015	0.012
120	0.024	0.020	0.019	0.018	0.014
150	0.030	0.026	0.024	0.022	0.018

Suppose that for a given vertical stirrup the allowable $f_s =$ 16,000 lb. per square inch and the allowable $u = 80$ lb. per square inch. Also, let the depth of the beam in question be 20 in. (d). Then $i = 0.24$ in.—practically 1/4 in. For deformed bars the bond may be increased to 100, or even to 150 lb. per square inch, varying with the character of the bar. Using 150 for u and 16,000 for f_s, a beam 20 in. deep to center of steel, making no allowance for the value of a bent end (called prong or hook), would require stirrups not to exceed practically 1/2 in. in diameter. Deformed bars are therefore useful for stirrups to permit larger diameters, although the total quantity of stirrup steel required throughout a beam is not changed except where the spacing would tend to exceed the allowable. In other words, with deformed bars the stirrups do not need to be spaced as closely as with plain bars.

Recent tests show that either a right angle or a semi-circular bend of 5 diameters is sufficient to stress the steel to its elastic limit, provided the hook is well embedded in the concrete so that it cannot kick out. To rely upon such increase in strength over the straight stirrup, an embedment is required in all directions equal to 8 diameters of the bar.

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 454. Copyright, 1905, 1909, by Frederick W. Taylor.

Illustrative Problem.—A concrete beam is 9 in. $\times$ 16 in. in cross-section and the tension reinforcement is 2 in. above the lower face of the beam. Span of the beam is 8.5 ft. Uniform load of 1800 lb. per foot. If necessary, the web is to be reinforced against diagonal tension using vertical stirrups. The working stresses recommended by the Joint Committee for a 2000 lb. concrete will be used.

$$\frac{V}{bd}=\frac{7650}{(9)(14)}=61 \text{ lb. per square inch.}$$

The allowable average shear v_o is 35 lb. per square inch, hence stirrups are necessary.

The diameter of a stirrup without any prong or hook should not exceed (see table)

$$\begin{aligned} i &= (0.012)(14) \\ &= 0.17 \text{ in.} \end{aligned}$$

We will use stirrups bent at the upper end and because of this bending 1/4-in round bars may be considered secure against slipping.

Stirrups are unnecessary at a distance from support (assuming $j=7/8$) equal to

$$x_1=\frac{8.5}{2}-\frac{(40)(9)(7/8)(14)}{1800}=1.80 \text{ ft.}$$

The minimum spacing of stirrups (U-shape) will occur at the supports and will be equal to

$$s=\frac{3}{2}\cdot\frac{(2)\ (0.049)(16{,}000)\ (7/8)(14)}{7650}=3.75 \text{ in.}$$

Let x = distance from the left support. Then for $x = \frac{l}{8} = 1.1$ ft., the spacing is $(4/3)(3.75) = 5.0$ in.; and for $x = \frac{l}{4} = 2.1$ ft., the spacing is $(2)(3.75) = 7.5$ in. A smooth curve drawn through the points determines the spacing at any part of the beam. In monolithic construction, the first stirrup may be placed one-half the minimum spacing from the edge of the support, and the last stirrup should not be farther distant from the limiting point, where stirrups are unnecessary, than half of the distance between the last two stirrups. In beams simply supported, the first stirrup should be placed one-half the minimum distance from the *center* of support.

---

43. Horizontal Bars Bent Up for Web Reinforcement.—The ends of the horizontal bars in a reinforced concrete beam may often be bent up to assist in providing for the diagonal tension. In some cases these bent rods may take all such stresses, and vertical stirrups then are not theoretically needed—though they are desirable, as shown by tests.

Plain rods bent up to provide web reinforcement often lack sufficient bond strength to render them fully effective. Where bent up at a considerable angle they should be turned again horizontally and extend some distance along the upper part of

the beam, as shown in Fig. 48. In heavy construction the ends of all bars should be bent into a hook. The most convenient method of using reinforcement is to bend up two rods at a time and make all the bars inclined at an angle of 45 degrees with the horizontal. The bars bent should theoretically be such as to keep the center of gravity of the beam cross-section in the line drawn vertically through the center of the section. An exception occurs to the bending of two rods at a time, in the case of an odd number of horizontal rods. Here, one of the bends may consist of either one or three rods.

If bent rods are not required to provide for diagonal tension, then the horizontal rods may be dispensed with at the points,

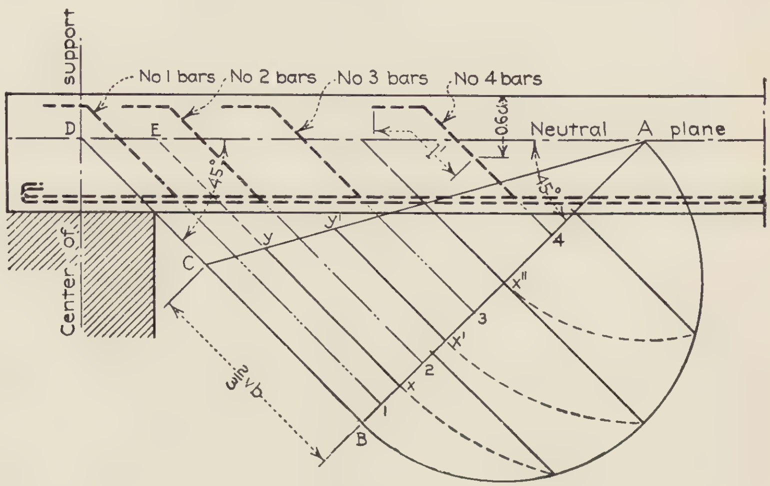


Fig. 48.

beyond which they are not needed to provide for tension due to bending. This method of stopping off the horizontal rods is not desirable, however, as the bond in the concrete near the middle of the beam is not as good as would be the case near the end where the moments are smaller. Also, when a bar is discontinued, the stress in those which remain is immediately increased tending still further to impair the bond between the steel and the concrete. This is true whether or not a hook is employed on the discontinued rods. With bent up rods a more ideal condition exists. The horizontal components of the upturned bars act with the bars unbent in taking the tension due to bending, and

so the tension in the horizontal rods decreases gradually toward the end of beam as it should.

At the bend in a horizontal rod, the stress is transferred gradually to the concrete as compression, and, if the bend is too abrupt, the unit stress in the concrete in this vicinity may become excessive. Theory tends to indicate that a radius of bend equal to 12 diameters is satisfactory. Cracks following the line of the rod are seldom seen in tests of beams having even sharper bends in the reinforcement.

The distance from the support to the point where web reinforcement is not needed is determined in the same manner as for vertical stirrups. The bent rods, if of the same diameter, should be so arranged that each rod will take an equal part of the diagonal tension—that is, if they can be bent in this way and still provide satisfactorily for the horizontal tension. If the rods cannot be bent at the desired points, vertical stirrups must be used to provide for the diagonal tension either toward the center, or toward the end of the beam.

First of all, we shall assume that the rods *can* be bent up conveniently to provide for all the diagonal tension. Consider uniform loading. The maximum shear V should be calculated at the support, point D, Fig. 48. One-third of V will be taken by the concrete and two-thirds by the bent bars. Point A should also be determined—namely, the point where the web reinforcement is not needed. From this point to the left support, the shear to be taken by the bent bars increases from zero to its maximum value of $2/3V$ at the support, and may be represented by the triangle ABC. To construct this triangle draw a line AB from point A at 45 degrees with the horizontal, and from point D draw a line DB perpendicular to the line AB. Then, consider the maximum shear per inch length of beam to be represented by BC; in other words, $BC = 2/3vb = \frac{2/3V}{jd}$. Now, suppose we intend to bend two rods at a time and to bend in all 8 rods, all of the same diameter. Then each of them will take an equal part of the diagonal tension. Divide the area of the triangle into four equal parts, find centers of gravity of each part, and from these centers of gravity draw lines to represent the location of points to bend up the bars in the beam. The method of division of the triangle into an equal number of parts is clearly shown in the drawing, where the line AB is divided into equal

parts and dotted arcs of circles are drawn with centers at A.[1]

The reason for projecting the neutral axis AD upon a plane making 45 degrees with the horizontal may need some explanation. Our formula for the required total area of steel in each bend is

$$a_s = \frac{2}{3} \cdot \frac{0.7Vs}{f_s jd}$$

Hence the sum of the stresses in the inclined bars at each bend is given by the following formula

$$a_s f_s = \frac{2}{3} \cdot \frac{0.7Vs}{jd}$$

But any ordinate as $BC = \frac{2}{3} \cdot \frac{V(\text{at}D)}{jd}$. Also, $xy = \frac{2}{3} \cdot \frac{V(\text{at}E)}{jd}$, and $Bx = 0.7s$. Then, the

$$\text{area } BC\ yx = \frac{2}{3}\frac{V(\text{at No. 1 bars})0.7s}{jd}$$

(the bars being practically half way between x and B), and represents the sum of the stresses in the No. 1 bars. Similarly, the area $xyy'x'$ represents the sum of the stresses in the No. 2 bars.

In practice the line AC need not be drawn. For example, no matter what angle the line AC makes with the line AB, the points 1, 2, 3, and 4 will be the same; the points 1, 2, and 3 will remain practically half way between B and x, x and x', and x' and x'', respectively, while $A4$ will remain as $2/3\ Ax''$.

The bond strength in these inclined bars must now be investigated. This strength should be provided in the upper portion of the beam. As with vertical stirrups, we shall arbitrarily assume that no stress is transmitted from the steel to the concrete below a point which is 0.6 d below the upper surface of the beam.

We will assume that the stress in an inclined bar is its working stress. This gives the maximum condition. Using the notation of the preceding article and l' for length,

$$l'ou = a_s f_s$$

and for round or square bars,

$$l'u = \frac{if_s}{4}$$

or

$$l' = \frac{f_s}{4u} \text{ diameters.}$$

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 475. Copyright, 1905, 1909, by Frederick W. Taylor.

The following table gives the values of $\frac{f_s}{4u}$ for different working values of tension and bond, as developed by Messrs. Taylor and Thompson in "Concrete, Plain and Reinforced."[1]

Allowable unit bond stress in pounds per square inch (u)	Allowable unit tension in inclined bar, in pounds per square inch (f_s)				
	12,000	14,000	15,000	16,000	20,000
80	37	44	47	50	62
100	30	35	38	40	50
120	25	29	31	33	41
150	20	23	25	27	33

The length of embedment may be obtained by multiplying the value selected from this table by the diameter of the bar.

The bond of deformed bars may be figured the same as the bond of plain bars except using for their diameter, the diameter of a cylinder based on the longest projections—that is, of a cylinder which would be sheared out by the deformed bar. For smooth metal of the nature of tool steel not over 40 lb. per square inch should be permitted for allowable bond strength. Flat steel should be given a similarly low value per square inch.

If the allowable $f_s = 16{,}000$ lb. per square inch and the allowable $u = 80$ lb. per square inch, then l' (as shown at No. 4 bars in Fig. 48) should equal 50 diameters by the above table. Suppose a deformed bar is used with $u = 150$ lb. per square inch. The length l' should then equal 27 diameters, or 1 ft. 8 1/4 in. for a 3/4 in. rod. Hooks should be provided at the ends of the bars to provide additional safety.

44. Vertical Stirrups and Bent Rods Combined.—Usually the diagonal tension in a simply supported beam of rectangular section can all be provided for by bent up rods and the *theoretical* combination of stirrups with bent up rods need not be considered. This combination, however, is a common one in T-beam design and may just as well be treated at this time.

Consider uniform loading and, in Fig. 49, let ABC be the diagonal tension triangle. Assume that four rods may be bent near the end of beam, but not so that they can take any diagonal

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 454. Copyright, 1905, 1909, by Frederick W. Taylor.

tension near to the point A. It will be considered feasible that the bent rods should be placed to take as much diagonal tension as possible from D toward A. The area $BCEF$ should be made equal to the allowable tensile stress in the two No. 1 bars; likewise area $EFGH$ should represent the tensile strength of the two No. 2 bars. This would not be true, however, if the distance s should turn out to be greater than $3/4d$. For such a case, the No. 2 bars should be placed the limiting distance from the No. 1 bars and the area toward the center of beam (representing the diagonal tension which the No. 2 bars may be assumed to take in that direction) should be made equal to $rtGH$. Stirrups may also be supplied to take the stress between e and A. The spacing near the point e would be obtained by the formula

$$s = \frac{3}{2} \cdot \frac{a_s f_s jd}{V}$$

in which V is the total vertical shear at the point where the

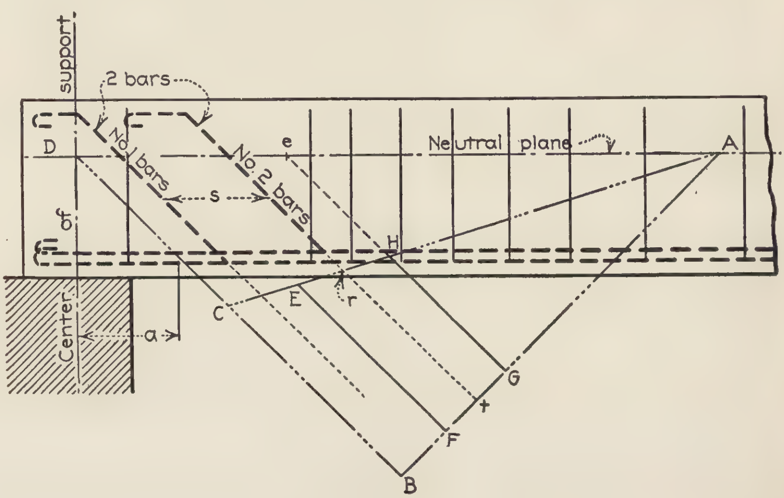


Fig. 49.

stirrup is placed. It will be well on the safe side to place the first stirrup one-half the computed spacing to the right of point e.

If possible, when bent up rods cannot take all the diagonal tension, it would be better design to bend up the rods as far from the end of beam as possible so that stirrups may be employed toward the end of beam where the diagonal tensile stresses are greatly inclined, rather than toward the center of beam as in Fig. 49. This method would be followed if the distance a were

considerable. Even in the case assumed in Fig. 49, at least one stirrup should be employed in this distance, as shown.

45. Points to Bend Horizontal Reinforcement.—If some of the horizontal bars are bent up at a given point to provide for diagonal tension, those remaining should have sufficient sectional area to carry the tension beyond this point.

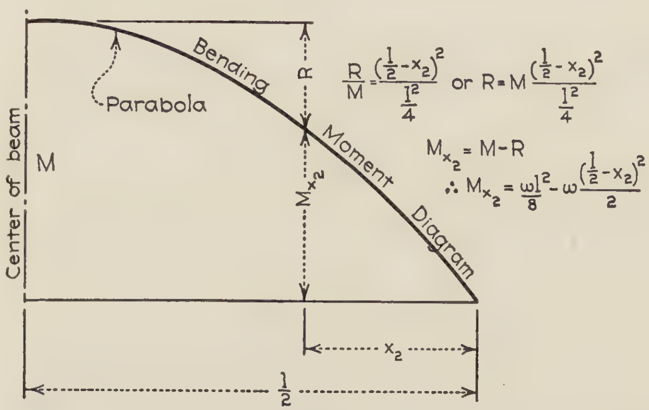


FIG. 50.

In determining the horizontal length of the various bars necessary to resist the bending moment, the same method, as developed by Messrs. Taylor and Thompson in "Concrete, Plain and Reinforced,"[1] may be used as in the design of plate girder flanges. Consider uniform loading.

Let m = number of bars at the center of the beam.
m_2 = number of bars to be bent.
l = span of beam in feet.
M = maximum moment $\frac{wl^2}{8}$, in which
x_2 = distance from support to point where m_2 bars may be bent up leaving sufficient steel to carry the tension.

The ratio of stress at the center of the beam to that at the point under consideration equals the ratio of moments at these points. Thus, if the steel is stressed equally at both points,

$$M_{x_2}:M=(m-m_2)\text{(area of one bar)}:m\text{(area of one bar)}.$$

Substituting

$$M=\frac{wl^2}{8} \text{ and } M_{x_2}=\frac{wl^2}{8}-\frac{w\left(\frac{l}{2}-x_2\right)^2}{2}$$

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 458. Copyright, 1905, 1909, by Frederick W. Taylor.

and solving for x_2 (see Fig. 50), we have

$$x_2 = \text{ or } < \frac{l}{2}\left(1 - \sqrt{\frac{m_2}{m}}\right)$$

For any bending moment $M = \frac{wl^2}{e}$,

$$x_2 = \text{ or } < \frac{l}{2}\left(1 - \sqrt{\frac{8m_2}{em}}\right)$$

(the meaning of this formula will be clear after studying Art. 54).

If it is desired to bend up a number of rods two or more at a time, then x_2 should be determined for each bend. After this is done, the remaining horizontal bars should be secure against slipping.

For concentrated and unsymmetrical loading, the maximum moments at various sections will need to be determined, in order

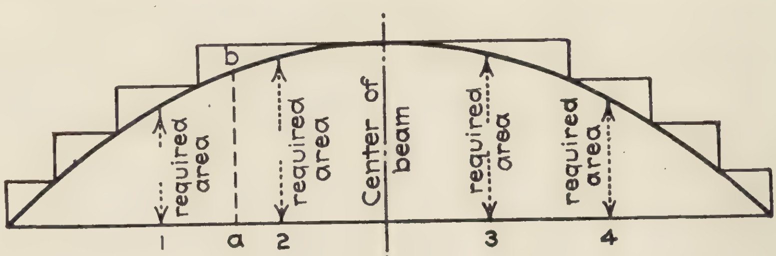


Fig. 51.

to ascertain the points where the horizontal bars may be bent up. From these maximum moments obtain the required area of horizontal rods at the different points (1, 2, 3, and 4, Fig. 51). Plot a curve to scale, as shown. Thus, ab represents the area required at the point a. On the center ordinate lay off the required areas of the rods, and draw horizontals as shown. The rods may be bent up where these horizontals cut the curve but it would be better, however, to carry them a short distance beyond the theoretical points.

Problems

(Working stresses recommended by the Joint Committee to be used throughout.)

33. A simply supported beam of 12 ft. span (*c.* to *c.* of supports) and 18 in. breadth is to carry a uniform load (live plus dead) of 6000 lb. per foot and rest upon concrete supports. Considering the reactions as uniformly

distributed over the bearing surface, how far should the beam extend beyond the edge of the supports, taking the allowable crushing strength of the concrete at 450 lb. per square inch?

34. In the beam of Problem 33, what is the distance from the left reaction to the point beyond which stirrups are unnecessary, assuming the moment calculations to give $b = 18$ in. and $d = 26$ in.

35. (a) Could four 1-in. plain square bars be used in the beam of Problem 34? Give reasons. (b) If eight 3/4-in. square bars were used, how many could be bent up to take diagonal tension (allow about 30 per cent more bond stress on the horizontal rods than would be considered safe by formula if three rods can be bent up and at least two bends made at each end of beam)?

36. In the beam of the preceding problems, at what points may the horizontal rods be bent up if eight 3/4-in. square rods are used?

37. At what points in the beam of Problem 36 should the horizontal rods be bent to provide for the diagonal tension in the best possible manner, assuming the resultant reactions 3 in. from the edge of supports (ends of beam 6 in. from edge)? Submit sketch neatly drawn.

38. In Problem 36 if the horizontal bars were run straight to the end of beam what size and spacing of vertical stirrups would be required to provide thoroughly for diagonal tension? Employ double-looped, square stirrups. Graphical work to be submitted.

39. (a) What should be the grip of the inclined rods of Problem 37 considering the rods stressed in tension to the allowable value of 16,000 lb. per square inch? (b) What should be the grip of these rods for the stress they are actually called upon to withstand?

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46. Transverse Spacing of Reinforcement.—The amount of concrete between the horizontal bars in a beam should be sufficient to transmit to the upper part of the beam the stress which the bars give over to the concrete below them; in other words, the shearing stress along *ab*, Fig. 52, should equal the amount of stress transmitted by bond along *bcd*. If bond and shearing strengths were equal, *ab* should equal *bcd*, and the clear space between bars should be $\frac{3.1416}{2}$ diameters = 1.57 diameters. But shearing strength here employed is controlled by the diagonal tension and is approximately $\frac{120}{80} = \frac{3}{2}$ the bond stress, using the values recommended by the Joint Committee. Hence, *ab* should be (2/3) (1.57) diameters = 1.05 diameters. That is, the minimum net distance in the clear between rods is approximately

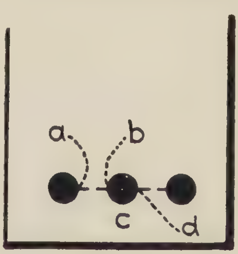


Fig. 52.

equal to the diameter of the rod. There is likely to be more or less tension in the concrete surrounding the rods and, besides, since the concrete is not easily placed between the rods, it may have a lower strength in that vicinity. A clear spacing of 1 1/2 diameters is advisable unless it is determined by computation that the bond stress is very much lower than the bond strength allowed. In the above discussion plain rods only have been considered. Deformed bars, if stressed to their full bond value, should be spaced farther apart than plain bars.

For a beam uniformly loaded, the bond stress near the center of the beam is low and the bars may be placed as closely together as the proper placing of the concrete between them will permit. Near the end of beam the bond stress should be calculated if it is desired to space the rods so as to obtain the minimum width of beam possible. In beams having the horizontal rods bent up, the bond stress at the bending points should be considered. This stress may be less than the maximum and, if so, the corresponding spacing may be made less than 1 1/2 diameters in the clear. With rods bent up, more liberal spacing can readily be made toward the end of beam.

The Joint Committee recommends that "the lateral spacing of parallel bars should not be less than 2 1/2 diameters, center to center, nor should the distance from the side of the beam to the center of the nearest bar be less than 2 diameters." In order that concrete may be readily placed between the rods and also give sufficient concrete on the sides of the beam for fire protection, it is also advisable to require that the spacing of rods be not less than 1 in. in the clear (if the maximum size of aggregate does not exceed 1 in.) and that 1 in. in the clear be also considered the minimum distance of the rods from the sides of the beam. Thus, the least width of beam should be the greater of the two values determined from the following formulas:

$$b = [2.5(n-1) + 4]d_1$$
$$b = a_g(n+1) + nd_1$$

in which b = least width of beam.

d_1 = thickness of the rods.

n = maximum number of rods which occurs in a horizontal layer.

a_g = maximum size of aggregate in inches.

It should be clear that, for a 1-in. maximum aggregate, the width

of beam for all rods greater than 5/8 in. in diameter will be governed by the first formula and for 5/8-in. rods and less, by the second formula.

Where two layers of rods are used, there is less danger of vertical splitting if the rods are placed directly over each other, and with sufficient space simply to permit the mortar to run between them. The Joint Committee specifies a limiting clear space of 1/2 in.

47. Depth of Concrete Below Rods.—Prof. Charles L. Norton of the Insurance Engineering Experiment Station has made a careful study of the thickness of concrete which is essential to thoroughly protect embedded steel from the direct action of flames, and recommends 2 in. for maximum conditions. An excessive thickness of concrete, however, adds to the danger of cracking, because the tension in the concrete increases with the depth below the steel and with but slight corresponding gain in strength to the beam. Also, flat slabs are found to be affected to a less depth than projecting members such as beams and columns. The Joint Committee suggests that "the metal in girders and columns be protected by a minimum of 2 in. of concrete; that the metal in beams be protected by a minimum of 1 1/2 in. of concrete; and that the metal in floor slabs be protected by a minimum of 1 in. of concrete."

The following depths of concrete below the steel may be employed under ordinary conditions, but wherever conditions are especially hazardous, the recommendations of the Joint Committee should be followed:

SLABS

Depth to steel (d)	Depth below center of steel
$3\frac{1}{4}$ in. and under....................	$\frac{3}{4}$ in.
Between $3\frac{1}{4}$ in. and $4\frac{3}{4}$ in............	1 in.
$4\frac{3}{4}$ in. and over....................	$1\frac{1}{4}$ in.

BEAMS AND GIRDERS

Depth to steel (d)	Depth in the clear below steel
10 in. and under....................	1 in.
Between 10 in. and 20 in.............	$1\frac{1}{2}$ in.
20 in. and over....................	2 in.

48. Ratio of Length to Depth of Beam for Equal Strength in Moment and Shear.—With given working stresses in concrete and steel, there is a definite ratio of length to depth of beam which will give equal strength in moment and shear. Let us first consider beams simply supported.

In the case of a single concentrated load at the center of span, the shear V, due to a given load W, is $1/2W$, and the moment M is $1/4Wl$. Hence,

$$W = 2V = \frac{4M}{l}$$

But from preceding formulas, we have $V = v'bjd$ and $M_s = pf_s jbd^2$, in which v' = allowable shearing stress and f_s = working stress in steel. Substituting, we have

$$2v'bjd = \frac{4pf_s jbd^2}{l}$$

from which

$$\frac{l}{d} = \frac{2f_s p}{v'}$$

For a uniformly distributed load, a similar process gives the ratio

$$\frac{l}{d} = \frac{4f_s p}{v'}$$

For beams loaded with equal loads at the third points,

$$\frac{l}{d} = \frac{3f_s p}{v'}$$

Taking for example, $v' = 40$ lb. per square inch, $f_s = 16{,}000$, $f_c = 650$, $n = 15$, and, using an average value of 7/8 for j, we have the following ratios for $\frac{l}{d}$.

For concentrated load at center of span................ $\frac{l}{d} = 6.16$

For uniformly distributed load........................ $\frac{l}{d} = 12.32$

For equal loads at the third points.................... $\frac{l}{d} = 9.24$

It should be clear that the strength of beams of greater relative length than obtained by the formulas will be determined by their moment of resistance, while that of shorter beams by their shearing resistance.

In the case of continuous beams the above formulas will apply if l is taken as the length between points of inflection. It is often convenient to know the extreme limit in design. The Joint Committee recommends 120 lb. per square inch for the shearing strength of concrete when adequately reinforced against diagonal tension. This is a low figure but is adopted in order to prevent any likelihood of cracks opening up in the concrete. Suppose then, it is required to know the minimum value of $\frac{l}{d}$ for a given

beam, uniformly loaded. Our formula readily gives us the result, using the working stresses given above.

$$\frac{l}{d}=\frac{(4)(16,000)(0.0077)}{120}=4.11$$

At the same time that the ratio of length to depth is being investigated for moment and shear, there are other conditions which must be considered. For instance, the ratio of length to breadth of beam should not exceed a value of about 25 if the beam is not supported laterally. The reason for this is found in the fact that the upper part of the beam is a column, and to prevent additional stress due to side bending the length should not exceed about 25 times the width. On the other hand, the best shaped beam is one in which b lies between $1/2d$ and $3/4d$. In any given case, to satisfy all requirements and arrive at a satisfactory design, two or three trials may be required.

49. Notation.—The notation used in this course in the design of rectangular reinforced concrete beams is summarized as follows:

f_c = unit compressive stress in outside fiber of concrete.
f_s = unit tensile stress in steel.
n = ratio of modulus of elasticity of steel in tension to modulus of elasticity of concrete in compression.
a_s = area of cross-section of steel.
b = breadth of beam.
d = distance from compression surface to axis of reinforcement.
M_c = resisting moment as determined by concrete.
M_s = resisting moment as determined by steel.
M = bending moment or resisting moment in general.
p = steel ratio $=\frac{a_s}{bd}$.
k = ratio of depth of neutral axis to depth of steel.
j = ratio of lever arm of resisting couple to depth of steel.
V = total shear.
v = unit shear.
v' = unit working shear.
$v_o = \frac{V}{bd}$ = average unit shear.
u = unit bond.
o = circumference of one bar.

Σ_o = total circumference of bars.
s = horizontal spacing of stirrups.
i = diameter of stirrup bar.
w = uniform load in pounds per foot.
l = span of beam in feet.
x_1 = distance in feet from support to point beyond which stirrups are unnecessary.
l' = required length of bent bars for bond strength (above a point $0.6d$ from upper surface of beam).
x_2 = distance from support to point where m_2 bars may be bent up leaving sufficient steel to carry the tension.
m = number of bars at center of beam.
m_2 = number of bars to be bent.

50. Formulas.—For convenience in working the problems which follow, the formulas which apply to rectangular reinforced concrete beams will be re-stated.

MOMENT

$$p=\frac{a_s}{bd} \qquad p=\frac{1/2}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)}$$

$$k=\sqrt{2pn+(pn)^2}-pn. \qquad j=1-1/3\,k$$

$$M_c=1/2\,f_ckjbd^2,\ \text{or } bd^2=\frac{2M}{f_ckj},\ \text{ or } f_c=\frac{2M}{kjbd^2}$$

$$M_s=pf_sjbd^2,\ \text{or } bd^2=\frac{M}{pf_sj},\ \text{ or } f_s=\frac{M}{a_sjd}$$

$$f_c=\frac{2f_sp}{k}$$

Note.—If a beam is over-reinforced, its resisting moment depends on M_c, and if under-reinforced on M_s. If it is desired to find the fiber stresses in concrete and steel for a beam already designed [with an amount of steel differing from the amount determined by the formula $p=\frac{1/2}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)}$], the formulas $f_s=\frac{M}{a_sjd}$ and $f_c=\frac{2M}{kjbd^2}$ (or $f_c=\frac{2f_sp}{k}$) should be used, where M is the external bending moment in each case. For a given external

M, either $bd^2 = \frac{2M}{f_c kj}$ or $bd^2 = \frac{M}{pf_s j}$ may be used to determine cross-section, when the p used is obtained from the formula $p = \frac{1/2}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)}$.

SHEAR

$$v = \frac{V}{bjd} \qquad v_o = \frac{V}{bd}$$

BOND

$$u = \frac{V}{\Sigma ojd}$$

VERTICAL STIRRUPS

$$s = \frac{3a_s f_s jd}{2V} \qquad a_s = \frac{2}{3}\,\frac{Vs}{f_s jd} \qquad x_1 = \frac{l}{2} - \frac{v'bjd}{w}$$

(uniform loading only)

$$i = 2.4\frac{u}{f_s}\,d \text{ (see table)}$$

INCLINED RODS

$$l' = \frac{f_s}{4u} \text{ diameters (see table)} \qquad x_2 = \text{ or } < \frac{l}{2}\left(1 - \sqrt{\frac{m_2}{m}}\right)$$

(uniform loading only)

$$s = \frac{3a_s f_s jd}{1.4V} \qquad a_s = \frac{2}{3}\,\frac{0.7Vs}{f_s jd} \text{ (45 degrees inclination)}$$

Note.—Throughout the following illustrative problems, those working stresses will be used which have been recommended by the Joint Committee for a 2000 lb. concrete. Allowable stress in the steel will be taken at 16,000 lb. per square inch. The beams are assumed to be simply supported and to rest upon concrete supports. Slide rule will be used wherever possible.

Illustrative Problem.—Design a beam to span 40 ft. and to support 600 lb. per foot (including weight of beam). The breadth and depth may be varied at will as far as the conditions of design are concerned, the only condition being that the beam must have the required strength.

$$p = \frac{1/2}{\frac{16,000}{650}\left(\frac{16,000}{15\times 650}+1\right)} = 0.0077$$

$$k = \sqrt{(2)(0.0077)(15) + (0.0077)^2(15)^2} - (0.0077)(15) = 0.378$$

$$j = 1 - 1/3\ (0.378) = 0.874$$

$$M = \frac{wl^2}{8} = \frac{(600)(40)(40)(12)}{8} = 1,440,000 \text{ in.-lb.}$$

$$bd^2 = \frac{M}{pf_s j} = \frac{1,440,000}{(0.0077)(16,000)(0.874)} = 13,400$$

If necessary to prevent side bending (not a rigid condition, since a beam is not similar to a column in all respects)

$$\frac{(12)(40)}{b} = \text{or} < 25, \text{ or } b = \text{or} > 19 \text{ in.}$$

In the best shaped beam b lies between

1/2 d and 3/4 d.

Assume $b = 18$ in.

$$d^2 = \frac{13{,}400}{18} = 745, \text{ or } d = 27.3 \text{ in., say } 27\ 1/2 \text{ in.}$$

$$v_0 = \frac{V}{bd} = \frac{12{,}000}{(18)(27.5)} = 24 \text{ lb. per square inch.}$$

Web reinforcement is not theoretically needed and we will take $b = 18$ in. and $d = 27\ 1/2$ in. Area of cross-section, $bd = (18)(27.5) = 495$ sq. in.

$$a_s = (495)(0.0077) = 3.81 \text{ sq. in.}$$

We shall select four 1 1/8-in. round rods = 3.98 sq. in. (See following table.)

Round rods				Square rods		
Size inches	Area square inches	Perimeter inches	Weight per foot pounds	Area Square inches	Perimeter inches	Weight per foot pounds
1/4	.0491	.785	.17	.0625	1.000	.21
5/16	.0767	.982	.26	.0977	1.25	.33
3/8	.1104	1.178	.38	.1406	1.50	.48
7/16	.1503	1.374	.51	.1914	1.75	.65
1/2	.1963	1.571	.67	.2500	2.00	.85
9/16	.2485	1.767	.85	.3164	2.25	1.08
5/8	.3068	1.964	1.04	.3906	2.50	1.33
11/16	.3712	2.160	1.26	.4727	2.75	1.61
3/4	.4418	2.356	1.50	.5625	3.00	1.91
13/16	.5185	2.553	1.76	.6602	3.25	2.25
7/8	.6013	2.749	2.04	.7656	3.50	2.60
15/16	.6903	2.945	2.35	.8789	3.75	2.99
1	.7854	3.142	2.67	1.0000	4.00	3.40
1 1/8	.9940	3.534	3.38	1.2656	4.50	4.30
1 1/4	1.2272	3.927	4.17	1.5625	5.00	5.31
1 3/8	1.4849	4.320	5.05	1.8906	5.50	6.43
1 1/2	1.7671	4.712	6.01	2.2500	6.00	7.65

The spacing adopted is shown in Fig. 53. The maximum

$$u = \frac{V}{\Sigma o j d} = \frac{12{,}000}{(4)(3.534)(7/8)(27.5)} = 35 \text{ lb. per square inch.}$$

Hooks will be provided at the ends of the rods for additional safety. Spacing of bars is greater than necessary. Depth of concrete below the rods is practically 2 in.

Minimum distance from the end of beam to edge of support is $\frac{12{,}000(2)}{(18)(450)} =$

3.0 in., allowing 100 per cent for unequal distribution of the reactions over the bearing surface. A practical limit is about 6 in. on concrete supports.

Let us now review the beam we have designed. It is not necessary to do this unless the values of bd^2 and a_s are made considerably different than the

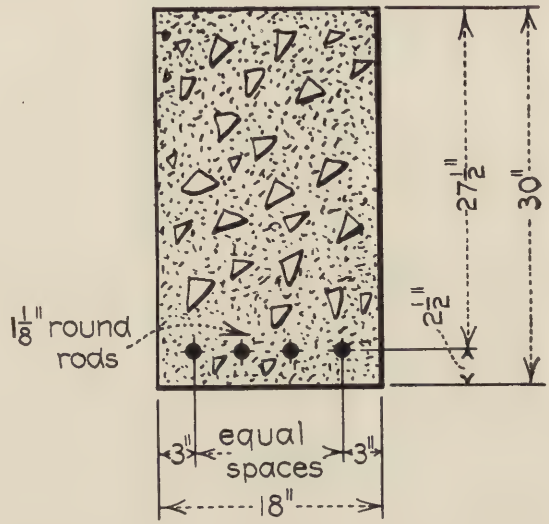


FIG. 53.

calculated values. The method will be shown here, however, for the benefit of the student.

$$p = \frac{a_s}{bd} = \frac{3.98}{495} = 0.0080$$

$$k = \sqrt{(2)(0.0080)(15) + (0.0080)^2(15)^2} - (0.0080)(15)$$
$$= 0.384$$

$$j = 1 - 1/3(0.384) = 0.872$$

$$f_s = \frac{1{,}440{,}000}{(3.98)(0.872)(27.5)} = 15{,}100 \text{ lb. per square inch.}$$

$$f_c = \frac{(2)(15{,}100)(0.0080)}{0.384} = 630 \text{ lb. per square inch.}$$

In long beams, it frequently occurs that stock bars are too short to extend the full length of the beam. In such cases welding may be resorted to. Since the maximum moment is at or near the middle of the span in most beams, the welding should be near the ends and not near the middle of the beam. This will require two welds instead of one, but welding near the section of maximum moment should never be permitted.

Illustrative Problem.—It is not always possible to commence the design of a beam with an accurate enough idea of its weight to proceed as in the previous problem. The usual way is to make an intelligent guess of the

weight, design the beam, compute the weight, and if necessary revise the design. Suppose a beam is to span 10 ft. and is to support 600 lb. per foot (not including weight). We shall assume the weight of beam at 85 lb. per foot.

$$p=0.0077$$
$$k=0.378$$
$$j=0.874$$
$$M=\frac{wl^2}{8}=\frac{(685)(10)(10)(12)}{8}=103{,}000 \text{ in. lb.}$$
$$bd^2=\frac{103{,}000}{(0.0077)(16{,}000)(0.874)}=960$$

Assume $b=7$ in.

$$d^2=\frac{960}{7}=137, \text{ or } d=11.7 \text{ in.}$$

We will take $b=7$ in., and $d=11\ 1/2$ in. Thus, web reinforcement will be needed.

$$v_o=\frac{V}{bd}=\frac{(685)(5)}{(7)(11.5)}=43 \text{ lb. per square inch.}$$

Allowable average shear is 35 lb. per square inch. We shall provide the web reinforcement by means of vertical stirrups. Area of cross-section, $bd=(7)(11.5)=80.5$ sq. in.

$$a_s=(80.5)(0.0077)=0.62 \text{ sq. in.}$$

We shall select three 1/2-in. square bars $=(3)(0.250)=0.750$ sq. in. No difficulty will be encountered in the spacing of these bars with a 7-in. width.

Weight of designed beam per linear foot is (total depth 13 in.)

$$=\frac{(7)(13)(150)}{(12)(12)}=95 \text{ lb., or } \frac{(7)(13)(145)}{(12)(12)}+(0.85)(3)=94 \text{ lb.}$$

The assumed and calculated weights are close enough so that the beam need not be redesigned. We shall, however, consider the loading as 695 lb. per foot in the following computations. The maximum

$$u=\frac{(5)(695)}{(3)(4)(0.5)(7/8)(11.5)}=58 \text{ lb. per square inch.}$$

The bond stress is satisfactory.

Minimum distance from end of beam to edge of support is $\frac{(695)(5)(2)}{(7)(450)}=2.2$ in., allowing 100 per cent for unequal distribution of reactions over supporting surface. A distance of 6 in. will be taken.

Reviewing the beam we have designed,

$$p=\frac{0.75}{(7)(11.5)}=0.0093$$
$$k=0.406$$
$$j=0.865$$
$$f_s=\frac{(695)(10)(10)(12)}{(8)(0.75)(0.865)(11.5)}=14{,}000 \text{ lb. per square inch.}$$
$$f_c=\frac{(2)(14{,}000)(0.0093)}{0.406}=640 \text{ lb. per square inch.}$$

which is satisfactory.

$$i=(0.012)(11.5)=0.138 \text{ in.}$$

We will use U-shaped stirrups bent at the upper end and 1/4-in. round rods

may be considered secure against slipping. Stirrups are unnecessary at a distance from the center of support equal to

$$x_1 = \frac{10}{2} - \frac{(40)(7)(0.865)(11.5)}{695}$$

$$= 5 - 4.0 = 1.0 \text{ ft.} = 12 \text{ in.}$$

The minimum spacing of stirrups will occur at the supports and will be equal to

$$s = \frac{(3)(2)(0.049)(16{,}000)(0.865)(11.5)}{(2)(5)(695)} = 6.7 \text{ in.}$$

The stirrups are needed for such a short distance from each end of the beam that two stirrups near the ends will suffice, as shown in Fig. 54.

Perhaps we may be able to provide web reinforcement by means of bent up rods. Let us investigate.

The total stress to be taken by the inclined rods is represented by the

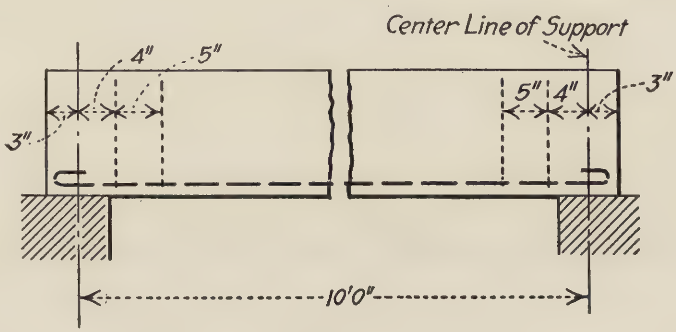


Fig. 54.

triangle *ABC*, Fig. 55. *BC* represents two-thirds of the horizontal shear at the support per 1-in. length of beam.

$$BC = \frac{2}{3} \cdot \frac{Vb}{bjd} = \frac{2}{3} \cdot \frac{V}{jd} = \frac{2}{3} \cdot \frac{(695)(5)}{(0.865)(11.5)} = 233 \text{ lb.}$$

$$r = (0.7)(AD) = (0.7)(12) = 8.4 \text{ in.}$$

Hence total stress to be taken by the rods

$$= \frac{BC}{2}(r) = \frac{(233)}{2}(8.4) = 980 \text{ lb.}$$

But the area of one rod multiplied by 16,000 gives its tensile value, or

$$\text{tensile value of one rod} = (0.250)(16{,}000) = 4000 \text{ lb.}$$

Thus, only one bent up rod is required. For the two straight rods the maximum bond stress

$$u = \frac{(695)(5)}{(2)(4)(0.5)(7/8)(11.5)} = 86 \text{ lb. per square inch, which is satisfactory.}$$

$$x_2 = \text{ or } < \frac{10}{2}\left(1 - \sqrt{1/3}\right) = 2.12 \text{ ft.}$$

The distance from the support to the point where web reinforcement is unnecessary is only 12 in. The rod can thus be bent up at the required place between these two points. Fig. 55 shows the construction necessary to locate the place where the rod should be bent. If the inclined rod received a stress of 16,000, the length l' should equal 50 diameters (from table) or

$$l' = (50)(1/2) = 25 \text{ in.}$$

but this rod receives only

$$\frac{980}{0.25} = 3920 \text{ lb. per square inch.}$$

and the length l' necessary is

$$\frac{f}{4u}(i) = \frac{3920}{(4)(80)} \cdot 1/2 = 6\ 1/4 \text{ in.}$$

In this case this length can easily be obtained.

It should be clear from the sketch that, if a greater length than about 9 in. were required for l', some kind of anchorage at the end of rod should be

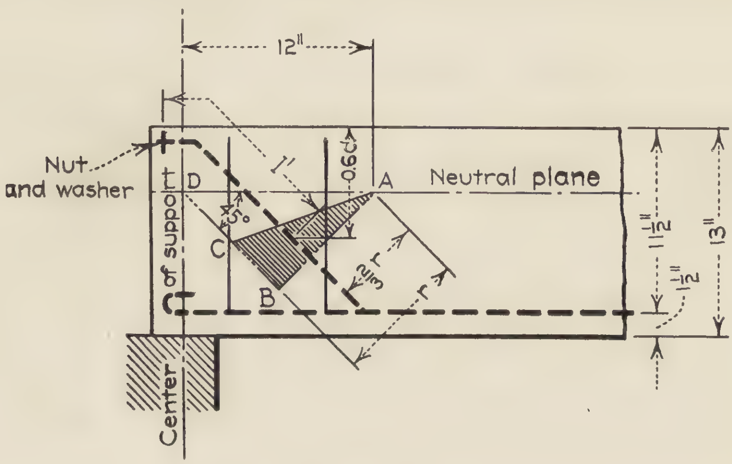


FIG. 55.

provided. A possible method would be to use a large nut and washer on the end of the rod, as shown. This would perhaps give sufficient bond but the work of construction would be troublesome. A hook as shown at the end of the horizontal bars is generally considered more satisfactory.

Tests seem to indicate (Art. 39) that too much reliance should not be placed upon one or two bent rods. For this reason, in the preceding problem it would be good design to use also the two stirrups as shown.

Illustrative Problem.—Design a beam to span 10 ft. and to support a load of 4900 lb. per foot (not including weight). We shall assume the weight of beam at 400 lb. per foot.

$p = 0.0077$
$k = 0.378$
$j = 0.874$

$$M=\frac{wl^2}{8}=\frac{(5300)\ (10)\ (10)\ (12)}{8}=795{,}000 \text{ in.-lb.}$$

$$bd^2=\frac{795{,}000}{(0.0077)(16{,}000)(0.874)}=7380.$$

Assume $b=14$ in.

$$d^2=\frac{7380}{14}=527, \text{ or } d=23 \text{ in.}$$

We shall take $b=14$ in., and $d=23$ in.

$$v_o=\frac{V}{bd}=\frac{(5300)(5)}{(14)(23)}=82 \text{ lb. per square inch. (Allowable 35.)}$$

Thus web reinforcement is needed. Area of cross-section, $bd=(14)(23)=$ 322 sq. in.

$$a_s=(322)(0.0077)=2.48 \text{ sq. in.}$$

We shall select ten 9/16-in. round rods $=2.485$ sq. in. If all the rods extended straight to end of beam, the maximum bond would be

$$u=\frac{(26{,}500)}{(1.77)(10)(7/8)(23)}=74.5 \text{ lb. per square inch.}$$

The spacing of the rods is shown in Fig. 56. Weight of designed beam per linear foot is

$$\frac{(26)(14)(150)}{(12)(12)}=379 \text{ lb., or } \frac{(26)(14)(145)}{(12)(12)}+(0.85)(10)=375 \text{ lb.}$$

Minimum distance from end of beam to edge of support is $\frac{(5280)(5)(1.5)}{(14)(450)}=$ 6.3 in., allowing 50 per cent for unequal distribution of reactions over bearing surface. A distance of 6 in. will be taken.

Reviewing the beam we have designed,

$$f_s=\frac{(5280)(10)(10)(12)}{(8)(2.485)(0.874)(23)}=15{,}800 \text{ lb. per square inch.}$$

$$f_c=\frac{(2)(15{,}800)(0.0077)}{0.378}=647 \text{ lb. per square inch.}$$

which is perfectly satisfactory.

Web reinforcement is unnecessary at a distance from support

$$x_1=\frac{10}{2}-\frac{(40)(14)(0.874)(23)}{5280}$$
$$=5-2.13=2.87 \text{ ft. or } 34.4 \text{ in.}$$

If inclined rods are employed, the total stress to be taken is represented by triangle ABC, Fig. 57. BC represents two-thirds of the horizontal shear at the support per 1 in. length of beam.

$$BC=\frac{2}{3}\cdot\frac{Vb}{bjd}=2/3\ \frac{(5278)\ (5)}{0.874\ (23)}=877 \text{ lb.}$$

$$r=(0.7)(AD)=(0.7)(34.4)=24.1 \text{ in.}$$

Hence total stress to be taken by the rods

$$=\frac{BC}{2}(r)=\frac{877}{2}(24.1)=10{,}600 \text{ lb.}$$

But the area of one rod multiplied by 16,000 gives its tensile value, or tensile value of one rod $=(0.2485)(16{,}000)=3980$ lb. Thus, only three rods are

required if they can be bent up at the proper points. The unit bond along the seven rods at end of beam will be but 33 per cent greater than the allowable, which seems reasonable for a beam with this number of rods bent up in two planes.

The rods may be bent up in the following order.

$$x_2 = \text{ or } < \frac{10}{2}\left(1-\sqrt{\frac{1}{10}}\right)(12)=41 \text{ in.}$$

$$x_2 = \text{ or } < \frac{10}{2}\left(1-\sqrt{\frac{3}{10}}\right)(12)= 27\ 1 \text{ in.}$$

Fig. 57 shows the construction necessary to locate the points where the rods may be bent. The drawing shows that the rods can be bent at the desired points to take all the diagonal tension and also that the longitudinal spacing

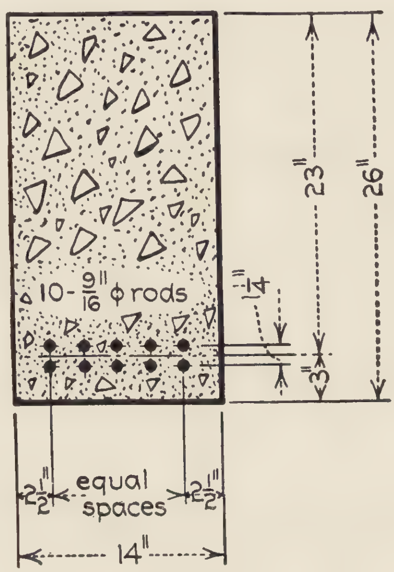


Fig. 56.

of these rods does not exceed three-fourths the depth of the beam. The length l' should equal nearly 50 diameters (from table) or

$$l' = (50)(9/16) = 28 \text{ in.}$$

This length cannot be obtained with the bars bent up nearest the support. These bars will be provided with a hook as shown. It would be good design to add one or two stirrups, as shown, to aid diagonal tension at the end of beam where the stresses have a greater inclination with the horizontal.

If stirrups were employed, their diameter and spacing would be determined by the method of Art. 42.

Where web reinforcement is used, it is a good plan to require the rods to be made up into frames and have them placed in the forms as such. If loose rods must be used, the reinforcing elements should be wired together to insure their being in place. A good plan is to have the main bars held in place by metal spacers.

Illustrative Problem.—Design a beam to span 15 ft. and to support the loads shown in Fig. 58. The reactions are readily found and are given in the sketch. The maximum moment occurs at the center load since the shear

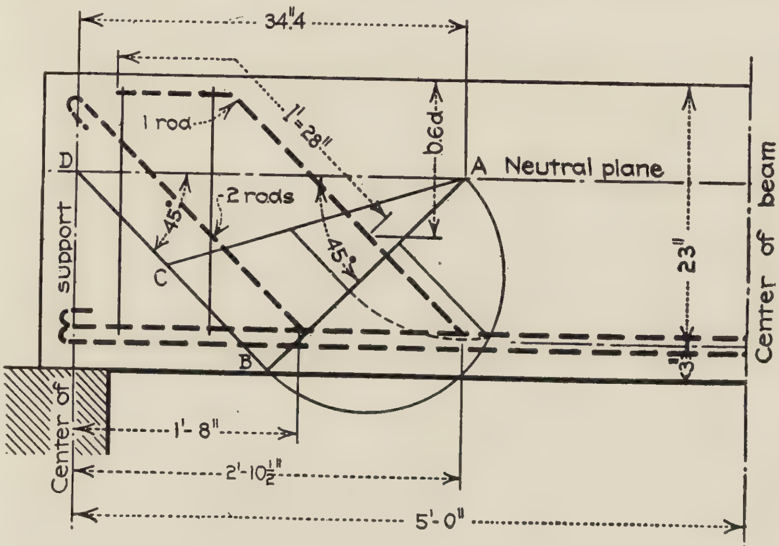


FIG. 57.

passes through the value zero at this point. We shall assume the weight of beam included in the uniform load of 1000 lb. per foot.

$$M = (25{,}500)(8)(12) - (23{,}000)(4)(12) = 1{,}344{,}000 \text{ in. lb.}$$

$$bd^2 = \frac{1{,}344{,}000}{(0.0077)(16{,}000)(0.874)} = 12{,}500.$$

Assume $b = 16$ in.

$$d^2 = \frac{12500}{16} = 782, \text{ or } d = 28 \text{ in.}$$

We shall take $b = 16$ in., and $d = 28$ in.

$$v_o = \frac{V}{bd} = \frac{25{,}500}{(16)(28)} = 57 \text{ lb. per square inch.}$$

Thus web reinforcement is needed.

$$a_s = (16)(28)(0.0077) = 3.45 \text{ sq. in.}$$

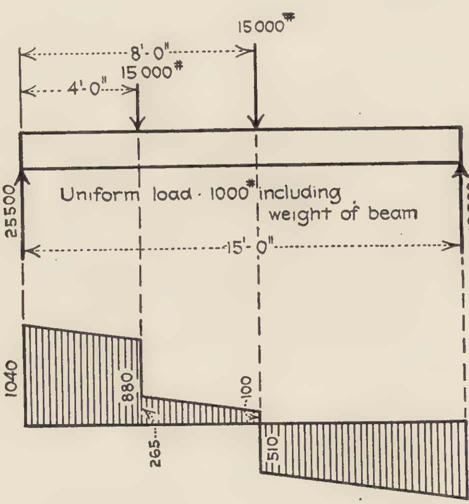
We shall select eight 3/4-in. round rods = 3.53 sq. in. Bond for one rod at the left end of beam is

$$u = \frac{25,500}{(2.356)(7/8)(28)} = 443 \text{ lb. per square inch.}$$

For plain rods, the number which must extend straight to the left end of beam is

$$\frac{443}{(1.5)(80)} = 4. \qquad \text{(See page 73.)}$$

Thus, at this end of beam four rods may be bent up. In a similar manner it will be found that four rods may be bent up at the right end.



FIGS. 58, 59.

The concrete will be found to take care of any diagonal tension between the concentrated loads. Horizontal shear (which measures diagonal tension) at the support is

$$\frac{V}{jd} = \frac{25,500}{(7/8)(28)} = 1040 \text{ lb. per linear inch.}$$

and to the left of the adjacent concentrated load, it is

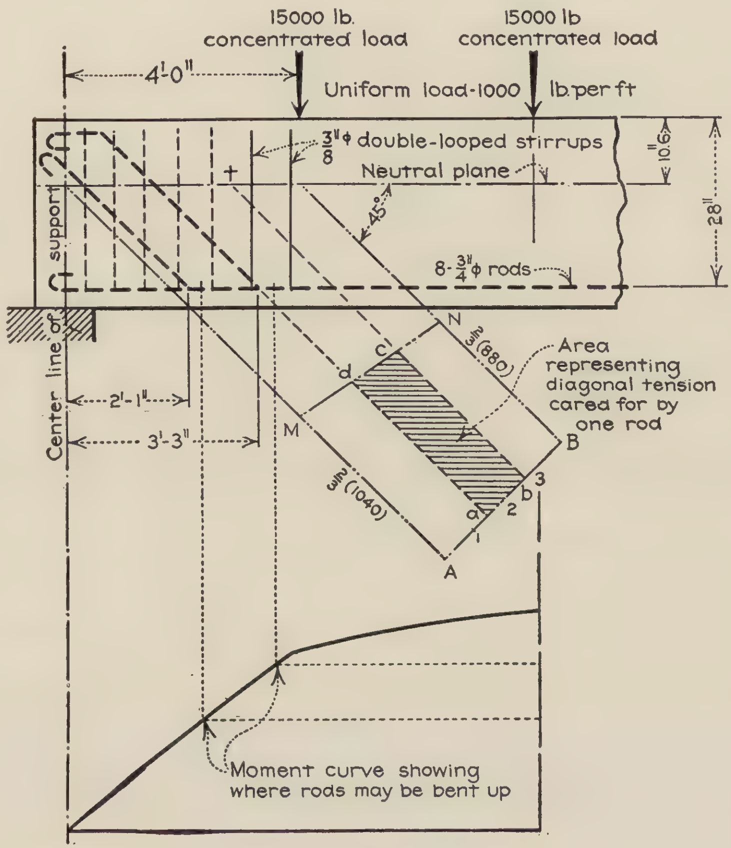
$$\frac{21,500}{(7/8)(28)} = 880 \text{ lb. per linear inch.}$$

The total diagonal tension is represented by a trapezoid (Fig. 59), the parallel sides of which are 1040 lb. and 880 lb. and the length 4 ft. Hence,

total stress in this part of beam to be taken by the concrete and by the web reinforcement is

$$\frac{1040+880}{2}\times 4\times 12=46{,}080 \text{ lb.}$$

One-third of the shear, or 15,360 lb., is assumed to be taken by the concrete, hence the amount to be taken by the web reinforcement is 30,720 lb.



FIGS. 60, 61.

Since four rods are to be bent, their comparative tensile value is

$$\frac{(4)(0.4418)(16{,}000)}{0.7}=40{,}400 \text{ lb.}$$

Thus, the tensile value of the rods is in excess of the stress to be provided for.

Now shear is nearly uniform between the support and the adjacent concentrated load and, as regards diagonal tension, it would be sufficiently

accurate to give nearly equal spacing to the inclined rods—closer spacing, however, being favored toward the support. The four rods will be bent up two at a time. To investigate for diagonal tension, the line AB, Fig. 60, should be divided into four equal spaces and the position of two of the rods determined by a line projected up at an inclination of 45 degrees with the horizontal, and beginning at about the point marked 1. The place to bend up the other two rods should be determined by a parallel line passing practically through the point 3. The points to bend up the rods between the right support and the adjacent concentrated load, to provide for diagonal tension, is determined in a similar manner to the above.

An investigation must now be made to see whether the tensile stresses in the bottom of the beam will permit the bending of the rods as required for diagonal tension. Fig. 61 shows the bending moment curve plotted to scale, and the points where the rods may be bent up are determined by the method described in Art. 45. It is clear that the rods cannot be bent up as desired to provide tho oughly for diagonal tension. The points where the rods are actually bent up are about 2 in. beyond the theoretical points as determined by moment.

The area $ABNM$, Fig. 60, represents the amount of diagonal tension to be taken by inclined rods. We have just found, however, that the rods cannot be bent up at just the proper points to take all of this. If each bent-up rod is assumed to take diagonal tension to the amount of one-half its tensile value on each side of itself, then the area $bBNc$ remains unprovided for. Stirrups will be provided to take diagonal tension between the point t, where the line bc produced meets the neutral line, and the adjacent load. Only two stirrups are required, but it would seem advisable in a design of this kind to also place stirrups at the positions indicated by the dotted lines. The spacing of the stirrups is determined by the formula

$$s=\frac{3a_sf_sjd}{2V}$$

and the student should be familiar with the method of procedure. The length of embedment of the inclined rods should be 50 diameters, or 37 1/2 in.

In the above problem the web reinforcement could be designed to take considerably less than 2/3 the diagonal tension with safety. The bent-up rods would not change but the number of stirrups theoretically required might be reduced. In a problem of this kind is found the maximum deviation from the assumption of Art. 41 pertaining to the coefficient 2/3 in the formulas for the design of web reinforcement due, of course, to the trapezoidal shape of the shear diagram.

51. Deflection of Beams.—Very little has been accomplished with respect to the determination of formulas for the deflection of reinforced concrete beams. The difficulty which has been experienced has been due to the fact that the beam action is complicated as far as it pertains to deflection. During the early loading, the concrete assists the steel in taking tension and the

deflection is quite uniform during this period. But as the concrete fails in tension near the center of the beam, there occurs a second or readjusting stage: the steel carries more and more of the tensile stresses and the deflection diagram is a curve. From this point on till the steel reaches its yield point, the deflection is again quite uniform. Since deflection depends on the stress at all sections, the deflection formulas for homogeneous beams cannot be used for reinforced concrete without modification because of the variable action of the concrete in tension throughout the length of the beam.

Fig. 62 gives the general form of a deflection diagram for a reinforced concrete beam. The portion AB shows the deflection before the concrete has begun to fail in tension, BC shows the deflection during the readjusting stage, and CD the deflection with the steel near the center of beam carrying practically all the tension.

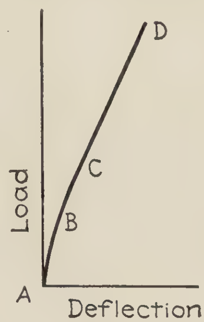


FIG. 62.

The deflection formulas presented in "Principles of Reinforced Concrete Construction" by Turneaure and Maurer yield results in fair agreement with actual measured deflections and undoubtedly are the best so far proposed. These formulas have been obtained semi-rationally from the deflection formulas for homogeneous beams and thus assume the material of the beam to obey Hooke's law (stress is proportional to deformation). Concrete in compression obeys the law very closely up to working stresses, but for concrete in tension the assumption is far from the actual conditions. Since tension in the concrete is considered in the above mentioned deflection formulas, the assumption of a linear stress-deformation relation, which is made for simplicity, should be regarded as a rough approximation.

The modulus of elasticity of the concrete in the type of formulas mentioned above should be taken as about the average or secant modulus up to the working compressive stress in the same. Although initial moduli of concrete for compression and tension are nearly equal, the deflection of a beam depends on the elongations and shortenings of *all* the fibers, and hence not upon the initial modulus but on some sort of a mean value. The resulting value of n should also be governed by the value as chosen from

experiments on actual deflection, so that this term becomes to some extent a sort of empirical coefficient-making correction for various errors in the deduction of the deflection formulas.

Turneaure and Maurer[1] recommend that 8 to 10 be used for n in the formulas which they have derived, and which are given below. They also state that the formulas presented are the result of modifying the deflection formulas for homogeneous beams in accordance with the following assumptions:

1. The representative or mean section has a depth equal to the distance from the top of the beam to the center of the steel.
2. It sustains tension as well as compression, both following the linear law.
3. The proper mean modulus of elasticity of the concrete equals the average or secant modulus up to the working compressive stress.
4. The allowance for steel in computing the moment of inertia of the mean section should be based on the amount of steel in the mid-sections, since stirrups and bent-up rods do not affect stiffness materially for working loads.

The following are the deflection formulas for rectangular reinforced concrete beams:

$$D=\frac{c_1}{E_s}\cdot\ \frac{Wl^3}{bd^3}\cdot\ \frac{n}{\alpha} \tag{1}$$

$$\alpha=1/3[k^3+(1-k)^3+3np(1-k)^2] \tag{2}$$

$$k=\frac{1+2np}{2+2np} \tag{3}$$

From equations (2) and (3), the value of α for any values of p and n may be computed, and then the deflection from equation (1). The notation employed in the above formulas is as follows:

D = maximum deflection (if desired in inches, the units specified below should be used).

b = breadth of the beam (inches).

d = depth of the beam to the center of the steel (inches).

W = total load (pounds).

l = span (inches).

p = steel ratio.

E_s = modulus of elasticity of the reinforcing steel (pounds per square inch).

n = ratio of the moduli of elasticity of steel and concrete.

[1] In Turneaure and Maurer's "Principles of Reinforced Concrete Construction," 2nd edition, pages 116 to 123.

α = a numerical coefficient depending on p and n.

k = proportionate depth of the neutral axis.

c_1 = the numerical coefficient in the formula for deflection of homogeneous beams, $c_1\frac{Wl^3}{EI}$, depending on the loading and support. For example,

for a cantilever loaded at the end, $c_1=\frac{1}{3}$

for a cantilever uniformly loaded, $c_1=\frac{1}{8}$

for a simple beam loaded at center, $c_1=\frac{1}{48}$

for a simple beam uniformly loaded, $c_1=\frac{5}{384}$

for a beam with fixed ends, load at the center, $c_1=\frac{1}{192}$

for a beam with fixed ends, uniformly loaded, $c_1=\frac{1}{384}$

The following are the deflection formulas for reinforced concrete T-beams (referred to later):

$$D=\frac{c_1}{E_s}\cdot\frac{Wl^3}{bd^3}\cdot\frac{n}{\beta}$$

$$\beta=1/3[k^3-\left(1-\frac{b'}{b}\right)\left(k-\frac{t}{d}\right)^3+\frac{b'}{b}(1-k)^3+3pn(1-k)^2]$$

$$k=\frac{np+1/2\left[\frac{b'}{b}-\frac{b'}{b}\left(\frac{t}{d}\right)^2+\left(\frac{t}{d}\right)^2\right]}{np+\frac{b'}{b}-\frac{b'}{b}\left(\frac{t}{d}\right)+\frac{t}{d}}$$

in which β is a coefficient depending upon the steel ratio and n, and other symbols as before.

Illustrative Problem.—A rectangular reinforced concrete beam with $b=18$ in. and $d=27\ 1/2$ in. has a span of 40 ft. and is reinforced with four 1 1/8 in. round rods (and stirrups) extending along the whole length. What is its probable deflection when sustaining a uniform load of 600 lb. per foot,

including its own weight, if the beam is considered as simply supported? A value of 8 will be taken for n.

$$p=\frac{a_s}{bd}=\frac{3.98}{495}=0.0080$$

$$k=\frac{1+2np}{2+2np}=\frac{1+(2)(8)(0.008)}{2+(2)(8)(0.008)}=0.530$$

$$\alpha=0.098$$

$$D=\frac{5}{(384)(30{,}000{,}000)}\cdot\frac{(24{,}000)(480)^3}{(18)(27.5)^3}\cdot\frac{8}{0.098}=0.25 \text{ in.}$$

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52. Economical Proportions.—In the designing of a reinforced concrete beam, the expression bd^2 appears. If one of the dimensions represented in this expression is assumed, then the other is also determined. Now the cost of a beam to resist a given bending moment will vary with the proportions adopted for breadth and depth, and it is useful to investigate this variation in cost for different conditions.

Without taking the matter of shear or web reinforcement into consideration, it is found from a mathematical study along this line that the cost of a rectangular reinforced concrete beam to support a given moment varies inversely with the depth, directly with the square root of the breadth, and also directly with the cube root of the ratio of breadth to depth. The depth will, however, be limited in many ways. It may be limited by the shearing stress fixing the value of bd, or it may be limited by the head room required, or it may need to be so chosen as to give a beam of satisfactory proportions. In the case of floor slabs, the breadth (and likewise the depth) becomes fixed, as the breadth of beam to carry the load coming upon a strip one foot wide is also one foot.

In the case where the cross-section of beam is determined by shear, the maximum depth theoretically permissible is that for which bd is just large enough to carry the shear. With a beam designed for moment alone, the cost decreases as the depth increases, but the area of the cross-section becomes less. A point must be reached when the beam will be of just the required strength in moment and shear. (Art. 48.) The question which now arises is whether or not a still greater depth will result in greater economy.

The quantity bd must now remain constant for the greater depths. But bd^2, on the other hand, is increased and the concrete stress (f_c) decreased. A smaller value for f_c permits the

use of a smaller percentage of steel, and the cost is still further reduced. Thus it should be clear that the proportions of a beam will not be determined by shear excepting as to minimum cross-section—an increase in depth always resulting in a gain in economy. It should be noted in this connection, however, that although deep beams are economical of concrete, the wooden forms cost more than they do for shallow beams.

53. Restrained Beams.—The discussion thus far in the course has related mainly to rectangular reinforced concrete beams, reinforced for tension and shear, and simply supported. It must be known by the student, however, that the ends of a reinforced concrete beam are often fixed, as in all-concrete construction. Such beams sometimes span only one opening, but more often they are continuous over several supports. It is sometimes difficult to provide a sufficient amount of restraint at the ends of such beams to be able to consider the ends entirely fixed, and good judgment should be used in the design for all such cases. Restrained beams of one span only will be considered under this heading, leaving the treatment of continuous beams, both fixed and supported ends, for a later discussion.

A smaller beam may be used when the beam is considered restrained than when considered simply supported, so that a beam computed as simply supported is always on the safe side. For example, the moment at the center of a restrained beam of one span, with both ends fixed, due to a uniform load over the entire beam, is $1/24\ wl^2$, where w is the load per linear foot of beam (maximum shear $0.5\ wl$). Also, a restrained beam with one end fixed and the other free, and under the same conditions as above, has a moment of $9/128\ wl^2$ (maximum shear $5/8\ wl$). A simply supported beam, on the other hand, has the much greater moment at the center of $1/8\ wl^2$.

Before one can appreciate the design of beams in all-concrete construction, it will be necessary to have a clear understanding of what is meant by a restrained beam, and the amount of restraint which is necessary to make the beam fixed. A restrained beam may be defined as a beam fastened at one or both ends in such a manner that the beam is not free to deflect at these points. A beam cannot be considered entirely fixed at either point, however, unless the restraint is sufficient to cause the neutral surface at that point to be horizontal.

An example of a beam completely fixed at the ends is shown

in Fig. 63. The fixing at the ends is effected in this case by building the beam for some distance into the wall. The same result, as far as the effect on the beam is concerned, might be effected as follows: having merely supported the beam and placed upon it the loads it has to bear, load the ends outside of the supports just enough to make the tangents at the supports horizontal. In reinforced concrete construction the designer must use his judgment of how near any particular case approaches the case above mentioned, the bending moment formulas for fixed beams being based upon the direction of the neutral surface as just described.

With restrained beams, a negative bending moment occurs over a fixed end; that is, at such a point the upper surface of the beam is in tension, and the bottom is in compression. The negative moment at each support, for a beam uniformly loaded

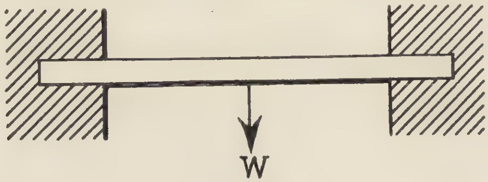

Fig. 63.

and fixed at both ends, is $-1/12\ wl^2$. For one end free, the negative moment at the fixed end is $-1/8wl^2$. If a beam in all-concrete construction runs into a column or heavy wall girder, the end in question is practically fixed. Top reinforcement will be required for the negative moment, and rods running into the support must be bent or otherwise anchored.

54. Continuous Beams.—The bending moments and shears on beams which are continuous over one or more intermediate supports cannot be derived from the principles of statics, since the unknown conditions in the case of the reactions are greater in number than the statical conditions of equilibrium. To find the reactions for continuous beams, an additional condition is to be introduced from the theory of elasticity, by means of the equation of the elastic curve. After the reactions are computed, the shears and moments due to given loads are readily determined, either analytically or graphically. Only the results of such study will be stated here, since the study itself is not important in this connection.

In general practice, such as building construction, the live

loading is usually indefinite, and is generally considered uniformly distributed except where the exact conditions of panel loading or concentrated loads are known in advance. Fig. 64 shows the variation in shear and moment along a continuous beam due to a uniformly distributed load. The diagram shows that if a beam is built continuously, pull or tension is bound to occur over each intermediate support, with compression at these points at the bottom of the beam. This is shown by the bending moment

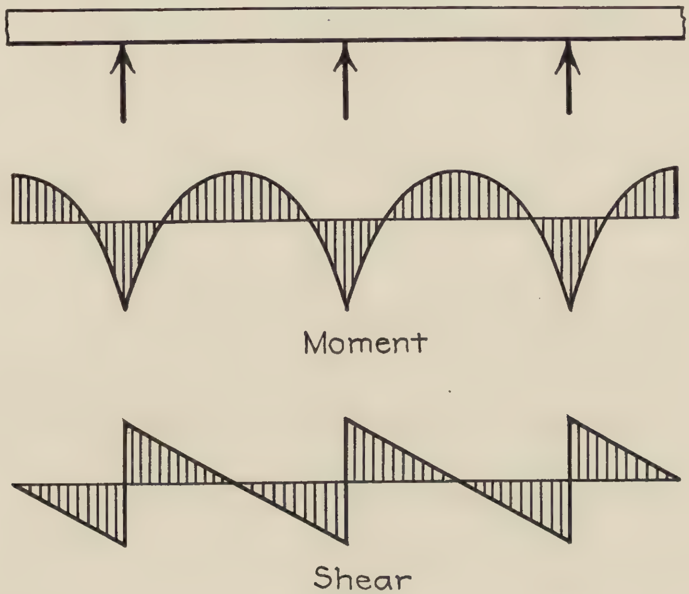

FIG. 64.

being negative at these supports. The same is also true at the ends of such beams when fixed.

Many of the foremost authorities have reached the conclusion that for a continuous beam of any number of spans, the maximum positive moment in the middle of all but the end spans and the negative bending moment over the supports (with the exception of the second support from each end) may be taken as numerically equal to each other and represented by the formula

$$M=\frac{wl^2}{12}$$

The center of the end span and the adjoining support between this and the next span should be designed for a moment

$$M=\frac{wl^2}{10}$$

A special case, however, occurs with continuous beams of two spans. Such beams should be designed for a positive bending moment of $\frac{wl^2}{10}$ in the middle of each span and a negative moment of $\frac{wl^2}{8}$ over the center support.

The shear at each support of continuous beams with fixed ends may be taken at one-half the span load. If the ends are simply supported the shear in the end spans near the second support will be approximately six-tenths of a span load.

Many of the building laws in the United States, to provide for the possibility of poor construction and other conditions, give the more conservative figure $\frac{wl^2}{10}$ to be used throughout. It must be understood that when $\frac{wl^2}{12}$ is used, it is absolutely necessary that the beam be really continuous both in design and construction.

In applying the values for bending moment given above to the various cases in all-concrete construction, the assumption is made that the moment of inertia of the beam is constant throughout its length. While this is not strictly true, extensive studies of various cases in reinforced concrete show that a large change in the moment of inertia makes a very small change in the bending moment, so that the relations are substantially correct.

Some engineers consider that the use of more steel is necessary between supports than the amount just sufficient to resist the bending moment $\frac{wl^2}{12}$. They consider that by figuring for a moment of $\frac{wl^2}{8}$ at the center of span, the stresses over the supports are relieved, and the design is more economical. It is true that the negative bending moment over the support decreases with the increase of steel at the center of span, but it is also true that the consequent decrease of steel over the supports does not offset the increase in steel needed throughout the beam.

Mr. Sanford E. Thompson in an article published in the issue of *Engineering News* of January 13, 1910, under the title "Continuity in Reinforced Concrete Beams," shows that a beam designed for a moment $\frac{wl^2}{8}$ in the middle of span requires 25

per cent more steel than the beam designed with $\frac{wl^2}{12}$, for both positive and negative moments.

Building laws, however, often require the beams and girders to be figured as simply supported. In such cases to obtain a safe structure and at the same time one that is as economical as possible under the conditions, the beam over the supports should have about six-tenths of the amount of steel that is employed in the middle of the beam.

The spans of a continuous beam are generally taken as the distance between the centers of supports. This is the simplest plan to follow in the majority of cases and is always on the side of safety. If the support is exceptionally wide, an arbitrary length of span may be taken which should not be less than the net span between supports plus the total depth of the member which is being designed.

For occasional concentrated loads which act in connection with uniform live and dead loads, and for loads produced by beams running into girders, the maximum moment may be computed with sufficient accuracy by considering the beam or girder simply

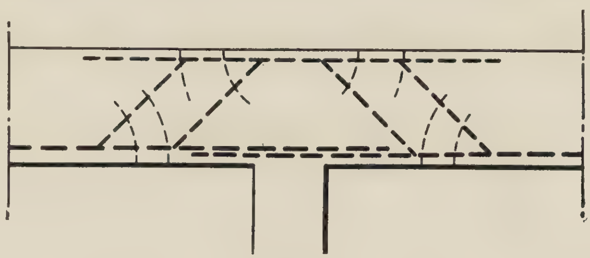

FIG. 65.

supported, and then reducing this maximum moment by the same ratio used in the distributed loading. For example, suppose the maximum moment due to given concentrated loads is K (considering the beam supported), then if $1/12\ wl^2$ is used in distributed loading instead of $1/8\ wl^2$ required for the supported beam, $8/12$ of K, or $2/3K$, may be used for the concentrated loads. The negative bending moment with concentrated loading may be taken the same as the maximum positive moment due to concentrated loading, reduced as already explained. If the principal live loads on a beam are concentrated as, for example, upon a girder bridge, the moments and shears at all points must

be specially computed. We will not consider such loading for the present.

The negative moment at the supports of continuous beams is provided for by bending up a sufficient amount of horizontal steel in the beams on each side of the support, and carrying it across over the support to about the third point of the span. The rods that are bent up for negative moment can be figured to take diagonal tension.

The moment at the supports being negative, the tendency is for diagonal cracks to start at the top while farther along the cracks tend to start at the bottom, as shown in Fig. 65. Stirrups at points of negative moment should loop about the upper bars, and at points of positive moment should loop about the lower bars. The student should satisfy himself with regard to the direction of the diagonal stress lines in such beams.

Problems

Make the best design of beam possible for the conditions given below. Use $f_c=600$, $f_s=16,000$, and the allowable unit stresses for shear and bond as recommended by the Joint Committee. Beams are to be considered as simply supported. $p=0.00675$. $k=0.360$. $j=0.880$.

40. Span 30 ft., load 800 lb. per ft. (including wt. of beam). Take $b=16$ in. Use 6 plain round rods. Determine also the maximum deflection.
41. Span 20 ft., load 1800 lb. per ft. (including wt. of beam). Take $b=16$ in. Use 6 plain round rods.
42. Span 15 ft., load 3200 lb. per ft. (including wt. of beam). Take $b=16$ in. Use 7 plain square rods.
43. Span 12 ft., load 5000 lb. per ft. (including wt. of beam). Take $b=16$ in. Use 10 plain round rods in two rows.
44. Span 10 ft., load 8000 lb. per ft. (*not* including wt. of beam). Take $b=16$ in. Use 14 plain round rods and bend up six.
45. Change the concentrated loads shown in Fig. 58 to 25,000 lb. each, and design a beam for the conditions shown, but with the allowable unit stresses as specified above. Consider the concrete to take its allowable value of shear. Take $b=20$ in. Include weight of beam in given loading.

Note.—In Problem 44 given above and in all slab and beam problems which follow in this course, the student is required to make his designs so that the assumed weight of beam checks within 10 per cent of the actual weight. All designs must be submitted neatly drawn in ink with the accompanying computations.

CHAPTER V

SLABS, CROSS-BEAMS, AND GIRDERS

55. Slabs.—Only the style of slab occurring in all-concrete construction will be considered at this time, in order to bring forward the treatment of T-beams clearly. Fig. 66 shows a finished floor slab in this type of construction in a building. A somewhat similar construction will be found in beam and girder bridges. The beams, girders, and slabs are continuous. The

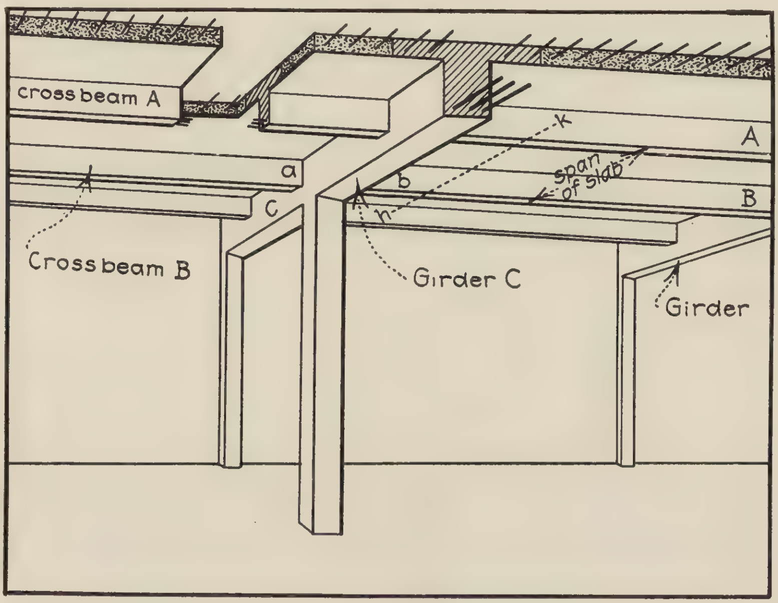

Fig. 66.

girders form intermediate supports for the beams, and the columns for the girders. The beam *BB*, for example, is supported by the girder *CC*, between the points *a* and *b*. Fig. 67 shows a cross-section along the line *hk*, Fig. 66. The slab constitutes part of the beams and girders, being built simultaneously with them. However, for a slab to be considered a part of a beam in

figuring stresses, the beam and slab must be well tied together by the steel reinforcement.

With cross-beam and girder construction, the span of the slab will usually range from 4 to 6 ft. In such short spans, the slabs become rigidly supported if they are well bonded to the beams, by reason of the action of the adjoining floor-panels. Even where the slabs are simply supported on the tops of steel beams, the adjoining slabs prevent any lateral motion and render the slab partially continuous. With supports in monolithic construction, therefore, the strengthening effect is especially great and reinforcement against negative moment is hardly necessary. For such short spans, a reinforcement of rods near the bottom will be effective. They should be laid with lapped and broken joints to give continuity and to prevent the localization of contraction cracks. (Fig. 67.)

Fig. 67.

In the case of spans longer than 5 or 6 ft. it becomes necessary to reinforce against negative moment. This may be done in the same manner as already suggested—by bending up a part of the rods and extending the bent ends beyond the beam. The bend in the bars should be near the 1/4 points in the span, and usually at an angle of 30 degrees with the horizontal. Too sharp an angle may tend to crack the slab. Fig. 68 illustrates two arrangements of rods. For clearness the rods are shown one above the other but, with the exception of the bends, they are actually in the same horizontal plane at the different points and are spaced alternately. Bars may extend over several spans if the spans are short, but in such cases they should be arranged to break joints so as to accomplish the same results as shown in Fig. 68.

Fig. 69 is another arrangement to accomplish the same object, using separate straight rods. Either arrangement in Fig. 68 is preferable to that shown in Fig. 69 for very heavy loads, in order to provide against shearing failures. Shearing failures are not usually important in slabs, but in special cases of heavy loading the same care should be used as in the design of large beams.

When placing slab reinforcement in long spans, a top reinforcement at least to the third point will be desirable. In ordinary spans the steel should at least be lapped a sufficient distance over supports to provide adequate bond strength. The use of a webbing for reinforcing the slab is to be preferred, since the spacing of the bars is not likely to be deranged, as occurs when separate rods are used.

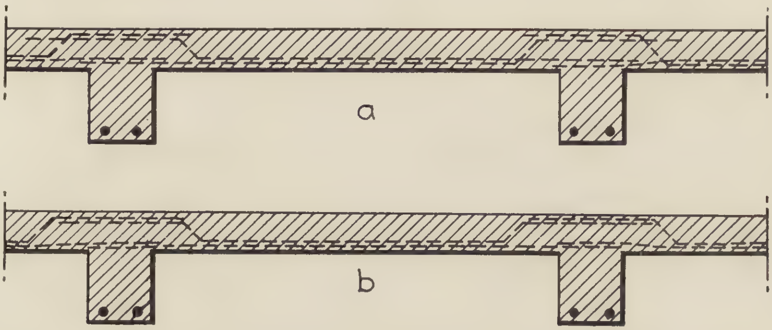

Fig. 68.

A slab should be figured in the same manner as a rectangular beam, and the depth and percentage of steel are therefore obtained by the formulas given in Art. 33—the bending moment being figured for a width of slab equal to one foot. For short uniformly loaded spans, fully continuous over two or more intermediate supports, a moment of $1/12\ wl^2$ may be used both in the centers of all spans and also over all supports, for both

Fig. 69.

dead and live loads. For slabs of long span, and for all slabs over two bays only, the moments should be taken as for continuous beams. (Art. 54.) The ratio of steel in a slab is most readily found by dividing the cross-section of one bar by the area between two bars, this area being the spacing of the bars multiplied by the depth of steel below the top of slab.

Slab bars should not be spaced too far apart to properly take stress directly nor yet should they be spaced so close that

the concrete cannot properly be placed between them. The reinforced concrete regulations of New York City adopted Dec. 28, 1911, require that the reinforcement shall not be spaced farther apart than two and one-half times the thickness of the slab. The minimum limit should be about the same as in beams.

When a floor panel is square, or nearly so, the slab may advantageously be reinforced in both directions. Exact analysis of stresses in such a case is impossible, but some important facts have been brought out by approximate solutions for uniform loading. The theory[1] applied in such an analysis depends upon the fact that the load at any point on the slab is distributed to the two systems of reinforcing bars at that point, in proportion to the stiffness of the beam elements lying in those directions. The

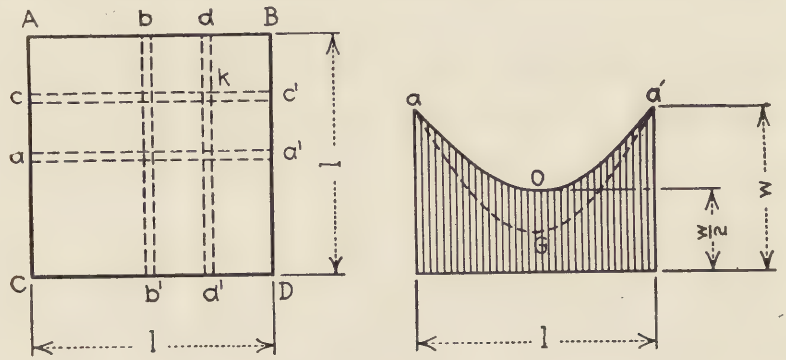

Fig. 70.

distribution of load on each system of reinforcement of a square slab, either along aa' or bb', Fig. 70, has been found to approximate the curve of a parabola aOa'. The center bending moment along aa' or bb' will be found by this assumption to have a slightly greater value than a slab reinforced in only one direction and carrying one-half the uniform load. Also the distribution of load along cc' (or dd') is shown by the curve aGa', and when AB (or BD) is reached by an element parallel to aa' (or bb'), the load supported by the reinforcement becomes zero.

The assumption usually made is that one-half the load is carried uniformly by each system of reinforcement, and the rods consequently for such an assumption have an equal spacing throughout the slab. The center bending moment resulting from an analysis, as above described, is so near the center bending moment for uniform distribution of load to each system, that

[1] From Turneaure and Maurer's "Principles of Reinforced Concrete Construction," 2nd edition, page 309.

practically it is accurate enough to consider these bending moments identical. A greater economy, however, may be obtained from the spacing of rods than is generally obtained in practice. The spacing at the center—namely, at aa' or bb'—may be determined on the equally distributed basis but at points intermediate between the center and the edge, the rods might well be spaced so that the number per foot would vary from the required number at the center to zero at the edge, following the law of the parabola. Or, the spacing for the center half of the slab may be the same and then gradually reduce the number of rods per foot to the edge of the slab, using one-half as many rods for the remaining two quarters.

From a similar approximate analysis regarding the stresses in rectangular slabs of greater length than breadth and reinforced in both directions, it seems proper to vary the spacing of the reinforcement for each system as already described, provided the panels are not far from square. As a slab becomes oblong in form, however, the relative amount of load carried by the longitudinal system becomes rapidly less.

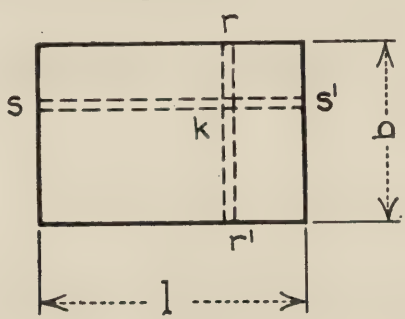

FIG. 71.

Consider uniform load over the slab represented in Fig. 71. It is required to find w_1 and w_2, the parts of the whole unit load w that is carried by the reinforcement parallel to the strips rr' and ss'. The deflections of these strips of unit width at k are the same, and are proportional to the fourth power of the length of the strip. Thus,

$$w_1b^4 = w_2l^4 \text{ and, since } w_1 + w_2 = w$$

$$\frac{w_1}{w-w_1} = \frac{l^4}{b^4}$$

or

$$\frac{w_1}{w} = \frac{l^4}{b^4+l^4}$$

which represents the proportion of load carried by the reinforcement parallel to the shorter axis. The proportion of the total load carried by the shorter system for various ratios of length (l) to breadth (b) is approximately as follows:

Ratio $\frac{l}{b}$	1	1.1	1.2	1.3	1.4	1.5	2.0
$\frac{w_1}{w}$	0.50	0.59	0.67	0.75	0.80	0.83	0.89

When the length of a floor panel is large compared to its breadth, the longitudinal reinforcement (that is, reinforcement parallel with the length) is of little value in carrying loads, but a small amount is nevertheless generally desirable in preventing shrinkage and temperature cracks and in binding the entire structure together. It is more important for wide beam spacing than when the beams are closely spaced. The amount of steel to use is usually selected somewhat arbitrarily, and 1/4-in. or 3/8-in. rods spaced 18 to 24 in. apart is common practice. The top of the slab over a girder should be reinforced transversely, not only for stiffening the girder, but also to provide for the negative bending moment produced with the bending of the slab at right angles to the direction of the principal slab steel.

56. Distribution of Slab Load to Cross-beams.—If a floor slab is reinforced in one direction only, the load will practically all be transmitted to the beams at right angles to the direction of the reinforcing rods. A small part, however, will be transferred directly to the girders at the sides of the panels, but this may well be neglected in the calculations for cross-beams. In fact, even with reinforcement in two directions, the load should be assumed as all transferred to the cross-beams unless the panel is nearly square.

If panels, nearly square, are reinforced in both directions, the loads carried to the cross-beams and girders will not be uniformly distributed over the length of such beams and girders, but may be assumed to vary in accordance with the ordinates of a triangle. This assumption is surely on the safe side in regard to moment, if the area of the triangle is made equal to that part of the total load on the panel which is transmitted to the beam in question—as determined by the table of the preceding

article. Assumptions of this load being either uniformly distributed, or varying as the ordinates of a parabola, give a lower resulting moment than the triangle method and for this reason are not so much in use.

Let w be the uniform load per unit of area on the slab, and w_2 and w_1 the parts of this unit load that go to the shorter and longer beams respectively. Applying the loads in the form of a triangle having its apex at the middle of the beam, the maximum moment will be for the longer beam, and, if this beam is considered simply supported and as carrying the load from one panel only,

$$M = w_1 \frac{bl}{4}\left(\frac{l}{2} - \frac{l}{6}\right)$$

$$= \frac{1}{12} w_1 b l^2.$$

and for the shorter beam $M = \frac{1}{12} w_2 b^2 l$

If the slab is square, w_1 is $\frac{w}{2}$, and $M = 1/24\ wl^3$. For beams made continuous, the bending moment may be multiplied by the proper coefficient as already described. (Art. 54.) With beams or girders common to two panels, the bending moment should be multiplied by 2.

Unless the panel is nearly square, floor slabs should not be reinforced in two directions, as it is evident that no economy results from double reinforcement when the ratio of length to breadth of panel is greater than about 1.2. If the length of the slab exceeds 1.5 times its breadth, the entire load should surely be carried by the transverse reinforcement.

57. Distribution of Beam and Slab Loads to Girders.—Fig. 66 shows cross-beams running into girders, with the one-way type of reinforcement. The load upon the girders in such construction consists of the concentrated live and dead loads from the cross-beams acting at their points of intersection with the girder, the uniformly distributed weight of the girder itself, and the unsymmetrically distributed weight of a small portion of the floor slab with its live load which bears directly upon the girder. (Fig. 72.) In order to avoid computing several moments, it has been found[1] that the maximum bending moment on a girder supporting cross-beams may be obtained without appreciable error by considering, as a uniformly distributed load, the weight of the stems of the girder and cross-beams plus the weight of the slab

[1] Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 432.

and its live load, for an area whose length is the length of the girder and whose width is the average length of the beams running from each side into the girder. The sum of these loads divided by the length of the girder gives a uniformly distributed load for which the ordinary formula for bending moment may be used.

Thus, in Fig. 72, instead of computing the moment on the girder as the sum of the moments produced by loads of the triangles *abc* plus the concentrated loads from the beams at *c* plus the weight of the girder, the entire live and dead load of the area *dddd* may be considered as uniformly distributed over the girder in the length *aa*.

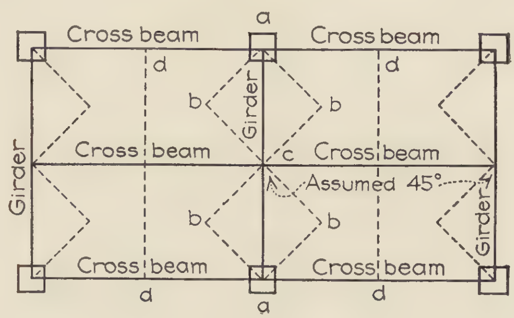

FIG. 72.

An exception occurs, however, in the case where two beams run into a girder at the one-third points. Here the maximum moment obtained by the uniformly distributed method gives slightly too conservative results, and may be reduced by 10 per cent. Moments in a girder other than the maximum must be computed for individual conditions.

58. Arrangement of Beams and Girders.—The spacing of columns and girders will be determined largely by architectural considerations and consequently the best spacing of cross-beams will vary for different cases. The designer must decide whether to omit all cross-beams, to insert them only at columns so as to form a square or nearly square panel, or to space them at much closer intervals using two or more to a girder panel, as shown in Fig. 66. For economy, slabs with square or nearly square panels should have both transverse and longitudinal reinforcement.

For the sake of lateral stiffness, it is generally desirable to place cross-beams at columns. In some cases, however, where such stiffness is not needed, the cross-beams may be entirely

omitted. The amount of concrete will be more, but the amount of steel required will be less and little saving, if any, is effected by using cross-beams when their use is in doubt. Heavy loads and low stresses, however, call for large weights of concrete, and cross-beams will undoubtedly be needed under such conditions, as the deeper a beam, the greater its moment of resistance for a given volume. The economical spacing of cross-beams will vary between 4 or 5 ft. to 10 or 12 ft. Architectural considerations will generally play an important part, but frequently designs must conform to building regulations relative to ratio of span to depth.

59. Design of Beams and Girders. *T-Beams.*—When a slab and beam (or girder) are built at the same time and thoroughly tied together, a part of the slab may be considered to act with the upper part of the beam in compression. Cross-beam *A* and a portion of the slab which may be considered to aid in resisting stress (Fig. 66) is shown cross-hatched; a like representation is given for girder *C*. This form of beam is called a T-beam, and the extra amount of concrete in the compressive part of such a beam makes possible a considerable saving over the rectangular form. The thickness of the flange is fixed by the thickness of slab required to support its load, but the width of slab which can be taken as effective flange width must be estimated.

This width should not be too great in proportion to the slab thickness, otherwise the shearing stresses on the vertical sections through the slab at the edge of beam will be excessive and greater than those on the horizontal section between stem and flange. Experiments show that it would be difficult to crush a flange as much as four times the width of web without failure taking place by the excessive shearing stresses in the web.

The arbitrary rules which have been adopted in the assumption of the flange width may be divided into two classes. The first class makes the flange width a factor of the width of stem or thickness of slab, or else of the distance between adjacent beams. The second class makes the flange width a factor of the length of span of the beam itself. The Joint Committee, however, combines the two and has recommended a width not exceeding one-fourth the span length of the beams and, in addition, limits the width to use on either side of the web to four times the thickness of the slab. In no case should the breadth of the flange be taken greater than the distance between beams.

The web and flange of a T-beam can be considered well tied together when slab reinforcement crosses the beam and when the web reinforcement extends well up into the slab. The bonding should be especially well looked after near the end of beam, and this is generally accomplished by means of the bent rods brought up as high as possible, in addition to the slab reinforcement (as mentioned) acting at right angles to the length of the beam. Along the center of the beam the differential stresses between the beam and slab are not large, but it is better to insert vertical stirrups extending up into the slab at occasional intervals since shrinkage of the concrete is apt to part the slab from the beam if there is not some means to hold the two together mechanically. The thinner the sections, the more thorough should be the bonding.

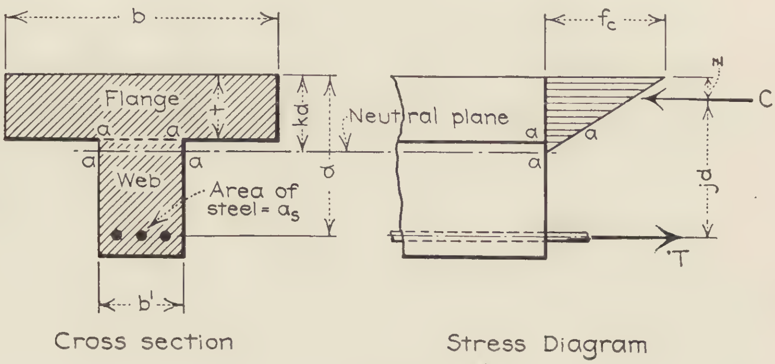

Fig. 73.

When frames are not used for beams and girders, successful results can be obtained by wiring the main rods together and employing U-shaped stirrups with hook ends to act as hangers by which to suspend the rods in the form. The length of the hook should be sufficient to permit the stirrup to rest on the top of the slab form. The hook has the further advantage of increasing the bond between the beam and slab.

With a T-beam it is necessary to distinguish two cases when applying formulas for design; namely, (1) the neutral axis in the flange, and (2) the neutral axis in the web.

Case I. The Neutral Axis in the Flange.—All formulas for "moment calculations" of rectangular beams apply to this case. It should be remembered, however, that b of the formulas de-

notes flange-width, not web-width, and p (the steel ratio) is $\frac{a_s}{bd}$, not $\frac{a_s}{b'd}$. (Fig. 73.)

Case II. The Neutral Axis in the Web.—The amount of compression in the web (*aaaa*, Fig. 73) is commonly small compared with that in the flange, and is generally neglected. The method of procedure in determining formulas for "moment calculations" of T-beams is identical with that for rectangular beams and the resulting equations only will be given.

The formulas to use, assuming a straight line variation of stress and neglecting the compression in the web, are:

$$k=\frac{1}{1+\frac{f_s}{nf_c}} \tag{1}$$

$$kd=\frac{2nda_s+bt^2}{2na_s+2bt} \tag{2}$$

$$k=\frac{pn+\frac{1}{2}\left(\frac{t}{d}\right)^2}{pn+\frac{t}{d}} \tag{3}$$

$$z=\frac{3kd-2t}{2kd-t}\cdot\frac{t}{3} \tag{4}$$

$$jd=d-z \tag{5}$$

$$j=\frac{6-6\left(\frac{t}{d}\right)+2\left(\frac{t}{d}\right)^2+\left(\frac{t}{d}\right)^3\left(\frac{1}{2pn}\right)}{6-3\frac{t}{d}} \tag{6}$$

$$\left.\begin{aligned} M_c&=f_c\left(1-\frac{t}{2kd}\right)bt.\ jd.\\ M_s&=f_sa_sjd\end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} f_s&=\frac{M}{a_sjd}\\ f_c&=\frac{f_sk}{n(1-k)}\end{aligned}\right\} \tag{8}$$

Approximate formulas can also be obtained. From the stress diagram, Fig. 73, it is clear that the arm of the resisting couple is never as small as $d-1/2t$, and that the average unit compressive stress is never as small as $1/2f_c$, except when the neutral axis is at

the top of the web. Using these limiting values as approximations for the true ones,

$$M_c = 1/2\, f_c bt(d - 1/2t) \qquad \text{(a)}$$

$$M_s = a_s f_s (d - 1/2t), \text{ or } a_s = \frac{M}{(f_s)(d - 1/2t)} \qquad \text{(b)}$$

The errors involved in these approximations are on the side of safety.

Since a T-beam will usually have ample strength in compression for any ordinary depth of beam likely to be selected, the design of the stem of the T, or the beam below the slab, is therefore largely a question of providing sufficient concrete to take care of the shearing stresses and to give a good layout of the tension rods. The manner of providing reinforcement for shearing stresses in T-beams is similar to that already described for rectangular beams. In T-beams, however, the reinforcement for shear should run well up into the slab in order to tie the beam and slab together. Results from tests for both shear and moment in T-beams agree quite closely with theoretical results. It also appears that the shearing strength of a T-beam is about the same as that of a rectangular beam of the same depth and a width equal to the width of the stem of the T.

All re-entrant angles in concrete are points of weakness and such angles should, therefore, be avoided. Also, T-beams should not be made too deep in proportion to the width of stem as such forms are relatively weak at the junction of stem and flange. The width should preferably be from one-third to one-half the depth in ordinary cases. For large beams the width may be made one-third to one-fourth the depth.

Deflection formulas for T-beams are given in Art. 51.

In the following problems and also in those problems to be worked out by the student, the working stresses recommended by the Joint Committee for a 2000-lb. concrete will be assumed; namely,

$$f_c = 650, \quad f_s = 16{,}000, \quad n = 15.$$

In the illustrative problems following Art. 50 in which the above-mentioned allowable stresses were used, the following values were determined:

$$p = 0.0077, \quad k = 0.378, \quad j = 0.874.$$

It will be well to bear these results in mind when following through the computations in slab design which follow. The rule is followed that for a slab of 4 in. (total depth) and less, the depth is taken to the nearest 1/4 in. The depth of slabs over 4 in. are taken to the nearest 1/2 in.

Illustrative Problem.—What safe load per square foot (including dead weight) can be supported by a slab 6 in. deep ($d = 4\ 3/4$ in.) and 10 ft. span, reinforced with 1/2-in. round rods placed 8 in. apart? The slab is simply supported and reinforced in only one direction.

$$p = \frac{0.1963}{(8)(4.75)} = 0.0052$$

$$k = \sqrt{(2)(0.0052)(15) + (0.0052)^2(15)^2} - (0.0052)(15)$$
$$= 0.325$$

The percentage of steel is less than 0.0077 and, consequently, the resulting moment is determined by the steel.

$$M = 1/8wl^2(12) = pf_s(1 - 1/3k)bd^2$$
$$1/8(w)(100)(12) = (0.0052)(16{,}000)(0.892)(12)(4.75)^2$$
$$w = 134 \text{ lb. per square foot, safe load.}$$

Illustrative Problem.—Design a slab to span 6 ft. and to carry a live load of 250 lb. per square foot. Slab is to be fully continuous and reinforced in only one direction.

We shall assume the dead load at 50 lb. per square foot., making a total loading of 300 lb. per square foot.

$$M = \frac{wl^2}{12}, \text{ then } M = \frac{(300)(6)(6)(12)}{12} = 10{,}800 \text{ in. -lb.}$$

Also, $M = (0.0077)(16{,}000)(0.874)(12)(d^2) = 10{,}800$

or $d = 2.9$ in.—take 3 in.

Taking 3/4-in. concrete below steel, thickness of slab is 3 3/4 in.

Area of steel per foot of breadth $= (3.0)(12)(0.0077) = 0.277$ sq. in. We shall use 3/8-in. round rods for tensile reinforcement. Area of 3/8-in. round rod $= 0.1104$ sq. in. Required spacing $= \frac{0.110}{0.277}(12) = 4\ 3/4$ in. on centers.

$$\frac{(3.75)(12)}{(12)(12)}(150) = 47 \text{ lb. per square foot.}$$

Thus the assumed and calculated dead weights are close enough, and the slab need not be redesigned. The slab should be reinforced against negative moment at the supports. The slab should also be reinforced transversely in order to prevent shrinkage and temperature cracks. Shear at ends of slab in direction of reinforcement is $(300)(3) = 900$ lb. per foot of breadth. Allowable shear $= (12)(3)(40) = 1440$. Thus no web reinforcement is needed, as is usually the case except for excessive loading.

Illustrative Problem.—Design a slab for a 10 ft. by 10 ft. panel to carry a live load of 250 lb. per square foot. Slab is to be fully continuous and reinforced in both directions. The dead load will be assumed at 60 lb. per square foot.

For a strip one foot wide at the center,

$$M=\frac{wl^2}{24}, \text{ then } M=15{,}500 \text{ in. -lb.}$$

Also, $M=(0.0077)(16{,}000)(0.874)(12)(d^2)=15{,}500.$

or $d=3.5$ in.

Area of steel (at the center) per foot of breadth in each direction = (3.5)(12)(0.0077) = 0.323 sq. in. We shall use 3/8-in. round rods for tensile reinforcement. Area of 3/8-in. round rod = 0.1104 sq. in. The required spacing for center half of slab $=\frac{0.110}{0.323}(12)=4$ in. on centers.

The spacing of rods should be gradually increased to the edge of the slab, using one-half as many rods for the remaining two-quarters. The slab should be reinforced against negative moment at the supports.

The depth of the slab should be made 5 in. in order to have the upper reinforcement system at the minimum distance 3 1/2 in. from the surface of the slab. The lower system will then be slightly stronger than necessary. The dead weight of slab is 62 1/2 lb. per square foot, or approximately that assumed. For safety in construction, it is preferable to require the two systems of reinforcement to be fastened together at frequent intervals. Web reinforcement is not necessary.

Illustrative Problem.—A floor panel is to be 12 ft. by 13.2 ft. and the slab is to be fully continuous and reinforced in both directions. Design such a slab to carry a live load of 250 lb. per square foot.

We shall assume the dead load at 75 lb. per square foot making a total load of 325 lb. per square foot.

$$\frac{l}{b}=\frac{13.2}{12}=1.1$$

From table, Art. 55, the 12-ft. span reinforcement should be designed to carry 0.59 w per foot, leaving 0.41 w to be carried by the longitudinal reinforcement. The resisting moment and consequently the depth of the slab is always determined by the shorter span.

$$M=\frac{(0.59)(325)(12)(12)(12)}{12}=27{,}600$$

Also, $M=(0.0077)(16{,}000)(0.874)(12)(d^2)=27{,}600$

or $d=4.6$—say 4 3/4 in.

Taking 1 1/4-in. concrete below center of steel, the thickness of slab is 6 in.

Area of steel at the center per foot of breadth in direction of short length = (4.6)(12)(0.0077) = 0.425 sq. in. We shall use 1/2-in. round rods. Area of a 1/2-in. round rod = 0.1963 sq. in. Then, required spacing for center half of slab $=\frac{0.196}{0.425}(12)=5$ 1/2 in. on centers.

Let us see if the long span system of reinforcement may not be placed above the short span reinforcement and still carry its load with safety. ($d=4$ 1/4 in.) Using the same spacing and size of rods

$$p=\frac{a_s}{bd}=\frac{0.1963}{(4.25)(5.5)}=0.0084$$

$$k=\sqrt{(2)(0.0084)(15)+(0.0084)^2(15)^2}-(0.0084)(15)$$
$$=0.392$$

$$j=0.869$$

The percentage of steel is greater than 0.0077 and consequently the resulting moment is determined by the concrete.

$$M = 1/2(650)(0.392)(0.869)(12)(4.25)^2$$
$$= 24{,}000 \text{ in. -lb.}$$

It is assumed to carry a load of

$$\frac{(0.41)(325)(13.2)^2(12)}{12} = 23{,}200 \text{ in. -lb.}$$

Thus, this arrangement of the two reinforcing systems is satisfactory.

The assumed and calculated weights are identical and the slab need not be redesigned.

In practice it is always more convenient to use the same sized rods and to space them the same in both directions and, if the floor spans are nearly square, this should be done. In the construction of the floor slab, the spacing of the rods should vary as described for a square panel. It will be found that web reinforcement is not necessary.

Illustrative Problem.—Design the center cross-section of the 12-ft. supporting beam for the slab of the preceding problem. Assume a center span floor panel so that $M = \frac{wl^2}{12}$ may be used throughout. (Art. 54.) The beam receives its load from two floor panels.

The triangular load on the beam due to the live load plus dead weight of slab is equal to w_2bl (Art. 56) = (0.41)(325)(12)(13.2) = 21,100 lb. Assume dead weight of stem of beam at 110 lb. per linear foot. Then maximum shear occurs at either support and equals $\frac{21{,}000}{2} + (110)(6) = 12{,}200$ lb.

The only purpose of the concrete below the neutral axis is to bind together the tension and compression flanges, and consequently its section is determined by the shearing stresses involved and space for the necessary rods. The shearing stress v should not be greater than 120. The area $b'd$ (unless the value of j should turn out to be less than 7/8) should not be less than

$$\frac{12{,}200}{(7/8)(120)} = 116 \text{ sq. in.}$$

Some rough calculations show that if four rods are to be used and all in one row, the breadth of stem necessary for the rods controls. A breadth b' of 9 in. and a depth d of 16 in. (total depth 18 in.) will be tried.

The bending moment due to the live load plus weight of slab is given by the formula $M = 1/12\, w_2b^2l$ (Art. 57) for a simply supported beam the load from one panel only. For a beam fully continuous and carrying the load from two panels,

$$M = 1/12(0.41)(325)(144)(13.2)(2/3)(2)(12)$$
$$= 340{,}000 \text{ in.-lb.}$$

The moment due to the dead weight of stem (113 lb.) is 16,300 in.-lb. or a total maximum moment of 356,300 in.-lb.

The flange of the T-beam is 6 in. thick and its breadth is controlled by one-fourth the span, or 36 in. Using approximate formula (b) of Art. 59,

$$a_s = \frac{M}{(f_s)(d - 1/2t)} = \frac{356{,}300}{(16{,}000)(13.0)} = 1.71 \text{ sq. in.}$$

and $$p = \frac{1.71}{(36)(16.0)} = 0.0030.$$

Supposing this beam to fall under Case I, we find

$$k=\sqrt{2(0.0030)(15)+(0.0030)^2(15)^2}-(0.0030)(15)=0.258$$
$$j=0.914$$

Hence, $kd=(0.258)(16.0)=4.1$ in., and the neutral axis is in the flange; that is, the case was correctly guessed.

The corrected value of a_s, is

$$a_s=\frac{M}{(f_s)(jd)}=\frac{356{,}300}{(16{,}000)(0.914)(16.0)}=1.53 \text{ sq. in.}$$

and $$p=\frac{1.53}{(36)(16.0)}=0.0027$$

Hence, $$k=\sqrt{2(0.0027)(15)+(0.0027)^2(15)^2}-(0.0027)(15)=0.247$$

The stress in the concrete is then found to be

$$f_c=\frac{2f_sp}{k}=\frac{(2)(16{,}000)(0.0027)}{0.247}=350 \text{ lb. per square inch}$$

which is satisfactory. Four 3/4-in. round rods will give the required steel area.

It is quite evident in the above problem that the compressive stress in the concrete could not be a determining factor in the design, but the computations are all given to show the student the method to be followed in the design of T-beams of larger size.

Illustrative Problem.—The flange of a T-beam is 24 in. wide and 4 in. thick. The beam is to sustain a bending moment of 480,000 in.-lb. What depth of beam and amount of steel is necessary?

Try $d=18$ in. Approximately, $jd=16$ in. Then formula (8) or formula (b), Art. 59, gives:

$$a_s=\frac{M}{f_sjd}=\frac{480{,}000}{(16{,}000)(16)}=1.88 \text{ sq. in.}$$

and $$p=\frac{1.88}{(24)(18)}=0.0043$$

Then equation (6) gives $j=0.910$ and the corrected value of a_s is

$$\frac{480{,}000}{(16{,}000)(0.910)(18)}=1.83 \text{ sq. in.}$$

Then equation (2) gives

$$kd=5.61 \text{ in., or } k=0.312$$

and shows that the beam falls under Case II, as assumed.

The stress in the concrete is found from equation (8) to be

$$f_c=\frac{(16{,}000)(0.312)}{(15)(1-0.312)}=485 \text{ lb. per square inch, which is permissible.}$$

Problems

46. A floor slab is simply supported and reinforced in only one direction. The span in the direction of the reinforcement is 8 ft., $d=4$ in., and 1/2-in. round bars are used, placed 7 in. center to center. Determine the safe load per square foot including the weight of slab.
47. What thickness of slab and what sizes and spacing of bars will give a resistance of 4900 ft.-lb. per foot of width?

48. (a) Design a center span slab to carry a live load of 250 lb. per square foot. Slab is to span 8 ft. and is to be fully continuous, with the one-way type of reinforcement. (b) Design section of continuous cross-beams with span of 12 ft. Take $b' = 8$ in.
49. In a square panel, 14 ft. $\times$ 14 ft., of the two-way type of reinforcement, what thickness of slab and what size and spacing of tension rods will be required to carry a live load of 400 lb. per square foot; slab is to be in a center span and fully continuous?
50. A center span floor panel is to be 12 ft. $\times$ 14.4 ft., and the slab is to be fully continuous and reinforced in both directions. Design such a slab to carry a live load of 400 lb. per square foot.
51. Design the center cross-section of the 12-ft. supporting beams for the floor slab in the preceding problem. The beam receives its load from two floor panels. Consider the depth of beam (d) fixed at 18 in. (total depth 20 in.) and use four plain round rods.
52. The flange of a T-beam is 20 in. wide, and 5 in. thick. The beam is to sustain a bending moment of 550,000 in.-lb. What amount of steel is necessary? Take $d = 20$ in.

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60. Economical Proportions of T-Beams.—When a floor-slab forms the flange of a T-beam, it is possible to determine economical proportions for the stem.

Consider a portion of a rectangular beam one unit in length. Let c = cost of concrete per unit volume; r = ratio of cost of steel to cost of concrete per unit volume; C = cost of beam per unit length; d' = depth of beam below slab. Then

$$C = c\left[b'd' + \frac{rM}{f_s(d' + 1/2t)}\right]$$

using the approximate formula (b) of the preceding article.

When d' is fixed by the head room available, the cost will be a minimum when b' is made as small as possible, and its value will then be determined by the shearing stress or by the space required for the rods. The expression also shows that the cost will decrease with increased values of f_s, and that with a fixed value of $b'd'$ the cost decreases with increase in depth. If the value of b' is assumed as fixed, then there is a definite value of d which will give minimum cost. By calculus the following expression has been deduced from the preceding equation and will give the value of d for minimum cost when the value of b' is fixed:

$$d = \sqrt{\frac{rM}{f_s b'}} + \frac{t}{2}$$

From this expression the best depths for various assumed widths may readily be determined and the desirable proportions finally selected.

In the design of beams, and especially T-beams, it is important for the student to note that liberal spacing favors large rods and few in number, while good bond strength without waste of material favors small rods. Also, if bent rods are to be used for web reinforcement, then numerous small rods are also advantageous.

61. Conditions Met With in Design of T-beams.—In practice the design of T-beams will take one of the following forms: (1) The dimensions may be given, to find the safe resisting moment of the beam or the stresses in the steel and concrete under a given load; (2) the flange of the T-beam may form a portion of a floor slab which is already designed, in which case the dimensions of the flange are given—also the loading and specified working stresses—and the design comprises the determination of suitable web dimensions and steel area; (3) the loading and working stresses may be given, to determine suitable proportions for the entire beam—that is, the flange does not form a part of a floor system already determined.

(1) The values of k and j may be found from Eqs. (3) and (6), or from Eqs. (2), (4) and (5), of Art. 59, and then the values of the moment of resistance from Eqs. (7), or the fiber stresses from Eqs. (8). If the value of k is found to be less than $\frac{t}{d}$, then the problem falls under Case I and the formulas for rectangular beams apply.

(2) Depth and width of beam should be selected with reference to shearing strength, space for necessary rods, and other considerations. The depth having been selected, the amount of steel may be approximately determined by either Eq. (a) or Eq. (b). The amount of steel being known, the value of j may be determined by Eq. (7). The value of k should also be found from Eqs. (2) or (3) in order to ascertain if the beam falls under Case I or Case II. The stress in the concrete, corresponding to the allowable working stress in the steel, is then found from Eq. (8). (This method has been explained by illustrative problems following Art. 59.)

(3) First, select suitable proportions for the web. A flange thickness is then assumed such as to give satisfactory proportions

between t and d. (See Art. 59 for suitable proportions.) The value of $\frac{t}{d}$ is then known and k and j can be determined from Eqs. (1), (4), and (5). The area of steel and the breadth of flange are then found from Eq. (7).

(*Note.*—When making approximate computations for shear or bond stress along the horizontal tension rods, an average value of $j=7/8$ may be assumed, as for rectangular beams.)

Illustrative Problem.—A T-beam in a floor system is subjected to a maximum shear of 19,000 lb. and a maximum moment of 722,000 in.-lb. Determine the economical depth of beam. Use working stresses recommended by Joint Committee. Assume the ratio of unit cost of steel to cost of concrete = 70. Thickness of floor slab is 3 3/4 in.

Cross-section of web as determined by shear $b'd=\frac{19,000}{105}=181$ sq. in. From formula of Art. 60, for

$b'=8$ in. $d=21.7$ in.
$b'=9$ in. $d=20.6$ in.
$b'=10$ in. $d=19.7$ in.

If the object is to obtain suitable proportions, either of the last two breadths with corresponding depths may be selected but, for convenience in placing steel, the breadth of 10 in. and depth of 19.7 in. will most likely give the best design.

Illustrative Problem.—Design a T-beam with span of 40 ft. Assume dead load = 1400 lb. per foot. Live load = 3000 lb. per foot. The beam is to be simp'y supported at the ends and the flange is to be proportioned as well as the web; that is, the flange does not form a part of a floor system already determined. Use working stresses recommended by the Joint Committee.

This problem comes under Case (3) in the design of T-beams and the method previously suggested will be followed.

The total bending moment,

$$M=\frac{(4400)(40)^2(12)}{8}=10,560,000 \text{ in.-lb.}$$

The maximum shear

$$V=(4400)(20)=88,000 \text{ lb.}$$

The required net web area

$$=b'd=\frac{88,000}{105}=838 \text{ sq. in.}$$

This area can be supplied by a web

$b'=16$ in. $d=53$ in.
$b'=17$ in. $d=50$ in.
$b'=18$ in. $d=47$ in.
$b'=19$ in. $d=44$ in.
$b'=20$ in. $d=42$ in.

To give good space for the steel and still obtain satisfactory proportions for the cross-section, we will select $b'=18$ in. and $d=47$ in. for a preliminary value. A thickness of flange of 12 in. will be tried. For this thickness $\frac{t}{d}=\frac{12}{47}=0.256$. Then, by means of formulas (1), (4), and (5) of Art. 59, we find that $k=0.379$ and $jd=42.0$ in. Also, from formulas (7), a_s, $=15.7$ sq. in. and $b=48$ in.

To illustrate the effect of varying proportions, calculations will also be made for a flange thickness of 8 in., 10 in., 14 in., and 16 in. The results are as follows:

t	b	jd	a_s	Overhanging width of flange	Area of flange outside of web
8 in.	61 in.	43.3 in.	15.2 sq. in.	21.5 in.	344 sq. in.
10 in.	52 in.	42.8 in.	15.4 sq. in.	17.0 in.	340 sq. in.
12 in.	48 in.	42.0 in.	15.7 sq. in.	15.0 in.	360 sq. in.
14 in.	46 in.	41.5 in.	15.9 sq. in.	14.0 in.	392 sq. in.
16 in.	44 in.	41.2 in.	16.0 sq. in.	13.0 in.	416 sq. in.

It should be observed that the amount of concrete is less the thinner the slab and that the effect of variation of t upon the amount of steel is very small. However, a relatively thick flange is desirable considering the fact that the girder is not a part of a floor system and that the flanges

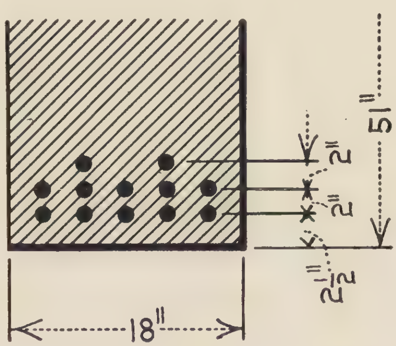

FIG. 74.

are unsupported at their outer edges. The 12-in. flange will be adopted. The dead load assumed is considerably on the safe side, but the design need not be changed.

The steel area required is 15.7 sq. in. Five round rods 1 3/8 in. diameter and seven round rods 1 1/4 in. diameter, giving a total area of 16.0 sq. in., will be used. To provide sufficient spacing between rods, the rods will be placed in three rows as shown in Fig. 74. The five 1 3/8-in. rods will

be placed in the lower row, five 1 1/4-in. rods above, and two 1 1/4-in. rods in the third row. Taking moments of areas about the center of the lowest row, the center of gravity of the group is found to be

$$\frac{(5)(1.23)(2)+(2)(1.23)(4)}{16}=1.4 \text{ in.}$$

above this row. Hence, the lower row should be placed about 48 1/2 in. below the top of the beam, thus giving a total depth, including the protective covering, of 51 in.

Bond for one rod (1 3/8 in.) at the end is

$$u=\frac{V}{\Sigma o j d}=\frac{88{,}000}{(4.3197)(42.0)}=485 \text{ lb. per square inch.}$$

For plain round rods, the number which must extend to the end of the beam is

$$\frac{485}{(1.5)(80)}=4$$

Six will be run to the end—namely, the entire lower row of 1 3/8-in. rods and also the center 1 1/4-in. rod of the second row.

Web reinforcement will be provided by means of bent rods and vertical stirrups. In bending up the rods, the two uppermost rods will be bent up nearest the center. Two rods will be bent up at a time since bending

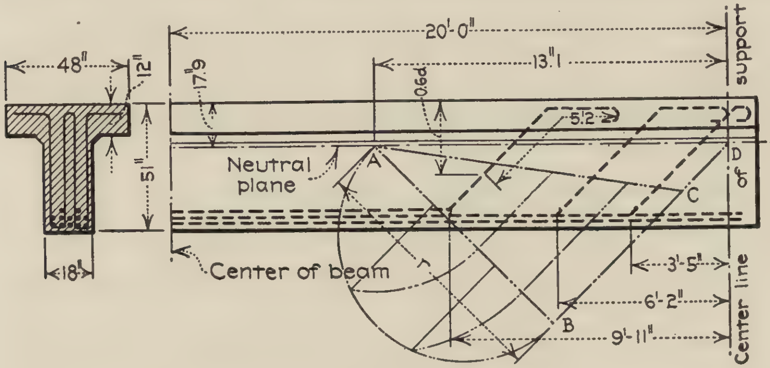

FIG. 75.

the rods singly greatly complicates the handling and prevents a symmetrical arrangement. The bends, also, will all be made at 45 degrees and the bent ends extended far enough to give ample strength of bond.

Web reinforcement is unnecessary at a distance from support equal to

$$x_1=\frac{40}{2}-\frac{(40)(18)(42.0)}{4400}=13.1 \text{ ft.}$$

The total stress to be taken by the inclined rods (and vertical stirrups, if necessary) is represented by triangle ABC, Fig. 75. BC represents two-thirds of the horizontal shear at the support per 1 in. length of beam.

$$BC=\frac{2}{3}\frac{Vb}{bjd}=\frac{2}{3}\frac{(88{,}000)}{(42.0)}=1395 \text{ lb.}$$

$$r=(0.7)(AD)=(0.7)(13.1)(12)=110 \text{ in.}$$

Hence, total stress to be taken by the rods

$$=\frac{BC}{2}(r)=\frac{1395}{2}(110)=76{,}700 \text{ lb.}$$

But the area of one rod multiplied by 16,000 gives its tensile value, or

$$\text{tensile value of one rod} =(1.2272)(16{,}000)=19{,}620 \text{ lb.}$$

Thus, only $\frac{76{,}700}{19{,}620}=4$ rods are required if they can be bent up at the proper points. Six rods, however, will be bent up as a trial.

The rods may be bent up at two at a time in the following order:

$$x_2 = \text{ or } < \frac{40}{2}\left[1-\sqrt{\frac{(2)(1.227)}{16.0}}\right]=12.18 \text{ ft.}$$

$$x_2 = \text{ or } < \frac{40}{2}\left[1-\sqrt{\frac{(4)(1.227)}{16.0}}\right]=8.94 \text{ ft.}$$

$$x_2 = \text{ or } < \frac{40}{2}\left[1-\sqrt{\frac{(6)(1.227)}{16.0}}\right]=6.44 \text{ ft.}$$

(*Note.*—Areas are used in the three preceding equations instead of the number of rods, since rods are of two different sizes. The location of the neutral plane, by formula (2) of Art. 59, is $kd=(0.38)(47)=17.9$ in. below the top of the beam.)

The length l' should equal 50 diameters (from table, Art. 43) or

$$(50)(1.25)=62.5 \text{ in.}=5.2 \text{ ft.}$$

Fig. 75 shows the construction necessary to locate the points where the rods should be bent to provide for diagonal tension. It is evident that the rods can be bent at the desired points, but notice that the spacing between two bends of the bent rods is greater than 3/4 d. Computations given above show that only 4 rods are required at the end of beam and the bending up of 8 rods should be tried unless it is intended to reinforce with vertical stirrups as well.

In the design at hand, with only six rods bent, the spacing of the bent rods can either be made not greater than $3/4d$ and stirrups supplied to take the diagonal tension which would remain unprovided for, or else the rods may be left as in Fig. 75 and stirrups employed throughout the length of the beam. Double-looped stirrups, as shown, are advantageous in a large beam of this kind and, even when stirrups are not theoretically necessary, it is good design to use them throughout since they support the tension rods, bind together the web and flange, and add greatly to the security of the construction. For the beam under discussion, the stirrups should be spaced about 2 ft. apart at the center of span varying to about 12 in. at the ends.

If it be possible in the design of a T-beam of this kind, that the live load may be applied to one flange rather than uniformly over the whole top, it may be necessary to provide transverse reinforcement in the upper side of the beam. Usually if the stirrups are extended transversely some distance beyond the width of web, as shown in Fig. 75, ample security will be given. The required area of this steel may be determined per foot of length of beam by treating the flange on either side of the web as a canti-

lever. Angles, such as those between the flange and the stem, are always a source of weakness and it is good practice to bevel off the corners of the forms, as shown.

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62. Beams with Steel in Top and Bottom.—Compressive stresses are generally carried by concrete more economically than by steel. It is sometimes desirable, however, to place steel in the compression as well as in the tension side of the beam. When a rectangular beam is limited as to size, double-reinforcement is sometimes the result, and in such cases the value of the steel reinforcement on the compressive side needs to be known. The effectiveness of steel in compression has sometimes been questioned, but the results of tests indicate that the steel does its share of the work.

Double-reinforcement is more commonly met with at the supports of continuous T-beams. At such places the bending moment is negative, the flange is under tension and is reinforced, and the lower part of the web is under compression. Also some of the center span steel is carried horizontally into the support and may be figured with the concrete to assist it in taking compression provided, of course, that its length beyond the center of support is sufficient to provide bond. A continuous beam at the supports is consequently double-reinforced, and the case is similar to double-reinforcement at the center of a span with the exception that the compressive and tensile stresses about the neutral axis are inverted.

A great deal of care is necessary in designing the supports of continuous beams. Many concrete buildings have been built with insufficient steel through the top of the supports to take the tension and insufficient concrete, or concrete and steel, at the bottom of beams at such points to take the compression. With such designs, large cracks have occurred at the supports.

The formulas which are used in the design of double-reinforced rectangular beams are derived by means of the same fundamental principles as already explained for beams with single reinforcement. The derivation of these formulas will not be given since the method followed is similar to that already outlined in Art. 33.

In deriving the following formulas the compression in the concrete is assumed to follow the linear law and the tension in it is neglected; the formulas then apply to working conditions only.

a'_s = cross-sectional area of compressive reinforcement. (Fig. 76).
d' = distance from the compressive face of the beam to the plane of the compressive reinforcement.
p' = ratio of cross-section of steel in compression to cross-section of beam above the tensile steel $= \frac{a'_s}{bd}$
f'_s = unit compressive stress in steel.

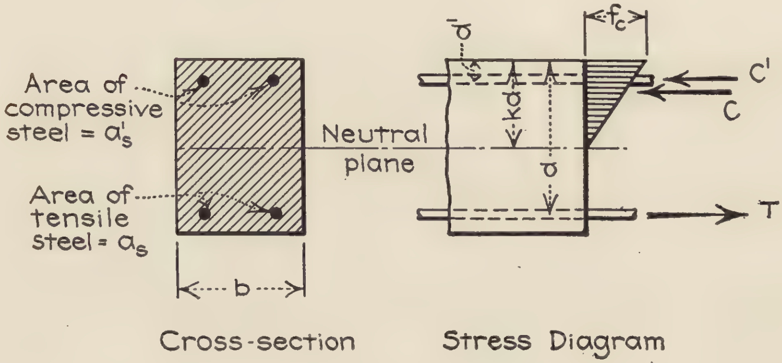


FIG. 76.

Then

$$f_s = f_c n \frac{1-k}{k} \tag{1}$$

$$f'_s = f_c n \frac{k - \frac{d'}{d}}{k} \tag{2}$$

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p+p')^2} - n(p+p') \tag{3}$$

$$M_c = bd^2 f_c L \text{ and } f_c = \frac{M}{bd^2 L}, \text{ in which} \tag{4}$$

$$L = \frac{k}{2}\left(1 - \frac{k}{3}\right) + \frac{np'}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right) \tag{5}$$

$$M_s = bd^2 f_s K \text{ and } f_s = \frac{M}{bd^2 K}, \text{ in which} \tag{6}$$

$$K = p\left(1 - \frac{d'}{d}\right) - \frac{k^2}{2n(1-k)}\left(\frac{k}{3} - \frac{d'}{d}\right) \tag{7}$$

$$f'_s = \frac{M}{bd^2 L'}, \text{ in which} \tag{8}$$

$$L'=p\frac{1-k}{k-\frac{d'}{d}}\left(1-\frac{k}{3}\right)+p'\left(\frac{k}{3}-\frac{d'}{d}\right) \qquad (9)$$

The cases which may be met with in practice, with the method of solution in each instance indicated, are as follows:

(1) To determine b and d.

Assume p, p', and $\frac{d'}{d}$.

Solve for k.

Substitute value of k in formulas for L and K.

Substitute L and K in formulas for M_c and M_s respectively.

Solve for bd^2 in each case and accept larger value.

(Remember in this case $M_c = M_s =$ exterior bending moment.)

(2) To determine moment of resistance.

Compute p, p', and $\frac{d'}{d}$.

Solve for k.

Substitute value of k in formulas for L and K.

Substitute L and K in formulas for M_c and M_s respectively.

Solve for M_c and M_s and take the smaller.

(3) To determine fiber stresses.

Obtain p, p', and $\frac{d'}{d}$.

Solve for k.

Substitute value of k in formulas for L, K, and L'.

Solve directly the formulas for f_c, f_s, and f'_s.

(*Note.*—When using formula for shear or for bond stress along horizontal tension rods of beams double-reinforced, an average value of $j=0.85$ may be taken.)

63. Design of a Continuous Beam at the Supports.—The formulas given in the preceding article apply directly to the design of continuous beams at the supports. If possible, half of the rods on each side should be bent up and extend along the top of the beams over the supports. The other rods should extend horizontally through the supporting columns.

In the design of continuous T-beams at the supports the student should realize that the flange being under tension, the stress in the concrete is negligible above the neutral axis and a rectangular section may be considered at such points. The method of design is thus similar to the design of a double-rein-

forced rectangular beam at the center of span. It should be noticed that the Joint Committee allows a higher compression in the concrete at the supports than at the middle of the beam, because of the fact that the negative moment decreases very rapidly and only a short section is under maximum stress. Also, a slight excess of stress at this point does not in any way endanger the structure but merely increases somewhat the positive moment on the beam. A unit stress of 750 lb. per square inch is permitted. (See Art. 40.)

There are three methods of reducing the compressive stress in the concrete at the bottom of the beam over supports: (1) by increasing the amount of compressive steel in the bottom of the beam; (2) by increasing the area of compressive concrete, which may or may not require a flat haunch depending upon the width of the support; and (3) by increasing the areas of both steel and concrete. In any case the increase must be made by trial. If the plan is to deepen the beam appreciably, thus forming a flat haunch, the new depth at the support must be chosen arbitrarily and the formulas for double-reinforced rectangular beams should be employed in order to determine if the new fiber stresses are satisfactory.

Under ordinary conditions the computations for a uniformly loaded beam need be made only at one point—that is, at the support—since the point to end the slope can be approximately figured by the following formula from Taylor and Thompson:[1]

$$x=\frac{l}{5}\cdot\frac{M_2-M_1}{M_2}$$

in which

M_2 = negative bending moment at the support.
M_1 = moment of resistance of the inverted T-beam without the haunch, governed by the concrete.
x = length of haunch.
l = span of beam.

The above formula is approximate (but sufficiently accurate for this work) due to the fact that the point of zero bending moment is considered to occur at a distance from the support equal to one-fifth the span. By similar triangles (Fig. 77)

$$\frac{M_2-M_1}{M_2}=\frac{x}{\frac{l}{5}}, \text{ or } x=\frac{l}{5}\cdot\frac{M_2-M_1}{M_2}$$

The bond stress along the horizontal rods at the top of a continuous beam over supports may be found by the same

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 430. Copyright, 1905, 1909, by Frederick W. Taylor.

formula as is employed for the tension rods at the end of a simply supported beam. However, if bent up rods are employed for web reinforcement and if these same rods are employed to take the tension over supports, the beam is greatly stiffened and the bond stress along the top rods is reduced appreciably below that given by the theoretical formula. This bond stress is affected by the amount of web reinforcement used in a somewhat similar manner to the way the bond stress is affected along the rods at the end of a simply supported beam. Art. 38 should be reviewed so that the student will have some guide

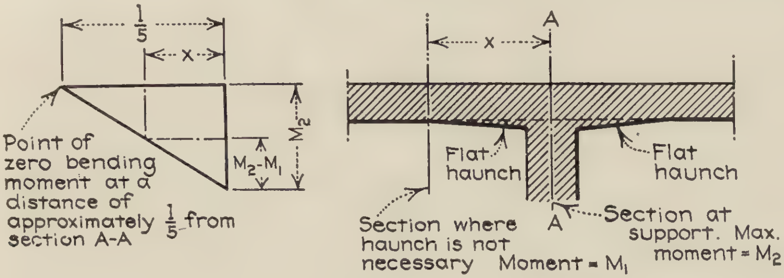


FIG. 77.

as to how much the allowable bond stress may be increased for different cases. A careful study of the problems following Art. 50 will also help to make the matter clear.

In beams considered uniformly loaded, the rods which are bent should extend beyond the center of support at least to about the third point (point of zero moment varies for different loadings) to provide thoroughly for negative moment, and this length should be increased if it is not sufficient to transfer to the concrete through bond, the greatest allowable tensile stress in the rods. Some designers consider the negative moment properly looked after if the top rods are extended only to the fourth point. The matter admits of much difference of opinion but it would seem well to be conservative in this part of the design.

If half of the rods from each span are used over the support, then half of the total number will extend to about the third point where the tension due to negative moment becomes zero. At this point the shear is only 1/3 of what it is at the end of span, if the beam is considered uniformly loaded. Since bond stress due to increment (or decrement) tension varies as shear,

more than a sufficient number of rods are thus run out to the third point, and with the bent rods being added gradually to this number until all the rods are acting in this manner at the support, it is clear that this method of design is satisfactory even when the bond stress at the support is the maximum allowable.

Rods should be bent in a position to take as much diagonal tension as possible, usually at an angle of 45 degrees, and the points where the rods are no longer needed at the bottom of beam to resist tension may be found for uniform loading by the formula (see Art. 45).

$$x_2 = \text{ or } < \frac{l}{2}\left(1-\sqrt{\frac{8m_2}{em}}\right)$$

It is also necessary to determine where the rods over supports may be bent down. It will be on the safe side, and sufficiently accurate, to consider the curve for negative moment as a straight line between the support and the point of zero moment at the third point. With this variation of the moment in mind, it is an easy matter to find where the rods may be bent down at the top of the beam. The designer must use his judgment in the matter, but this much may be said: if a 45 degree angle bend cannot be made in a rod, as proposed, due to the controlling points for bending at the top and bottom, a greater number of rods should be employed at the center of span in order to make this bending possible, and the design governed accordingly. It is evident from the above that it will not always be possible to place the rods so as to take all the diagonal tension; in fact, there will be less likelihood than in a simply supported beam. It will be advantageous in any case, however, due to the dangerous section for diagonal tension being near the support, to employ some stirrups even when they are not theoretically necessary. In Volume II will be given a design of a floor system illustrating continuous beam design for uniform loads as well as for concentrated loads due to beams running into girders.

Since a T-beam in the center of span becomes a rectangular beam over supports, the stress in the tensile steel at the support will generally be greater in ordinary designing than the corresponding stress at the center of beam; that is, this stress will be greater if half the rods are bent up on each side and lap over the support. For this reason, then, when selecting the steel at the center of span, a little more than the required amount at that

point should be chosen. It should be noticed, however, that the column has some strengthening action at the support and it will not be necessary to keep too closely to the allowable stress.

Another point to be noted in the design of a continuous beam at the supports is the bond stress of the compressive reinforcement. The general formula for horizontal shear at any point of a beam (see Art. 29) is

$$vb=\frac{VQ}{I}.$$

At the support, the quantities V and I are constant and the horizontal shear per linear inch at any horizontal plane is seen to be proportional to the statical moment of the effective area outside such plane about the neutral axis. At the neutral axis this shear per linear inch has been shown to be $vb=\frac{V}{jd}$, and the resulting bond per square inch of the surface is $\frac{V}{\Sigma ojd}$. The shear per linear inch between the compression rods and the concrete will, therefore, very closely equal

$$\frac{V}{jd}\cdot\frac{Q \text{ of equivalent compressive steel area}}{\text{Total } Q \text{ of compression area}}$$

(By the term equivalent steel area is meant the area of the steel multiplied by n and considered equivalent to concrete at the same horizontal plane.) If this quantity be divided by $\Sigma o'$, considering o' the circumference of a rod in compression, then the result will be the bond per square inch of surface along the compression rods.

Now the total statical moment of the compression area about the neutral axis equals the total moment of the tension area, which is $na_s(1-k)d$. The moment of the compressive steel is equal to $na'_s(kd-d')$. Hence, for bond stress of compressive steel per square inch,

$$u=\frac{V}{\Sigma o'jd}\times\frac{a'_s(kd-d')}{a_s(1-k)d}$$

If now we let d_c = diameter of compression rods, d_t = diameter of tension rods, and Σo = total circumference of the tension rods, then the above formula reduces to

$$u=\frac{V}{\Sigma ojd}\times\frac{d_c(kd-d')}{d_t(1-k)d}$$

This formula shows that the bond stress per square inch for the tension and compression rods will be proportional to the product

of the diameters by their distances from the neutral axis. Since the compressive steel will generally be nearer the neutral axis than the tensile steel, it follows that, if the compression bars are no larger in diameter than the tension bars, the bond stress per square inch will always be less than that of the tension bars and the above formula need not be used.

The formula derived above may be used in any special case in designing, but generally it will be sufficient to consider simply the compressive stress in the steel and provide a sufficient length from this point to the end of the bar to transmit this stress. The working strength of the steel in compression cannot be reached without exceeding the compressive strength of the concrete in which it is embedded. Consequently, in common design it will be satisfactory to provide a lap beyond the center of support sufficient to take care of compressive stress in the steel equal to the maximum as determined by the concrete.

Illustrative Problem.—A continuous T-beam, uniformly loaded, has a bending moment at the center of each span of 358,000 in.-lb. Negative bending moment at the supports and the positive bending moment at the center of span are figured by the formula $\frac{wl^2}{12}$. The tensile steel at the center of span consists of four 3/4-in. round rods. $b'=9$ in. $d=15.5$ in. Design the supports using working stresses recommended by the Joint Committee.

At the supports the flange of the T-beam, being in tension, is negligible and the T-beam changes into a rectangular beam with steel in top and bottom. Two of the tensile rods on each side of the supports will be bent up and made to lap over the top of the supports, while the other two rods on each side will be continued straight and lapped over supports at the bottom of beam. It will be assumed that the steel at the center of span has already been chosen so that this may be done.

The ratios of steel in tension and compression are the same, and are respectively:

$$p=p'=\frac{(4)(0.4418)}{(9)(15.5)}=0.013$$

With these values of p and p', and $n=15$, also assuming $\frac{d'}{d}=0.1$ (this value is assumed simply to make it easier for the student to check up the computations—in practice, the correct ratio should be taken) we obtain the following from equations (3), (5), (7), and (9), of Art. 62: $L=0.291$, $K=0.0115$, and $L'=0.0266$

Maximum pressure in concrete is

$$f_c=\frac{358{,}000}{(9)(15.5)^2(0.291)}=570 \text{ lb. per square inch, by formula (4)}$$

Also,

$$f_s = \frac{358,000}{(9)(15.5)^2(0.0115)} = 14,400 \text{ lb. per square inch by formula (6)}$$

$$f'_s = \frac{358,000}{(9)(15.5)^2(0.0266)} = 5600 \text{ lb. per square inch, by formula (8)}$$

Allowable compression in concrete at the support is thus satisfactory, and no haunch is necessary.

The value of f'_s can always be determined as above, but it is seldom desired as the working strength of the steel in compression cannot be reached without exceeding the compressive strength of the concrete in which it is embedded. With $n=15$, the allowable stress in the steel cannot exceed 15 times the compressive strength of the concrete.

Illustrative Problems.—A continuous T-beam with $b'=14$ in., and $d=26.5$ in. has a negative bending moment at the supports of 2,000,000 in.-lb., and has equal spans of 18 ft. Reinforcement at supports consists of 7/8-in. round rods. Eight bars are in tensile and four in compressive part of beam. Hence,

$$\text{Ratio tension steel, } p = \frac{4.81}{(14)(26.5)} = 0.0130$$

$$\text{Ratio compression steel, } p' = \frac{0.0130}{2} = 0.0065$$

With these values of p and p' and $n=15$, also assuming $\frac{d'}{d}=0.1$, we obtain from equations (3) and (5), $L=0.243$, and from equation (4).

$$f_c = \frac{2,000,000}{(14)(26.5)^2(0.243)} = 836 \text{ lb. per square inch.}$$

which is excessive, only 750 lb. per square inch being allowed, or a stress 15 per cent greater than at the center of span.

For depth of haunch assume $\frac{d'}{d}=0.1$ and try $d=29$ in. For this depth of beam the ratios of steel in tension and compression change to

$$p = 0.0130 \left(\frac{26.5}{29}\right) = 0.0119$$

$$p' = \frac{0.0119}{2} = 0.0059$$

The corresponding value of $L=0.233$ and $K=0.0104$. The maximum compression in the concrete

$$f_c = \frac{2,000,000}{(14)(29)^2(0.233)} = 729 \text{ lb. per square inch}$$

and maximum tension in steel

$$f_s = \frac{2,000,000}{(14)(29)^2(0.0104)} = 16,300 \text{ lb. per square inch}$$

This stress is allowable and the depth of haunch from top of beam of 29 in. will be accepted.

Using the formula (4)

$$M_1 = (750)(14)(26.5)^2(0.242) = 1,780,000 \text{ in.-lb.}$$
$$M_2 = 2,000,000 \text{ in.-lb.}$$

Hence, from formula in Art. 63, the length of haunch

$$x=\frac{18}{5}\times\frac{220,000}{2,000,000}(12)=4.75 \text{ in.}$$

Since maximum negative moment occurs in middle of column and necessary length of haunch is only 4.75 in., no haunch will be introduced outside of the column.

Problems

53. Determine the economical depth of a T-beam in a floor system. Thickness of floor slab is 4 in. Maximum shear = 16,000 lb. Maximum moment = 600,000 in.-lb. Assume the ratio of unit cost of steel to cost of concrete = 60. Use working stresses recommended by the Joint Committee.
54. A T-beam has the following dimensions: $b=48$ in., $t=6$ in., $d=22$ in., and $b'=10$ in.—the steel consisting of six 3/4-in. round rods. Take $f_s=15{,}000$, $f_c=600$, and $n=15$. What is the safe resisting moment of the beam?
55. Change t of Problem 54 to 4 in. and find the safe resisting moment.
56. Make a complete design of a T-beam with span of 30 ft. Live load = 3800 lb. per foot. Assume dead load = 1000 lb. per foot. The beam is simply supported and the flange does not form a part of a floor system already determined. Use working stresses recommended by the Joint Committee. Take $b'=17$ in. and $t=10$ in. Select required steel area from 1 1/8-in. and 1-in. round rods.
57. A beam of which $b=14$ in. and $d=20$ in. has 2 per cent of tensile steel and 1 per cent of compressive. What is the safe resisting moment of the beam? Assume $\frac{d'}{d}=0.1$ and use stresses recommended by the Joint Committee.
58. Suppose the beam in Problem 57 were subjected to a bending moment of 1,500,000 in-lb.; what would be the resulting fiber stresses, f_c, f_s, and f'_s?
59. Suppose the conditions met in Problem 58 occurred at the supports of a continuous beam with equal spans of 30 ft. Design the necessary haunch. Take $d=23$ in.

CHAPTER VI

COLUMNS

Concrete columns may be divided into five classes, as follows:

1. Plain concrete columns.
2. Columns reinforced with longitudinal rods only.
3. Columns reinforced with bands or spirally wound metal, called *hooped* columns.
4. Columns reinforced with both hoops and longitudinal rods.
5. Columns reinforced with structural steel shapes.

It is generally conceded that long columns should be avoided because of the fact that any variation in the quality of concrete affects the strength more seriously than in any other structural form, and also because of the fact that for such columns very little data is available from tests. The use of reinforced columns should be limited to a length of about 15 diameters (see recommendations of the Joint Committee, Art. 40) and for this reason the strength of longer columns will not be considered in the discussion which follows.

64. Plain Concrete Columns.—In the testing of plain concrete columns, two distinct forms of failure have been observed: (1) a diagonal shearing failure, and (2) a failure by gradual crushing. The failure by diagonal shearing is sudden, the columns breaking suddenly with a loud report and without warning. The failure by gradual crushing may be considered as due to simple compression, the breaking down not being discovered until the weighed load begins to decrease and the *point* of failure not being determined until the machine produces a further shortening of the column. For both forms of failure, however, the ultimate strength of a column could be predicted if a stress-deformation diagram were developed with the progress of the test. It has been found that rich mixtures tend to give the diagonal shearing type of failure, and columns with lean mixtures generally give a failure of the simple compression type.

But few comparative tests of cylinders and columns are available, but these indicate that the strength of the column and of the cylinder very nearly agree. (For the relation of crushing

strength of cylinders to crushing strength of cubes, see Art. 10). Strength of plain concrete columns of an average 1:2:4 mixture at 60 days may be taken at about 1800 lb. per square inch although tests made at the University of Wisconsin in 1908 indicate that, with careful workmanship and testing, an average value of about 2000 lb. per square inch can be obtained. Plain concrete is entirely suitable for short columns up to lengths of 6 to 10 diameters, but for more slender proportions hooped or banded columns are much to be preferred. The Joint Committee recommends that the length of a plain concrete column be limited to 12 diameters.

The compressive strength of concrete is approximately proportional to the amount of cement which it contains, so that increasing the richness of the concrete in either a plain or reinforced column is an effective means of strengthening the column to permit smaller section. Tests show that for a 1:2:4 plain concrete column there is no excessive variation in individual tests. For a weaker mixture than this, however, the individual tests are much more at variance, indicating greater unreliability. A rich mortar for the above reasons may often prove to be the more economical in column construction.

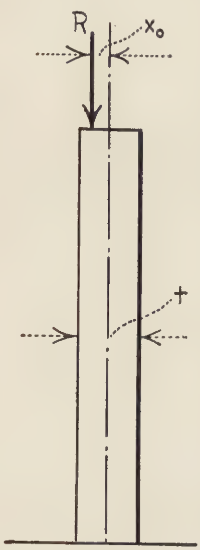


FIG. 78.

Bending stresses in columns due to eccentric loads must be provided for by increasing the section until the maximum stress does not exceed the allowable. A formula for homogeneous columns follows, and formulas applicable to reinforced concrete will be given under "Bending and Direct Stress."

The ordinary formula for the compressive fiber stress due to eccentric loading upon solid rectangular columns of homogeneous materials (Fig. 78), is as follows:

R = total load.

A = area of column.

x_o = eccentricity.

t = breadth of column.

f_c = total unit pressure on outer fiber nearest to line of vertical pressure.

Then

$$f_c=\frac{R}{A}\left(1+\frac{6x_o}{t}\right)$$

and the additional intensity of compressive stress due to eccentric loading is seen to be equal to $\frac{R}{A}\cdot\frac{6x_o}{t}\cdot$ The above formula may be used for columns of plain concrete.

65. Columns with Longitudinal Reinforcement.—Since the modulus of elasticity of a material is the ratio of stress to deformation, it follows that for equal deformations the stresses in the steel and concrete of a concrete column will be as their moduli of elasticity. Thus,

$$\frac{f_s}{f_c}=\frac{E_s}{E_c}, \text{ or } f_s=f_cn$$

Let A denote total cross-section of column.
A_c denote cross-section of concrete.
A_s denote cross-section of steel.
p denote steel ratio $=\frac{A_s}{A}\cdot$
f_c denote stress in concrete.
f_s denote stress in steel.
n denote $\frac{E_s}{E_c}\cdot$
P' denote total strength of a plain column for the stress f_c.
P denote total strength of a reinforced column for the stress f_c.

Then,
$$P'=f_cA$$
or
$$P=f_cA_c+f_sA_s=f_c(A-pA)+f_cnpA$$
Thus,
$$P=f_cA[1+(n-1)p] \tag{1}$$
from which also
$$\frac{P}{P'}=1+(n-1)p \tag{2}$$

The relative increase in strength caused by the reinforcement is

$$\frac{P-P'}{P'}=(n-1)p \tag{3}$$

Tests of columns made at the Massachusetts Institute of Technology, the Watertown Arsenal, the University of Wisconsin, and the University of Illinois, on columns with vertical steel bar reinforcement, indicate that the steel may be counted upon in design to take its portion of the loading as computed from equa-

tion (1). Also the value of 15 for n (see Art. 25) as recommended by the Joint Committee, is found to give very conservative results when employed in the above formulas. In this form of column the concrete fails suddenly and in a manner similar to the diagonal shearing failure of a plain concrete column.

The density and rigidity of the concrete when steel is employed is apt to be less than in the plain column, so that for small percentages of longitudinal reinforcement the gain in strength is small. On the other hand, it is undesirable to use high percentages of reinforcement as the strength of such columns is not well determined. If great strength is desired, an increase in the proportion of cement is preferable to a high percentage of steel. The riveted column unit has advantages over the separate rod reinforcement if large amounts of steel are required.

Where the concrete is depended upon to fireproof the steel in a column, a certain thickness should be deducted in calculating strength. As already mentioned, the necessary thickness for fireproofing is about 2 in., but if 1 1/2 in. be deducted all around in calculating strength this will amply provide for the weakening effect of fires. A less thickness than this should be sufficient where the contents of a building are not especially inflammable.

In the construction of columns the reinforcing bars should be straight, and great care should be exercised in keeping them in place during the pouring of the concrete. Assuming a load uniformly distributed over the cross-section of the column, the reinforcing rods should be arranged symmetrically over this area. Ultimate failure is sometimes due to the buckling of the reinforcement causing the outside concrete to scale and, for this reason, at least 1 1/2 in. of concrete should cover the rods on the outside. The rods should also be held securely in place by wiring or banding the rods together at intervals of about a foot, but such banding cannot be considered as hooping in the sense usually employed. As will be seen later, hooping to be effective must be spaced relatively close. Vertical reinforcement held in place by hoops is shown in Fig. 79.

In splicing columns, large rods (or structural shapes) should have their ends planed true and well spliced. The splicing of the large rods may be effected by placing a pipe-sleeve over the upper end of the lower bar and projecting above it, and then setting the lower end of the upper bar within the pipe and resting upon the lower bar. The rods should either be grouted in place

or else the diameter of the sleeve should not be over 1/16 in. larger than the diameter of the rod. All such splices should be made above the floor level but not more than 12 in. above the same. When small diameter rods are used, say up to 1 1/4 in., joints in the vertical steel may be provided for by overlapping a sufficient distance to develop the requisite bond strength, and the lapped rods should be securely wired or bolted to each other. It is much better to avoid splices in a column between lateral supports. In footings where the length of embedment is not sufficient to take all the stress, large rods (or shapes) should rest upon suitable base plates in the foundation concrete.

The economy of steel reinforcement is dependent upon the working stresses permissible in the concrete and the value of n, since the stress in the steel $= f_c n$. The following table gives the various values of $\frac{P}{A}$ and f_s for different stresses in the concrete and different moduli of elasticity:

f_c	n	Values of $\frac{P}{A}$ for				
		$p=0.01$	$p=0.02$	$p=0.03$	$p=0.04$	f_s
450	10	490	531	571	612	4500
	15	513	576	639	702	6750
	20	535	621	706	792	9000
550	10	599	649	698	748	5500
	15	627	704	781	858	8250
	20	654	759	863	968	11000
650	10	708	767	825	884	6500
	15	741	832	923	1014	9750
	20	773	897	1020	1144	13000
750	16	817	885	952	1020	7500
	15	855	960	1065	1170	11250
	20	892	1035	1177	1320	15000

It should be noted in the table that the stresses in the steel will be relatively low except in the unusual combination of high working stresses in the concrete with large value of n.

66. Columns with Hooped Reinforcement.—Whenever a material is subjected to compression along one axis, then, as a consequence, there will be an expansion of the material along

axes which are perpendicular to the one first considered. Thus, if the material of a column is held laterally, then lateral compressive stresses are developed which tend to neutralize the effect of the longitudinal compressive stresses and thus to increase the resistance against failure. This is the principle involved in the spiral or banded column. Fig. 80 shows a spiral form of

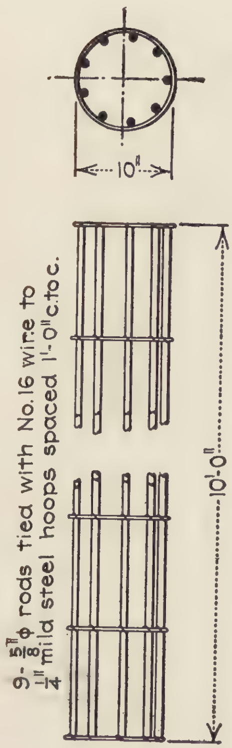


FIG. 79.

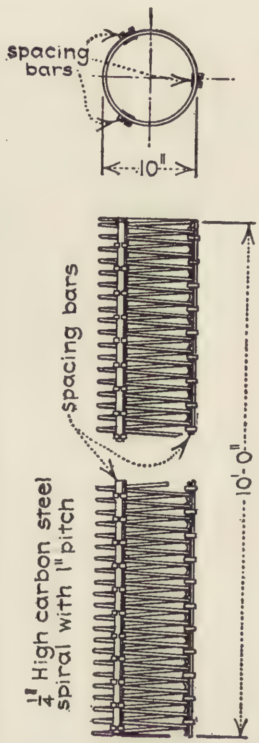


FIG. 80.

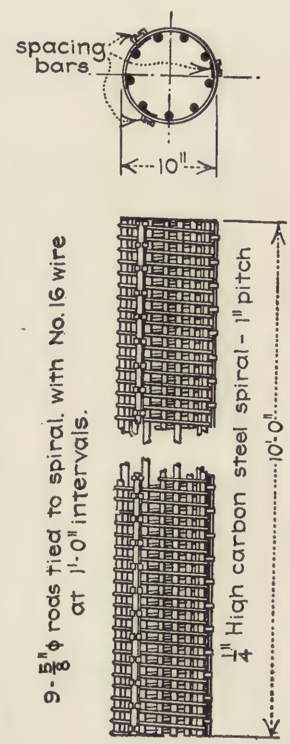


FIG. 81.

reinforcement. Longitudinal crimped spacing bars serve to maintain a uniform pitch. Banded reinforcement is similar to that shown in Fig. 79 with the exception that the clear spacing of the bands to be effective must not be greater than about one-fourth the diameter of the enclosed column. The same limit as regards spacing applies to spirals.

The yield-point is taken to be the point on the stress-defor-

mation diagram where the rate of deformation increases rapidly. It is very nearly the deformation at which plain concrete would fail. Theoretically it can be shown that within this limit of elasticity the hooped reinforcement is much less effective than longitudinal reinforcement, but that such reinforcement may be quite effective in increasing the ultimate strength of the column. The results of tests verify the theoretical conclusion. The ultimate strength is found increased, but the shortening of the column is so great at a comparatively early period in the loading, that the safe strength cannot be based directly on the ultimate strength. Concrete expands laterally only a very small percentage of its vertical deformation or shortening, so that the hoops do not come much into play until the concrete has shortened to an extent such that its elastic limit has been passed. Under further loading, however, the concrete is prevented by the hooping from actual failure, but continues to expand laterally until final failure occurs by the breaking of the wire or by its excessive stretching. It is also found that the shell of concrete outside of the hooping, which is necessary for fireproofing and for the protection of the steel, begins to crack and peel off at about the same load as that which causes complete failure in plain concrete columns. If hooping is not continuous or rigid, it will also peel off with the surface concrete so that the effective strength of the column will be no greater than a similar one of plain concrete.

67. Columns with Hooped and Longitudinal Reinforcement.— The addition of bands or spirals to columns having longitudinal reinforcement does not have much effect upon the deformation of such columns up to the point of failure without hooping. In fact the elastic limit and rigidity of the column appears to be decreased if anything. The effect of such hooping, however, raises slightly the ultimate strength and increases the capacity of the column to deform at loads beyond the elastic limit, so that a somewhat higher working stress may be employed on the concrete than for plain concrete columns. Tests show that about 1 per cent of a closely spaced spiral of high carbon steel is sufficient to prevent the longitudinal rods from bulging outward and will provide a satisfactory amount of toughness with a corresponding raising of the ultimate strength beyond the elastic limit. Vertical reinforcement combined with spiral hooping is shown in Fig. 81.

68. Columns Reinforced with Structural Steel Shapes.—If a structural steel column is designed to take all the load and then is simply fire-proofed with a covering of concrete, it cannot properly be called a reinforced concrete column. To be classed under this heading the steel must be designed so that it takes a

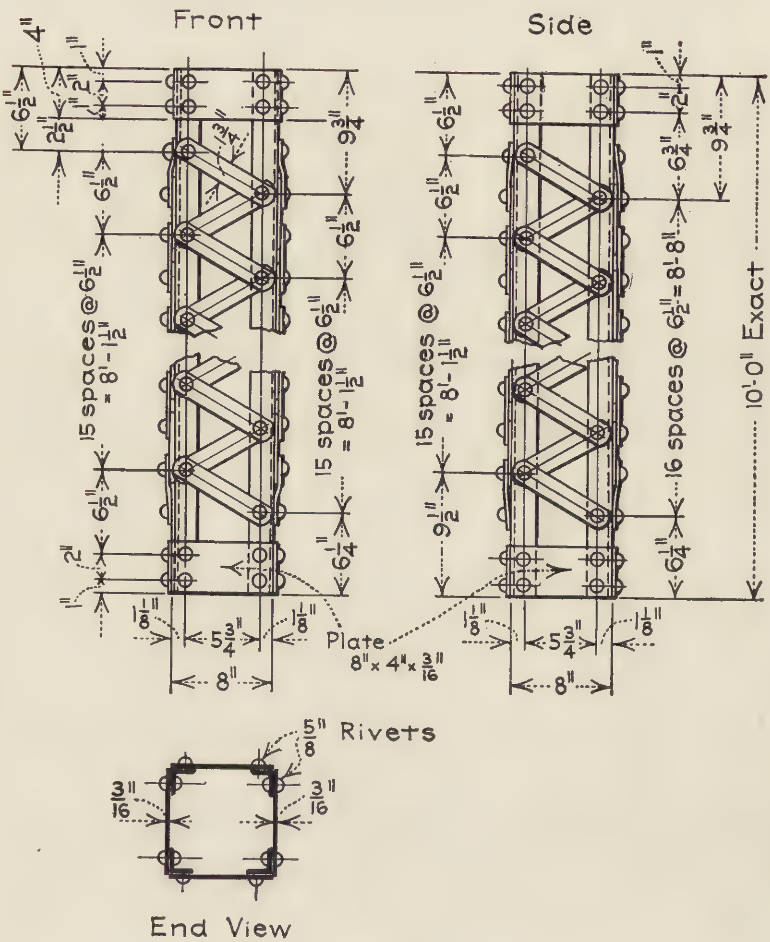


Fig. 82.

load in combination with the concrete; that is, the steel must be figured in the same way as vertical rods and the stresses determined by the formulas previously given.

Structural steel reinforcement is sometimes in the form of a cross in the center of the column or more often angles are

employed connected by riveted latticing. (Fig. 82.) Tests of columns of this character generally show lower ultimate strength than similar columns reinforced with the same quantity of steel in the form of vertical rods. This is most likely due to the difficulty of properly placing the concrete around the steel, and, furthermore, to the fact that the adhesion of concrete to steel where the latter presents broad flat surfaces is not good.

It is often advantageous to employ steel columns for reinforcement and arrange them to carry the false work and dead load of two or more floors, thus enabling the placing of concrete to proceed simultaneously on several floors. The initial dead load stress applied to the steel of the column in this way enables higher steel stresses to be used. In other words, this initial stress need not be counted with respect to the stress in the concrete, but the total stress in the steel must be looked to in order to make sure that the allowable stress per square inch is not exceeded.

To be able to count upon the concrete in columns reinforced with structural forms, the concrete should be well enclosed either by the steel form itself or by means of bands or hooping. However when the amount of steel becomes very large, the relative value of the concrete becomes more uncertain, and it would be good design to neglect its element of strength.

69. Tests on Plain and Reinforced Concrete Columns.—Important tests have been made on plain and reinforced concrete columns at the Watertown Arsenal, the Massachusetts Institute of Technology, the University of Illinois, and the University of Wisconsin. Similar conclusions have been reached in each case. Some of the tests made at The University of Wisconsin will be described as indicating in general what results may be expected from any series of tests.

All columns of the series to be described were 10 ft. long with a 1:2:4 concrete mix throughout. Columns were reinforced (Fig. 83) as follows:

A_1, A_2, A_3, A_4 . . . . none.
B_1, B_2, B_3, B_4 . . . . reinforcement shown in Fig. 82.
C_1, C_2, C_3, C_4 . . . . reinforcement shown in Fig. 80.
D_1, D_2, D_3, D_4 . . . . reinforcement shown in Fig. 81.
E_1, E_2, E_3. . . . . reinforcement shown in Fig. 79.

Columns A_1 to B_4 were squares 12 in. on a side, and C_1 to E_3 were octagons 12 in. on short diameters. The reinforced columns of this series were all made with a protective shell 2 in. thick. In order to study the behavior of this shell and its effect on the strength of the test piece, it was removed from columns B_1, B_4, and C_3 before testing. Thus B_1 and B_4 with the outside shell knocked off had a cross-section 8×8 in., and C_3 when stripped of its outside shell before testing had a cross-section in the form of a circle 10 in. in diameter.

Columns B_1 and B_4 exhibited considerably more toughness than did the plain columns. These columns, outside shell removed, sustained an average ultimate load of 239,50 lb. (3740 lb. per square inch). A plain concrete column of the same size, namely, 8×8 in. and composed of the same concrete,

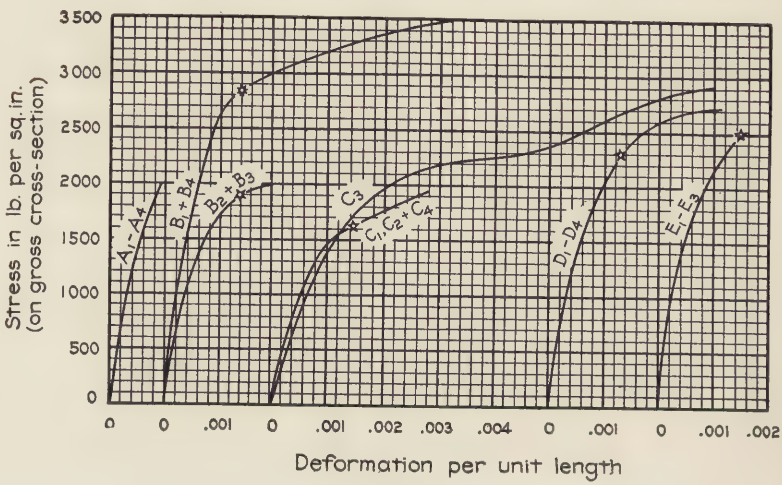


Fig. 83.

sustained 120,600 lb. (or 1880 lb. per square inch). Also, a steel column similar to the reinforcement in these columns failed under a load of 138,000 lb. To quote from Prof. M. O. Withey's paper before the American Society for Testing Materials, Volume IX, 1909: . . . . . . "A comparison of these figures indicates the value of the reinforcement and also shows a means of increasing the strengths of hollow steel columns by filling the inside with concrete. In the tests of columns B_2 and B_3, the area of the outside shell was so large in proportion to the area of the core that the value of this form of reinforcement

was obscured. The results of tests of these two columns, together with those of columns B_1 and B_4, indicate that the protective shell does not carry its share of the load in this type of column, especially when there is relatively little direct bond between it and the concrete core. . . . . . A comparison of the stresses sustained by the column before and after the shell was removed shows that the shell sustained only about 40 per cent of the average unit stress carried by the entire cross-section. However tests of these two columns revealed that the protective shell remained intact until the yield point of the steel was passed. The point on the stress-deformation curve at which this outside coating began to crack is indicated by a cross in Fig. 83. These tests demonstrate that this type of column possesses considerable toughness and a high load-carrying capacity; that the steel and concrete act together in carrying the load and in resisting deformation; and that the protective coating, although it remains intact until the yield point of the steel is passed, should not be counted upon to resist deformations or stresses.

"Columns E_1 to E_3 exhibited considerable stiffness when tested. No cracks of any consequence appeared until the deformations indicated that the yield point of the steel had been reached. At this point, however, owing to insufficient lateral support offered by the widely separated 1/4-in. ties, they failed very suddenly. The increase in strength for each per cent of longitudinal reinforcement for these columns, over the strength of the plain columns A_1 to A_4, is approximately 156 lb. per square inch. These columns were lacking in toughness and did not have a high ultimate strength.

"The preliminary tests of the spiral columns C_1 to D_4 demonstrated that this type of column has great toughness and a high ultimate strength, which is accompanied by large deformations and deflections. By comparing the test records and the stress-deformation curves for columns D_1 to D_4, which had vertical reinforcement, it was observed that the outside shell of concrete cracked at about the same time that the stress in the steel reached the yield point. As may be seen in Fig. 83, the stress-deformation curves for columns C_1 to C_4 are practically the same up to a stress of 1600 lb. per square inch. As the shell concrete in columns C_1, C_2, and C_4 cracked at about this stress, it seems evident that the shell and core act in unison up to this point in this type of column. However as the shell on a column in

a building is liable to be greatly weakened by a fire, its strengthening and stiffening value should be neglected in such design."

In the experiments above referred to, it was also found that the yield point of columns reinforced with both spirals and longitudinal rods is closely given by the formula

$$\frac{P_1}{A}=(1-p)f_c+pf_s$$

and the ultimate strength approximately by the formula

$$\frac{P}{A}=(1-p)f_c+pf_s+0.12f'_s\sqrt{p'}$$

In these formulas, P_1 = load at yield point, P = maximum load, A = area of column inside of spiral, p = percentage of lateral steel, f_c = ultimate compressive strength of concrete, f_s = yield point of longitudinal steel, and f'_s = yield point of lateral steel. The spiral steel is not included in the first formula as its effect on the yield point is very small.

From the above and other tests made at The University of Wisconsin during the year 1909, Prof. Withey draws the following conclusions:

"1. A small amount, 0.5 to 1 per cent of closely spaced lateral reinforcement, such as the spirals used, will greatly increase the toughness and ultimate strength of a concrete column, but does not materially affect the yield point. More than 1 per cent of lateral reinforcement does not appear to be necessary. The use of lateral reinforcement alone does not seem advisable.

"2. Vertical steel in combination with such lateral reinforcement raises the yield point and ultimate strength of the column and increases its stiffness. Columns reinforced with vertical steel only, are brittle, and fail suddenly when the yield point of the steel is reached, but are considerably stronger than plain columns made from the same grade of concrete.

"3. Increasing the amount of cement in a spirally reinforced column increases the strength and stiffness of the column. A column made of rich concrete or mortar and containing small percentages of longitudinal and lateral reinforcement, is without doubt fully as stiff and strong and more economical than one made from a leaner mix reinforced with considerably more steel. In these texts, doubling the amount of cement increased the yield point and ultimate strength of the columns without vertical steel about 100 per cent, and added about 50 per cent to the strength of those reinforced with 6.1 per cent vertical steel.

"4. From the behavior under test of the columns reinforced with spirals and vertical steel and the results computed, it would seem that a static load equal to from 35 to 40 per cent of the yield point would be a safe working load.

"5. The results obtained from tests of columns reinforced with structural steel indicate that such columns have considerable strength and toughness, and that the steel and concrete core act in unison up to the yield point of the former. The shell concrete will remain intact until the yield point of the steel is reached, but no allowance should be made for its strength or stiffness.

"6. As many of the blotters on the tops and bottoms of columns bore imprints of the vertical steel after failure, it would seem a safe precaution to use bed plates at the foundations for such columns, and thus prevent any possiblity of the steel punching through the concrete under an excessive load."

Further experiments made in 1910 at The University of Wisconsin were for the purpose of making a detailed study of the strength and elastic properties of columns reinforced with spirals and longitudinal rods. To quote from Bulletin No. 466 of the University written by Prof. Withey: . . . . . "The tests were made to obtain some data relating to 1, the effect of varying the percentage of spiral reinforcement; 2, the effect of varying the percentage of longitudinal reinforcement; 3, the effect of varying the richness of the mixture; 4, the effect of a small number of repeated loadings; 5, the effect of maintaining a constant load for different time intervals; 6, the behavior of columns eccentrically loaded; 7, the relative value of plain and deformed bars for longitudinal reinforcement; 8, the effects of differences in end conditions. All told, 66 columns of commercial size were made and tested.

"It is customary in good practice to encase all column reinforcement in a shell of concrete about 2 in. thick. This shell is provided to protect the reinforcement against fire and rusting. Since this coating is liable to be injured or destroyed in various ways its strength should not be considered in structural design. Consequently, the core, or portion of the column inside the spiral, should be proportioned to carry the entire load. It, therefore, seemed desirable to make the protective shell on these columns as thin as possible in order that the properties of the core might be more readily observed."

Prof. Withey draws the following general conclusions from the results of these tests:

"1. If materials can be obtained at average unit prices, rich mixtures are more economical than lean ones. Considering materials similar to those employed herein, the more economical mixtures will be produced if the proportion of cement to aggregate, by weight lies between 0.2 and 0.7.

"2. Although the yield point of a reinforced concrete column is practically independent of the percentage of spiral reinforcement, the ultimate strength and the toughness are directly affected by it. On account of the excessive deformations accompanying loads beyond the yield point, on account of the probability that both yield point and ultimate strength are less in repeated or long time load tests than in the progressive load tests ordinarily made in the testing machine, and on account of the uncertainties which always surround the hypothesis adopted in designing, good practice demands that only a portion of the stress producing disintegration of the outside shell be used as a working stress. Consequently, only enough lateral reinforcement is needed to prevent the longitudinal rods from bulging outward, and to provide an additional factor of safety against an overload by increasing the toughness and raising the ultimate strength somewhat above the yield point. From these tests 1 per cent of a closely spaced spiral of high carbon steel seems to be sufficient for this purpose.

"3. By the addition of longitudinal steel the yield point, ultimate strength, and stiffness of a spirally reinforced column can be considerably increased. If maximum economy in floor space is desired, if a column is so long or is so eccentrically loaded that tension exists on a portion of the cross-section, or if a large dead load must be sustained by the column while the concrete is green, a high percentage of longitudinal reinforcement may often be advantageously employed. Such reinforcement is also a valuable safeguard against failure due to flaws in the concrete. If the cost of cement is extremely high, it may be economical to use a leaner mixture than suggested in (1) and considerable longitudinal steel to increase the stiffness and strength; columns like those of Series 1 may profitably be used. In general, however, cement is a more economical reinforcement than steel. Therefore, for ordinary constructions it does not seem advan-

tageous to use in combination with a rich concrete more than 2 or 3 per cent of longitudinal steel.

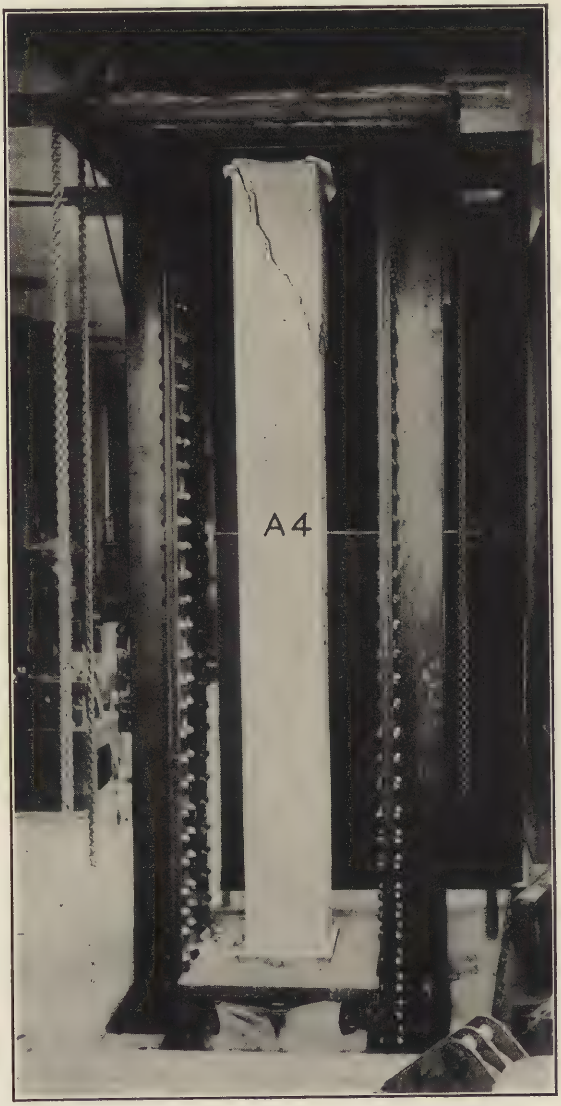
"4. The amount of data presented on tests of columns subjected to repeated or time loadings is far too small to warrant drawing definite conclusions as to the limiting stress for repeated loadings which will hold true for all kinds of columns and for an infinite number of repetitions, or for a prolonged loading. However, it does appear from the results presented that there is practically no increase in set or deformation after a few repetitions of loads equal to 40 to 50 per cent of the yield points of the columns tested. The results of the repeated load tests also plainly indicate that there is considerable additional strength and toughness afforded by the spiral after the yield point of the longitudinal steel has been passed. That all of this additional strength may not be permanent is suggested by the slopes of the deformation-time curves for two of the columns.

"5. The close agreement between theoretical values and values derived from test data shows that the formula commonly used in designing short homogeneous columns eccentrically loaded, may be applied to reinforced concrete columns, provided suitable allowance be made for the steel.

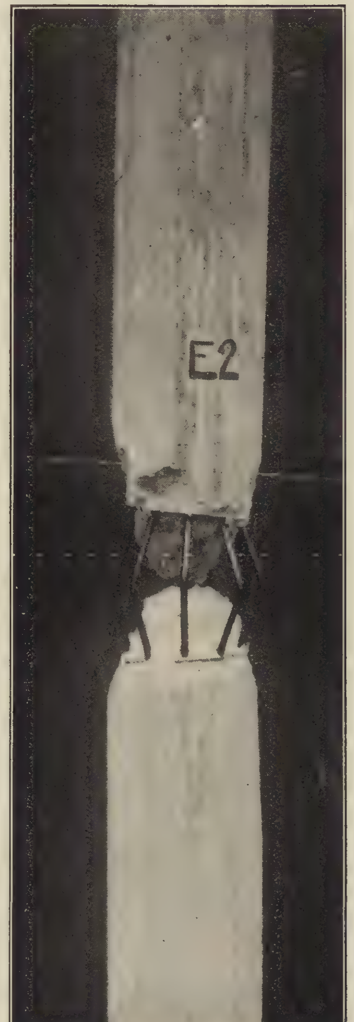
"6. The strength of a column resting upon a footing will be about as great as when bedded on a metal plate, provided considerable lengths of the longitudinal reinforcing bars are bent outward into the slab or footing. The results given show that the use of metal base plates and longitudinal steel milled to their required length leads to greater uniformity and strength.

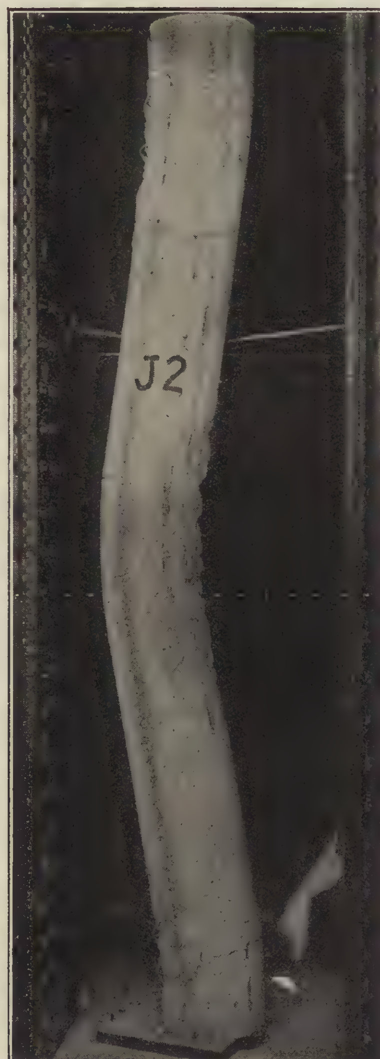
"7. Although only a few test pieces were reinforced with corrugated bars of high carbon steel, the results were so uniform and the strength so high that attention should be called to this type of reinforcement. It is quite evident that, with certain ratios of unit prices, the use of deformed bars with high elastic limits will be more economical than the use of plain round bars of mild steel.

"8. Briefly summarizing the foregoing, it seems economical to use for reinforced concrete columns a very rich mixture, and advantageous to employ about 1 per cent of closely spaced high carbon steel lateral reinforcement combined with two or three per cent of longitudinal reinforcement. From the test data presented it seems apparent that such columns, centrally loaded, may be subjected to a static working stress equal to one-third of the stress at yield point."



The column A4, shown above, illustrates a diagonal shearing failure in a plain concrete column. The failure was sudden—the column breaking with a loud report.





As soon as the yield point had been reached in the vertical steel of column E2, shown above, failure occurred suddenly owing to the insufficient lateral support offered by the widely separated bands. The concrete failed in the same way as in some plain columns. The reinforcement, of course, buckled between the 1/4-in. ties.

The above illustration (column J2) shows the great ductility of a column reinforced with both spiral and longitudinal steel.

70. Working Stresses.—Considering the sudden manner of failure of plain concrete columns, the factor of safety should be relatively large. The Joint Committee recommend a working stress of 22 1/2 per cent of the compressive strength at 28 days, or 450 lb. per square inch on 2000 lb. concrete tested in cylindrical form. For richer and stronger mixtures the working stress may be increased accordingly.

In a column reinforced with longitudinal steel only, the concrete fails in a manner similar to the failure of an unreinforced column. Failure takes place suddenly at approximately the same stress per square inch on the concrete as in a plain concrete column. Thus, the factor of safety employed for unreinforced columns might well be used with respect to the concrete in columns of this type. The value of 15 for n is also recommended. (See Art. 25.) With $n = 15$, the corresponding stress in the steel, for a working stress of 450 lb. per square inch on the concrete, is 6750 lb. per square inch.

The proper working stress for hooped columns should be selected mainly with reference to the elastic limit, but the greater toughness of the hooped column as compared with other types, insures a much larger and more certain margin of safety; in other words, the hooped column should permit the use of a somewhat higher working stress in the concrete than for similar columns without hoops. The Joint Committee recommends a working stress 20 per cent higher than for a plain concrete column, or 540 lb. per square inch on 2000 lb. concrete at the age of 28 days, when the amount of band or spiral reinforcement is not less than 1 per cent of the volume of the column enclosed. It is also recommended that the clear spacing of such bands or hoops shall not be greater than one-fourth the diameter of the enclosed column. The working stress allowed is to be applied to the concrete alone, and the hooping is not to be taken directly into account. With $n = 15$, the corresponding stress in the steel, for a working stress of 540 lb. per square inch on the concrete, is 8100 lb. per square inch.

For hooped columns containing an ordinary amount of longitudinal reinforcement, the elastic limit of the column tends to approach a point corresponding to the elastic limit of the longitudinal steel. One per cent of hooping appears from the results of tests to be sufficient to make possible very high working stresses. The Joint Committee recommends that columns reinforced with

not less than 1 per cent and not more than 4 per cent of longitudinal steel and with bands or spirals spaced not greater than one-fourth the diameter of the enclosed column, shall have an allowable stress on the concrete 45 per cent higher than given for a plain concrete column, or 650 lb. per square inch on 2000 lb. concrete at the age of 28 days. With $n=15$, the corresponding stress in the steel is 9750 lb. per square inch.

When columns reinforced with structural steel are designed in accordance with the principles already stated, and the steel and concrete receive their load simultaneously, the working stresses may be taken the same as for hooped columns with vertical steel rods. This is also in accordance with the Joint Committee's recommendations.

71. Value of Longitudinal Reinforcement in Columns.—When a column is of any considerable length something more than plain concrete is desired in practice. Bending moments in such columns are apt to occur from unforeseen conditions, and tensile stresses may be produced which would rupture a column if without reinforcement. In very large and relatively short columns, little is to be feared from bending stresses, as in such a case no resultant stress is likely to occur, but in small sections where the danger of imperfect spots in the concrete is greatest, steel reinforcement is of great value in producing a more reliable structure.

It should be understood, however, that from a theoretical standpoint the use of steel in columns is not economical. From equation (1) previously given, we see that with a value of $n=15$, the use of each 1 per cent of longitudinal steel adds 14 per cent to the strength of a column, while with 50 as the ratio of cost of steel to cost of concrete per unit volume, the increased cost of a column with 1 per cent of steel will be $50 \times 1 = 50$ per cent. That is, the relative economy of the reinforced column is ordinarily only $\frac{114}{150} = 76$ per cent that of the concrete column without reinforcement.

Illustrative Problem.—The effective area of a column is 144 sq. in.; load to be carried is 80,000 lb.; and working stress on the concrete is 450 lb. per square inch. What percentage of longitudinal bars without hooping will be required? Take $n=15$.

The safe strength of a plain concrete column would be

$$144 \times 450 = 64{,}800 \text{ lb.}$$

Hence, from equation (2),

$$\frac{P}{P'}=\frac{80}{64.8}=1+(15-1)p$$

$$p=1.7 \text{ per cent}$$

Illustrative Problem.—What sectional area of vertical steel will be required for a round column limited to 35 in. diameter, which has to bear 750,000 lb.; working stresses as recommended by Joint Committee?

Using 1 per cent of bands or spirals, a pressure of 650 lb. per square inch may be allowed on the concrete and 9750 lb. per square inch on the steel. One and one-half inch will be considered a sufficient protective covering for the hooped reinforcement. Taking the area within the hooping as effective, the safe strength of a corresponding plain concrete column would be

$$\frac{\pi d^2}{4}(650)=\frac{\pi(32)^2}{4}(650)=523{,}000 \text{ lb.}$$

Hence, from equation (2),

$$\frac{P}{P'}=\frac{750}{523}=1+(15-1)p$$

$$p=3.1 \text{ per cent}$$

Problems

60. What will be the safe strength of a column with a 12-in. × 12-in. effective cross-section, which is reinforced with 1.3 per cent of vertical steel,—the working stress in the concrete being 450 lb. per square inch? Take $n=15$.
61. The effective area of a column is 196 sq. in. and the load to be carried is 120,000 lb. What percentage of vertical steel is required if stresses as recommended by the Joint Committee for a 2000 concrete are employed?
62. Determine the size of square column, reinforced with 2 per cent of longitudinal steel, which will be required to support a load of 100,000 lb. Provide protective covering for steel, and use the working stresses as recommended by the Joint Committee.
63. Find the diameter of a round column, reinforced by 1 per cent of hooping, designed to support a load of 150,000 lb.; stresses as recommended by the Joint Committee.
64. Determine the required area of a column 15 ft. high, supporting 800,000 lb., and reinforced with 3 per cent of longitudinal steel and 1 per cent of hooping; working stresses as recommended by the Joint Committee.
65. What sectional area of vertical steel will be required for a round column limited to 30 in. diameter, which has to bear 500,000 lb.? Use the working stresses recommended by the Joint Committee.

CHAPTER VII

SLAB, BEAM, AND COLUMN TABLES

72. Illustrative Problems.—Some of the illustrative problems of preceding articles will be partly worked out under this heading to show the amount of numerical computations which may be avoided by the use of tables. Working stresses recommended by the Joint Committee will be employed throughout.

1. Design a rectangular beam to span 40 ft. and to support 600 lb. per foot (including weight of beam). Beam is assumed to be simply supported.

From Table 2, for $n=15$, $f_s=16{,}000$, and $f_c=650$.

$$K=107.4$$

$$M=\frac{wl^2}{8}=\frac{(600)(40)^2(12)}{8}=1{,}440{,}000 \text{ in.-lb.}$$

$$bd^2=\frac{1{,}440{,}000}{107.4}=13{,}400$$

Assume $b=18$ in.

$$d^2=\frac{13{,}400}{18}=745, \text{ or } d=27\ 1/2 \text{ in.}$$

Area of cross-section, $bd=(18)(27.5)=495$ sq. in.

$$a_s=(495)(0.0077)=3.81 \text{ sq. in.}$$

We shall select four $1\frac{1}{8}$-in. round rods $=3.98$ sq. in. (See Table 1 or Table 13.)

To make clear the method of reviewing beams, let us review the beam we have designed.

$$p=\frac{a_s}{bd}=\frac{3.98}{495}=0.0080$$

From table 3, for this value of p,

$$k=0.384 \quad j=0.872$$

Then,

$$f_s=\frac{1{,}440{,}000}{(3.98)(0.872)(27.5)}=15{,}100 \text{ lb. per square inch}$$

$$f_c=\frac{(2)(15{,}100)(0.0080)}{0.384}=630 \text{ lb. per square inch}$$

Column 4 of Table 3 shows that $f_s=15{,}600$ when $f_c=650$.

Table 8 could also be used in the design of above beam.

Table 3 may be employed to find minimum allowable depth of beam for a given percentage of steel and various assumed widths. Also, this table may be employed to determine the amount of steel for a beam with given loading, the stress in the concrete being limited to 650 lb. per square inch and the stress in the steel to 16,000 lb. per square inch.

14

To find the depth of beam for a given percentage of steel, select the lower value of K on line with the given percentage. This substituted in formula

$$M = Kbd^2, \text{ or } d = \sqrt{\frac{M}{bK}}$$

gives the smallest permissible depth for various assumed widths.

To find the amount of steel for a given beam under the conditions above stated, compute the value of K from formula $M = Kbd^2$. Locate this value either in column (5) or column (7), whichever satisfies the allowed stresses. Thus, if $K = 80$, it must be located in column (7) instead of column (5), because the latter would give a higher stress in the steel than is allowable. The desired ratio, therefore, is 0.0056. If $K = 112.0$, it must be located in column (5) because column (7) would give too high a stress in the concrete.

Table 3 may also be employed to determine the safe resisting moment of a given beam. The preceding discussion should make clear the method of procedure.

2. Design a rectangular beam to span 10 ft. and to support a load of 4900 lb. per foot. Beam is assumed to be simply supported.

We shall use Table 8 (based on $M = \frac{wl^2}{12}$) in the design of this beam. The weight of beam will be assumed at 400 lb. per foot.

Assuming a width of beam of 14 in., the total load per inch of width is $\frac{5300}{(14)(0.667)} = 567$ lb. per linear foot. Referring directly to the table, we find that the depth (d), corresponding to a 10-ft. beam with this load, is 23 in. The area of reinforcement (a_s) is shown to be equal to $(0.177)(14) = 2.48$ sq. in. We shall select ten 9/16-in. round rods $= 2.485$ sq. in. The spacing of the rods at the center of beam is shown in Fig. 56. At the bottom of Table 8 is given the weight of beam 1 in. wide per linear foot for two rows of steel with a total depth of 26 in., and the corresponding weight is $(27.1)(14) = 379$ lb. The assumed and calculated weights do not differ materially and the beam as designed will be considered satisfactory.

3. What safe load per square foot (including dead weight) can be supported by a slab 6 in. deep ($d = 4$ 3/4 in.) and 10 ft. span, reinforced with 1/2-in. round rods placed 8 in. apart? The slab is simply supported and reinforced in only one direction.

$$p = \frac{0.1963}{(8)(4.75)} = 0.0052$$

From Table 7,

for $p = 0.006$ – safe load for slab 6 in. deep ($d = 4$ 3/4 in.) = 193 lb. per sq. ft.

for $p = 0.004$ – safe load for slab 6 in. deep ($d = 4$ 3/4 in.) $= \frac{131}{62}$ lb. per sq. ft.

$$\begin{aligned} \frac{12}{20}(62) &= 37.2 \\ & 131.0 \\ & \overline{168.2} \text{ safe load, based on } M = \frac{wl^2}{10} \end{aligned}$$

$(168.2)(0.80) = 135$ lb. per square foot, safe load for slab simply supported.

4. Design a slab to span 6 ft. and to carry a live load of 250 lb. per square foot. Slab is to be fully continuous and reinforced in only one direction.

From Table 6 for a span of 6 ft., a 3 3/4-in. ($d = 3$ in.) slab, fully continuous, is seen to sustain a load of $(269)(1.2) = 323$ lb. per square foot. Corre-

sponding weight of slab is 47 lb. per square foot and the total load for the slab to carry is thus 297 lb. This depth of slab will be considered satisfactory and is on the side of safety.

The area of steel per foot of breadth may be taken directly from Table 6 and, for the depth of slab chosen above, $a_s = 0.277$ sq. in. From Table 4 we may use 3/8 round rods spaced 4 3/4 in. on centers, or 7/16-in. rods spaced 6 1/2 in., or 1/2-in. rods spaced 8 1/2 in., etc.

5. Design the center cross-section of a T-beam in a floor system; the beam is to have a span of 12 ft. and be fully continuous. Maximum shear (live plus dead) is closely equal to 12,200 lb. Maximum moment (live plus dead) $=356,300$ in.-lb. Supported slab is 6 in. thick.

The breadth of the flange is controlled by one-fourth the span, or 36 in. Using approximate formula (b) of Art. 59, assuming a depth (d) of 16 in.

$$a_s = \frac{M}{(f_s)(d-1/2t)} = \frac{356,300}{(16,000)(13.0)} = 1.71 \text{ sq. in.}$$

and
$$p = \frac{1.71}{(36)(16.0)} = 0.0030$$

also,
$$\frac{t}{d} = \frac{6.0}{16.0} = 0.375$$

Referring to Table 9, Part 1, it is seen at once that this beam falls under Case I; that is, the neutral axis is in the flange. For the above value of p, Table 3 gives $j = 0.914$. The corrected value of a_s is

$$a_s = \frac{M}{f_s jd} = \frac{356,300}{(16,000)(0.914)(16.0)} = 1.53 \text{ sq. in.}$$

and
$$p = \frac{1.53}{(36)(16.0)} = 0.0027$$

From Table 3 the stress in the concrete is found to be 350 lb. per square inch. The beam as designed is thus satisfactory.

6. The flange of a T-beam is 24 in. wide and 4 in. thick. The beam is to sustain a bending moment of 480,000 in.-lb. What depth of beam and amount of steel is necessary?

Try $d = 18$ in. Approximately, $jd = 16$ in. Then formula (8) or formula (b), Art. 59, gives:

$$a_s = \frac{M}{f_s jd} = \frac{480,000}{(16,000)(16)} = 1.88 \text{ sq. in.}$$

and
$$p = \frac{1.88}{(24)(18)} = 0.0043$$

also,
$$\frac{t}{d} = \frac{4}{18} = 0.222$$

Referring to Table 9, Part 2,

$$\text{for } p = 0.004 \text{ and } \frac{t}{d} = 0.222 \ldots\ldots\ldots\ldots \begin{cases} j = 0.911 \\ f_c = 456 \end{cases}$$

$$\text{for } p = 0.006 \text{ and } \frac{t}{d} = 0.222 \ldots\ldots\ldots\ldots \begin{cases} j = 0.905 \\ f_c = 619 \end{cases}$$

Thus,

$$\text{for } p = 0.0043 \text{ and } \frac{t}{d} = 0.222 \ldots\ldots\ldots\ldots \begin{cases} j = 0.910 \\ f_c = 480 \end{cases}$$

The corrected value of a_s is

$$\frac{480,000}{(16,000)(0.910)(18)} = 1.84 \text{ sq. in.}$$

Another and perhaps better method of arriving at the same results is as follows:

$$\frac{M}{bd^2}=\frac{480{,}000}{(24)(18)^2}=61.6$$

For this value of $\frac{M}{bd^2}$ and for $\frac{t}{d}=0.222$, Table 9, Part 2, shows p to have a value between 0.004 and 0.006.

$$\begin{array}{ll} 86.9 & 61.6 \\ \underline{58.4} & \underline{58.4} \\ 28.5 & 3.2 \end{array} \qquad p=0.004+\frac{3.2}{28.5}(0.002)=0.0042$$

$$f_c=456+\frac{3.2}{28.5}(163)=475 \text{ lb. per square inch.}$$

$$a_s=(24)(18)(0.0042)=1.82 \text{ sq. in.}$$

Suppose that the flange of the above beam had been made 5 in. thick and that the depth (d) had been taken at 16 1/2 in.
Then,

$$a_s=\frac{480{,}000}{(16{,}000)(14)}=2.14 \text{ sq. in.}$$

and

$$p=\frac{2.14}{(24)(16.5)}=0.0054$$

also,

$$\frac{t}{d}=\frac{5}{16.5}=0.30$$

Referring to Table 9, Part 2,

$$\text{for } p=0.0043 \text{ and } \frac{t}{d}=0.30 \ldots\ldots\ldots\ldots \begin{cases} j=0.901 \\ f_c=\ \ 457 \end{cases}$$

$$\text{for } p=0.0060 \text{ and } \frac{t}{d}=0.30 \ldots\ldots\ldots\ldots \begin{cases} j=0.888 \\ f_c=\ \ 565 \end{cases}$$

Thus,

$$\text{for } p=0.0054 \text{ and } \frac{t}{d}=0.30 \ldots\ldots\ldots\ldots \begin{cases} j=0.893 \\ f_c=\ \ 527 \end{cases}$$

The corrected value of a_s is

$$\frac{480{,}000}{(16{,}000)(0.893)(16.5)}=2.04 \text{ sq. in.}$$

7. Design a T-beam with span of 40 ft. Assume dead load $=1400$ lb. per foot. Live load $=3000$ lb. per foot. The beam is to be simply supported at the ends and the flange is to be proportioned as well as the web; that is, the flange does not form a part of a floor system already determined.

From Art. 61, $b'=18$ in. and $d=47$ in. are suitable dimensions and a thickness of flange of 12 in. is tried. The total bending moment on the beam is 10,560,000 in.-lb.

$$\frac{t}{d}=\frac{12}{47}=0.256$$

From Table 10.

$$\frac{z}{t}=0.414 \quad z=4.97 \quad jd=42.0$$

From Formula (7), Art. 59.

$$a_s=\frac{10{,}560{,}000}{(16{,}000)(42.0)}=15.7 \text{ sq. in.}$$

The detailed design of this beam has been given at the end of Art. 61.

8. A continuous T-beam, uniformly loaded, has a bending moment at the center of each span of 358,000 in.-lb. Negative bending moment at the supports and the positive bending moment at the center of span are figured by the formula, $M=\frac{wl^2}{12}$. The tensile steel at the center of span consists of four 3/4 in. round rods. $b'=9$ in. $d=15.5$ in. Design the supports.

Two of the tensile rods on each side of the supports will be bent up and made to lap over the top of the supports, while the other two rods on each side will be continued straight and lapped over supports at the bottom of beam.

The ratios of steel in tension and compression are the same, and are respectively:

$$p=p'=\frac{1.77}{(9)(15.5)}=0.013$$

(1.77 in above computations taken from Table 13.) We will assume $\frac{d'}{d}=0.1$ as before.

From Table 11, Part 2, knowing $p'=p$, we obtain

$$\text{for } p=0.010 \ldots\ldots\ldots\ldots \begin{cases} L=0.250 \\ K=0.0089 \end{cases}$$

$$\text{for } p=0.015 \ldots\ldots\ldots\ldots \begin{cases} L=0.318 \\ K=0.0133 \end{cases}$$

Thus,

$$\text{for } p=0.013 \ldots\ldots\ldots\ldots \begin{cases} L=0.291 \\ K=0.0115 \end{cases}$$

Maximum pressure in concrete is

$$f_c=\frac{358{,}000}{(9)(15.5)^2(0.291)}=570 \text{ lb. per square inch.}$$

Also,

$$f_s=\frac{358{,}000}{(9)(15.5)^2(0.0115)}=14{,}400 \text{ lb. per square inch.}$$

9. The effective area of a column is 144 sq. in.; load to be carried is 80,000 b.; and working stress on the concrete is 450 lb. per square inch. What percentage of longitudinal bars without hooping will be required? Take $n=15$.

The safe strength of a plain concrete column would be

$$144\times450=64{,}800 \text{ lb.}$$

Hence,

$$\frac{P}{P'}=\frac{80}{64.8}=1.235$$

From Table 12, for $n=15$, and $p=0.017$

$$\frac{P}{P'}=1.238$$

Thus, 1.7 per cent of steel is required, and

$$A_s=(144)(0.017)=2.45 \text{ sq. in.}$$

From Table 13, four 7/8-in. round rods will be seen to have about the required area.

When bands or spirals are used, Table 14 gives the sectional area of hooping for a maximum pitch of one-fourth the diameter of the enclosed concrete—also for a pitch somewhat less than this but which varies for the different column diameters. The amount of hooped reinforcement taken is 1 per cent of the volume of the column. This conforms to the recommendations of the Joint Committee.

Let $D =$ diameter of enclosed concrete.
$A_h =$ sectional area of one strand of hooping for given pitch.
$P =$ pitch allowed.
$l_h =$ length of hooping in 1 ft. in height of column.

Then, for banded reinforcement

$$(0.01)\left(\frac{\pi D^2}{4}\right)(P) = A_h \pi D$$
$$A_h = 0.0025\ PD$$

Also,

$$A_h l_h = \frac{\pi D^2}{4}(12)(0.01)$$
$$l_h = \frac{37.7D}{P}$$

Results for banded and spiral reinforcement will not differ appreciably and Table 14 may be used for both bands and spirals.

Problems

In a certain building, it was necessary to space the columns 12 ft. centers and to have beams in one direction only. The specifications called for the following items:

For slabs, $M = 1/12wl^2$; beams, $M = 1/10wl^2$.

Loadings:

Dead load, actual weight, concrete weighing 150 lb. per cubic foot.
Live load, 250 lb. per square foot.
Unit stresses as recommended by Joint Committee.

66. (a) What thickness (total) of floor slabs required?
(b) What will be the spacing of the steel reinforcement if 1/2 in. round bars are used?

67. Design the beams to support the floor, making $b' = 10$ in. and assuming weight of beam below slab $= 210$ lb. per linear foot. Make $b'd$ just large enough to resist shear due to load, considering the web properly reinforced. Required: b, b', d, a_s, and the number of 5/8 in. round rods required.

68. What will be the stress in the concrete at the bottom of the beam at the column if one-half of the rods from each side are bent up and the horizontal rods are lapped at the support? Use eight 5/8-in. round rods at center of span.

69. A beam 12 in. × 22 in. in cross-section has four 7/8-in. round rods placed 2 in. above the lower face of the beam. If the allowable stresses as recommended by the Joint Committee are employed
(a) What uniform load in pounds per linear foot could be applied to the span? Span $= 16$ ft. $M = 1/8\ wl^2$.
(b) What is the maximum stress developed in the steel?

70. Given a column load of 80 tons, what size of square column and what total area of steel cross-section is required if 3 per cent of longitudinal reinforcement is employed?

71 Given a column whose effective dimensions are 20 in.$\times$20 in. and whose cross-sectional area of steel amounts to 14 sq. in. What load will the column sustain?

72. Given a column load of 200 tons, what will be the design of hooped column required, if the area of longitudinal rods is taken equal to 3 per cent of the sectional area of the enclosed concrete?

73. (a) Using Tables 2 and 3 design a rectangular beam to span 15 ft. and to carry a live load of 1500 lb. per linear foot. Beam is to be simply supported.

(b) Design this beam using Table 8.

74. The flange of a T-beam is 48 in. wide and 4 in. thick. The beam is to sustain a bending moment of 800,000 in.-lb. Assume the ratio of unit cost of steel to cost of concrete = 60. Take $b' = 9$ in. and determine the economical depth of beam, also the amount of steel necessary. Consider the computed cross-section as sufficient to resist shear.

TABLE 1.—AREAS, PERIMETERS, AND WEIGHTS OF RODS

	Round rods			Square rods		
Size inches	Area square inches	Perimeter inches	Weight per foot pounds	Area square inches	Perimeter inches	Weight per foot pounds
1/4	.0491	.785	.17	.0625	1.000	.21
5/16	.0767	.982	.26	.0977	1.25	.33
3/8	.1104	1.178	.38	.1406	1.50	.48
7/16	.1503	1.374	.51	.1914	1.75	.65
1/2	.1963	1.571	.67	.2500	2.00	.85
9/16	.2485	1.767	.85	.3164	2.25	1.08
5/8	.3068	1.964	1.04	.3906	2.50	1.33
11/16	.3712	2.160	1.26	.4727	2.75	1.61
3/4	.4418	2.356	1.50	.5625	3.00	1.91
13/16	.5185	2.553	1.76	.6602	3.25	2.25
7/8	.6013	2.749	2.04	.7656	3.50	2.60
15/16	.6903	2.945	2.35	.8789	3.75	2.99
1	.7854	3.142	2.67	1.0000	4.00	3.40
1 1/8	.9940	3.534	3.38	1.2656	4.50	4.30
1 1/4	1.2272	3.927	4.17	1.5625	5.00	5.31
1 3/8	1.4849	4.320	5.05	1.8906	5.50	6.43
1 1/2	1.7671	4.712	6.01	2.2500	6.00	7.65
1 5/8	2.0739	5.105	7.05	2.6406	6.50	9.98
1 3/4	2.4053	5.498	8.18	3.0625	7.00	10.41
1 7/8	2.7612	5.891	9.39	3.5156	7.50	11.95
2	3.1416	6.283	10.68	4.0000	8.00	13.60
2 1/4	3.9761	7.069	13.52	5.0625	9.00	17.22
2 1/2	4.9087	7.854	16.69	6.2500	10.00	21.25
2 3/4	5.9396	8.639	20.20	7.5625	11.00	25.72
3	7.0686	9.425	24.03	9.0000	12.00	30.09

TABLE 2.—DATA FOR DESIGN OF RECTANGULAR BEAMS[1]

Formulas needed:

$$M = Kbd^2, \text{ or } bd^2 = \frac{M}{K}$$

$$a_s = pbd$$

Formulas used in preparing table:

$$p = \frac{1/2}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)}$$

$$k = \sqrt{2pn + (pn)^2} - pn \quad j = 1 - 1/3k$$

$K = pf_sj$, or $1/2 f_ckj$ (from formula $M = Kbd^2$)

Working stresses		Ratio of Moduli $n=12$				Ratio of Moduli $n=15$			
f_s	f_c	k	j	p	K	k	j	p	K
12,000	500	0.332	0.889	0.0069	73.6	0.384	0.872	0.0080	83.7
	550	0.354	0.882	0.0081	85.7	0.407	0.864	0.0093	96.4
	600	0.375	0.875	0.0094	98.4	0.428	0.857	0.0107	110.0
	650	0.394	0.869	0.0107	111.3	0.448	0.851	0.0121	123.6
	700	0.412	0.863	0.0120	124.4	0.467	0.844	0.0136	138.0
	750	0.429	0.857	0.0134	137.8	0.484	0.839	0.0151	152.0
	800	0.444	0.852	0.0148	151.3	0.501	0.833	0.0167	166.9
14,000	500	0.300	0.900	0.0054	67.5	0.348	0.884	0.0062	76.7
	550	0.320	0.893	0.0063	78.6	0.372	0.876	0.0073	89.5
	600	0.340	0.888	0.0073	90.6	0.391	0.870	0.0084	102.0
	650	0.358	0.881	0.0083	102.5	0.410	0.863	0.0095	114.8
	700	0.375	0.875	0.0094	114.8	0.428	0.857	0.0107	128.3
	750	0.391	0.870	0.0105	127.6	0.446	0.851	0.0120	142.3
	800	0.407	0.864	0.0116	140.4	0.462	0.846	0.0132	156.3
15,000	500	0.286	0.905	0.0048	64.7	0.334	0.889	0.0056	74.1
	550	0.306	0.898	0.0056	75.4	0.355	0.882	0.0065	86.1
	600	0.325	0.892	0.0065	86.7	0.375	0.875	0.0075	98.3
	650	0.343	0.886	0.0074	98.4	0.393	0.869	0.0085	111.3
	700	0.360	0.880	0.0084	110.3	0.411	0.863	0.0096	124.2
	750	0.376	0.875	0.0094	123.1	0.429	0.857	0.0107	137.9
	800	0.391	0.870	0.0105	135.7	0.445	0.852	0.0118	151.2
16,000	500	0.273	0.909	0.0043	62.0	0.319	0.894	0.0050	71.3
	550	0.292	0.903	0.0050	72.2	0.339	0.887	0.0058	82.3
	600	0.310	0.897	0.0058	83.2	0.358	0.881	0.0067	94.4
	650	0.328	0.891	0.0067	95.0	0.378	0.874	0.0077	107.4
	700	0.344	0.885	0.0075	106.2	0.397	0.868	0.0087	120.6
	750	0.361	0.880	0.0085	119.1	0.414	0.862	0.0097	133.8
	800	0.375	0.875	0.0094	131.3	0.429	0.857	0.0107	146.7
20,000	500	0.230	0.923	0.0029	53.1	0.272	0.909	0.0034	61.8
	550	0.248	0.917	0.0034	62.4	0.292	0.903	0.0040	72.2
	600	0.264	0.912	0.0040	72.2	0.311	0.897	0.0047	83.7
	650	0.280	0.907	0.0046	82.4	0.328	0.891	0.0053	94.4
	700	0.295	0.902	0.0052	93.3	0.344	0.885	0.0060	106.2
	750	0.309	0.897	0.0058	103.9	0.359	0.880	0.0067	117.9
	800	0.324	0.892	0.0065	115.6	0.374	0.875	0.0075	130.9

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 519.

TABLE 3.—DATA FOR REVIEWING RECTANGULAR BEAMS

$n=15$

Formulas needed:

$$p=\frac{a_s}{bd}$$

$$\frac{M}{bd^2}=K$$

Formulas used in preparing table:

$$k=\sqrt{2pn+(pn)^2}-pn$$

$$j=1-1/3k$$

$$f_c=\frac{2f_sp}{k}$$

$K=pf_sj$, or $1/2f_ckj$ (from formula $M=Kbd^2$)

p	k	j	$f_c=650$ lb. per square inch		$f_s=16{,}000$ lb. per square inch	
			f_s	K	f_c	K
(1)	(2)	(3)	(4)	(5)	(6)	(7)
.0020	0.217	0.928	35300	65.5	295	29.7
.0022	0.226	0.925	33400	68.0	310	32.4
.0024	0.235	0.922	31800	70.4	330	35.8
.0026	0.243	0.919	30400	72.6	340	38.0
.0028	0.251	0.916	29100	74.6	360	41.4
.0030	0.258	0.914	28000	76.8	370	43.6
.0032	0.265	0.912	26900	78.5	390	47.1
.0034	0.273	0.909	26100	80.7	400	49.6
.0036	0.279	0.907	25200	82.1	410	51.9
.0038	0.285	0.905	24400	83.9	430	55.1
.0040	0.292	0.903	23700	85.6	440	58.2
.0042	0.298	0.901	23100	87.4	450	60.4
.0044	0.303	0.899	22400	88.6	465	63.3
.0046	0.309	0.897	21800	89.8	480	66.5
.0048	0.314	0.895	21100	90.6	490	68.8
.0050	0.320	0.893	20800	92.8	500	71.4
.0052	0.325	0.892	20300	94.2	510	73.9
.0054	0.330	0.890	19900	95.6	520	76.4
.0056	0.334	0.889	19400	96.6	540	80.1
.0058	0.339	0.887	19000	97.7	550	82.7
.0060	0.344	0.885	18600	98.8	560	85.2
.0062	0.348	0.884	18300	100.3	570	87.7
.0064	0.353	0.882	18000	101.6	580	90.6
.0066	0.357	0.881	17600	102.3	590	92.8
.0068	0.361	0.880	17300	103.5	600	95.3
.0070	0.365	0.878	16900	103.9	610	97.7
.0072	0.369	0.877	16700	105.4	625	101.1
.0074	0.373	0.876	16400	106.3	635	103.7
.0076	0.377	0.875	16100	107.1	645	106.4
.0078	0.381	0.873	15900	108.3	655	108.1
.0080	0.384	0.872	15600	108.8	670	112.2
.0082	0.388	0.871	15400	110.0	680	114.9
.0084	0.392	0.869	15200	110.9	690	117.5
.0086	0.395	0.868	15000	112.0	700	120.0
.0088	0.398	0.867	14700	112.2	710	122.5
.0090	0.402	0.866	14500	113.0	720	125.3
.0092	0.405	0.865	14300	113.8	725	127.0
.0094	0.408	0.864	14100	114.5	740	130.4
.0096	0.411	0.863	13900	115.2	750	133.0
.0098	0.415	0.862	13800	115.6	760	135.9
.010	0.418	0.861	13600	117.1	770	138.6
.012	0.446	0.851	12100	123.6	860	163.2
.014	0.471	0.843	11000	129.8	950	188.6
.016	0.493	0.836	10000	133.8	1040	214.3
.018	0.513	0.829	9300	138.8	1120	238.1
.020	0.531	0.823	8600	141.6	1210	264.4

TABLE 4.—SPACING OF ROUND RODS IN SLABS

Diameter inches	Sectional area of steel per foot of slab when spaced as follows:													
	2″	2½″	3″	3½″	4″	4½″	5″	5½″	6″	7″	8″	9″	10″	12″
¼	.29	.23	.20	.17	.15	.13	.12	....	....	....	....	....	....	....
5/16	.46	.36	.31	.26	.23	.20	.18	.17	.15	.13	....	....	....	....
⅜	.66	.53	.44	.38	.33	.29	.26	.24	.22	.19	.17	.15	.13	....
7/16	.90	.72	.60	.51	.45	.40	.36	.33	.30	.26	.23	.20	.18	.15
½	1.18	.94	.78	.67	.59	.52	.47	.43	.39	.34	.29	.26	.24	.20
9/16	1.49	1.19	.99	.85	.75	.66	.60	.54	.50	.43	.37	.33	.30	.25
⅝	1.84	1.47	1.23	1.05	.92	.82	.74	.67	.61	.53	.46	.41	.37	.31
11/16	2.23	1.78	1.48	1.27	1.11	.99	.89	.81	.74	.64	.56	.49	.45	.37
¾	2.65	2.12	1.77	1.51	1.32	1.18	1.06	.96	.88	.76	.66	.59	.53	.44
13/16	3.11	2.48	2.07	1.78	1.56	1.38	1.24	1.13	1.04	.89	.78	.69	.62	.52
⅞	3.61	2.88	2.40	2.06	1.80	1.60	1.44	1.31	1.20	1.03	.90	.80	.72	.60
15/16	4.14	3.31	2.76	2.37	2.07	1.84	1.66	1.51	1.38	1.18	1.03	.92	.83	.69
1	4.71	3.77	3.14	2.69	2.36	2.09	1.88	1.71	1.57	1.35	1.18	1.05	.94	.78
1⅛	.....	4.77	3.98	3.41	2.98	2.65	2.39	2.17	1.99	1.70	1.49	1.33	1.19	.99
1¼	.....	.....	4.91	4.21	3.68	3.27	2.95	2.68	2.45	2.10	1.84	1.64	1.47	1.23
1⅜	.....	.....	.....	5.09	4.45	3.96	3.56	3.24	2.97	2.55	2.23	1.98	1.78	1.48
1½	.....	.....	.....	.....	5.30	4.71	4.24	3.86	3.53	3.03	2.65	2.36	2.12	1.77

TABLE 5.—SPACING OF SQUARE RODS IN SLABS

Dimension inches	Sectional area of steel per foot of slab when spaced as follows:													
	2″	2½″	3″	3½″	4″	4½″	5″	5½″	6″	7″	8″	9″	10″	12″
¼	.37	.30	.25	.21	.19	.17	.15	.13	.12	....	....	....	....	....
5/16	.59	.47	.39	.33	.29	.26	.23	.21	.19	.17	.15	.13	....	....
⅜	.84	.67	.56	.48	.42	.37	.34	.31	.28	.24	.21	.19	.17	.14
7/16	1.15	.92	.77	.66	.57	.51	.46	.42	.38	.33	.29	.25	.23	.19
½	1.50	1.20	1.00	.86	.75	.67	.60	.55	.50	.43	.37	.33	.30	.25
9/16	1.90	1.52	1.27	1.08	.95	.84	.76	.69	.63	.54	.47	.42	.38	.32
⅝	2.34	1.87	1.56	1.34	1.17	1.04	.94	.85	.78	.67	.59	.52	.47	.39
11/16	2.84	2.27	1.99	1.62	1.42	1.33	1.13	1.03	.94	.81	.71	.66	.57	.47
¾	3.37	2.70	2.25	1.93	1.69	1.50	1.35	1.23	1.12	.96	.84	.75	.67	.56
13/16	3.96	3.17	2.64	2.26	1.98	1.76	1.58	1.44	1.32	1.13	.99	.88	.79	.66
⅞	4.59	3.67	3.06	2.62	2.30	2.04	1.84	1.67	1.53	1.31	1.15	1.02	.92	.77
15/16	5.27	4.22	3.52	3.01	2.64	2.34	2.11	1.92	1.76	1.51	1.32	1.17	1.05	.88
1	6.00	4.80	4.00	3.43	3.00	2.67	2.40	2.18	2.00	1.71	1.50	1.33	1.20	1.00
1⅛	.....	6.08	5.06	4.34	3.80	3.37	3.04	2.76	2.53	2.17	1.89	1.69	1.52	1.27
1¼	.....	.....	6.25	5.36	4.69	4.17	3.75	3.41	3.12	2.68	2.34	2.08	1.87	1.56
1⅜	.....	.....	.....	6.48	5.67	5.04	4.54	4.12	3.78	3.24	2.84	2.52	2.27	1.89
1½	.....	.....	.....	.....	6.75	6.00	5.40	4.91	4.50	3.86	3.37	3.00	2.70	2.25

TABLE 6.—USE FOR DESIGNING SLABS

Based on $M=\frac{wl^2}{10}$. For supported ends, $\left(M=\frac{wl^2}{8}\right)$ deduct 20 per cent from load.

For fully continuous, $\left(M=\frac{wl^2}{12}\right)$, add 20 per cent to load.

$f_c=650$ lb. per square inch. $f_s=16{,}000$ lb. per square inch. $n=15$. $p=0.0077$. $k=0.378$. $b=12$ in.

$M=pf_sjbd^2=1290\ d^2$ (safe moment of resistance)

$a_s=bpd=0.0924\ d$ (steel area)

$\frac{wl^2}{10}(12)=pf_sjbd^2$, or $w=1074\frac{d^2}{l^2}$ (safe load)

Depth to steel	Depth below steel	Total depth of slab	Total safe load (w) per square foot including weight of slab. For safe live load deduct weight of slab												Weight of slab per square foot	Steel area in a section of slab 1 ft. wide	Safe moment of resistance
			Span in feet (l)														
in.	in.	in.	4	5	6	7	8	9	10	11	12	13	14	15	lb.	sq. in.	in.-lb.
1¾	¾	2½	205	132	91	67	51	40	...	...	...	...	...	...	31	0.162	3950
2	¾	2¾	268	172	120	88	67	53	43	35	...	...	...	...	34	0.185	5160
2¼	¾	3	339	217	151	111	85	67	54	45	...	...	...	...	38	0.208	6530
2½	¾	3¼	419	268	187	137	105	83	67	55	47	...	...	...	41	0.231	8060
2¾	¾	3½	507	325	226	165	127	100	81	67	57	48	...	...	44	0.254	9750
3	¾	3¾	605	387	269	197	151	119	97	80	67	57	49	...	47	0.277	11600
3¼	¾	4	709	454	315	232	177	140	113	94	79	67	58	50	50	0.300	13600
3½	1	4½	822	527	366	269	206	162	132	109	91	78	67	58	56	0.323	15800
4	1	5	1074	688	478	351	269	212	172	142	119	102	88	76	62	0.370	20600
4½	1	5½	1360	871	605	444	340	269	218	180	151	129	111	97	69	0.416	26100
4¾	1¼	6	1520	971	675	496	380	300	243	201	169	144	124	108	75	0.438	29100
5¼	1¼	6½	1850	1190	825	606	463	366	297	245	206	175	151	132	81	0.485	35600
5¾	1¼	7	2220	1420	990	726	556	438	356	294	247	210	182	158	87	0.531	42700
6¼	1¼	7½	2620	1680	1170	858	657	519	420	347	292	248	214	187	94	0.578	50500
6¾	1¼	8	3070	1960	1360	1000	766	605	490	405	340	290	250	218	100	0.624	58900
7¼	1¼	8½	3540	2260	1570	1150	884	698	566	467	392	335	289	252	106	0.670	67900
7¾	1¼	9	4040	2580	1800	1320	1010	798	646	534	449	382	330	288	113	0.716	77600
8¼	1¼	9½	4580	2930	2030	1490	1140	904	733	605	509	434	374	326	119	0.762	87900
8¾	1¼	10	5150	3300	2290	1680	1290	1020	824	680	572	487	420	366	125	0.808	98800

TABLE 7.—USE FOR REVIEWING SLAB DESIGNS

Based on $M=\frac{wl^2}{10}$.

$f_c =$ or $<$ 650 lb. per square inch.

$n=15$.

$f_s =$ or $<$ 16,000 lb. per square inch.

For supported ends $(M=\frac{wl^2}{8})$, deduct 20 per cent from loads.

For fully continuous, $(M=\frac{wl^2}{12})$, add 20 per cent to loads.

$$M = \text{lesser of } \begin{matrix} M_s = pf_sjbd^2 \\ M_c = \frac{1}{2}f_ckjbd^2 \end{matrix} \text{ (safe moment of resistance)}$$

$$a_s = bpd = 12pd \text{ (steel area)}$$

$$\left.\begin{matrix} \frac{wl^2}{10}(12) = pf_sjbd^2 \\ \frac{wl^2}{10}(12) = \frac{1}{2}f_ckjbd^2 \end{matrix}\right\} \text{and using Table 3}$$

For $p=0.002$, $w=297\,\frac{d^2}{l^2}$ (safe load)

For $p=0.004$, $w=582\,\frac{d^2}{l^2}$ (safe load)

For $p=0.006$, $w=852\,\frac{d^2}{l^2}$ (safe load)

For $p=0.008$, $w=1088\frac{d^2}{l^2}$ (safe load)

For $p=0.010$, $w=1171\frac{d^2}{l^2}$ (safe load)

p	Depth to steel	Depth below steel	Total depth of slab	Total safe load (w) per square foot including weight of slab. For safe live load deduct weight of slab — Span in feet (l)												Weight of slab per square foot	Steel area in a section of slab 1 ft. wide	Safe moment of resistance
	in.	in.	in.	4	5	6	7	8	9	10	11	12	13	14	15	lb.	sq. in.	in.-lb.
0.002	2¼	¾	3	94	60	41	....	....	....	...	...	...	...	...	...	38	0.054	1800
	3¼	¾	4	196	125	87	64	49	....	...	...	...	...	...	...	50	0.078	3760
	4	1	5	297	190	132	97	74	59	47						62	0.096	5700
	4¾	1¼	6	420	268	186	137	105	83	67	55	47	...	...	...	75	0.114	8060
	5¾	1¼	7	615	393	273	200	153	121	98	81	68	58	50	44	87	0.138	11800
	6¾	1¼	8	847	542	376	276	212	167	135	112	94	80	69	60	100	0.162	16300
	7¾	1¼	9	1120	715	496	364	279	220	179	147	124	106	91	79	113	0.186	21400
	8¾	1¼	10	1420	910	632	464	356	281	228	188	158	135	116	101	125	0.210	27300
0.004	2¼	¾	3	184	118	82	60	46	....	...	...	...	...	...	...	38	0.108	3540
	3¼	¾	4	384	246	170	125	96	76	61	51	...	...	...	...	50	0.156	7380
	4	1	5	582	372	258	190	145	115	93	77	65	55	47	...	62	0.192	11200
	4¾	1¼	6	823	526	366	268	206	162	131	109	91	78	67	58	75	0.228	15800
	5¾	1¼	7	1200	770	535	393	301	238	193	159	134	114	98	86	87	0.276	23100
	6¾	1¼	8	1660	1060	738	542	415	328	267	219	184	157	135	118	100	0.324	31800
	7¾	1¼	9	2190	1400	972	714	547	432	350	289	243	207	178	155	113	0.372	42000
	8¾	1¼	10	2790	1780	1240	910	697	551	446	368	310	264	228	198	125	0.420	53500

TABLE 7.—*Continued.*

p	Depth to steel	Depth below steel	Total depth of slab	Total safe load (w) per square foot including weight of slab For safe live load deduct weight of slab — Span in feet (l)												Weight of slab per square foot	Steel area in a section of slab 1 ft. wide	Safe moment of resistance
	in.	in.	in.	4	5	6	7	8	9	10	11	12	13	14	15	lb.	sq. in.	in.-lb.
0.006	2¼	¾	3	270	173	120	88	67	53	43	...	...	...	...	...	38	0.162	5160
	3¼	¾	4	562	360	250	183	140	111	90	74	62	53	...	...	50	0.234	10800
	4	1	5	852	545	379	278	213	168	136	113	95	81	70	...	62	0.288	16300
	4¾	1¼	6	1200	771	535	393	301	238	193	159	134	114	98	86	75	0.342	23100
	5¾	1¼	7	1760	1130	784	575	441	348	282	233	196	167	144	125	87	0.414	33800
	6¾	1¼	8	2430	1550	1080	793	607	480	388	321	270	230	198	173	100	0.486	46600
	7¾	1¼	9	3200	2050	1420	1040	800	632	512	423	356	303	261	228	113	0.558	61500
	8¾	1¼	10	4080	2610	1810	1330	1020	806	653	540	453	386	333	290	125	0.630	78300
0.008	2¼	¾	3	344	220	153	112	86	68	55	45	...	...	...	...	38	0.216	6610
	3¼	¾	4	718	459	319	234	179	142	115	95	80	68	59	...	50	0.312	13800
	4	1	5	1090	696	483	355	272	215	174	144	121	103	89	77	62	0.384	20900
	4¾	1¼	6	1540	984	682	502	333	303	246	203	171	145	125	109	75	0.456	29500
	5¾	1¼	7	2250	1440	1000	735	561	444	360	297	250	213	184	160	87	0.552	43200
	6¾	1¼	8	3100	1990	1380	1010	774	612	496	409	344	293	253	220	100	0.648	59500
	7¾	1¼	9	4080	2620	1820	1330	1020	807	654	540	454	387	334	291	113	0.743	78500
	8¾	1¼	10	5210	3340	2320	1700	1300	1030	834	688	579	493	426	370	125	0.840	100000
0.010	2¼	¾	3	370	237	165	121	93	73	59	49	...	...	...	...	38	0.270	7100
	3¼	¾	4	773	495	344	252	193	153	124	102	86	73	63	55	50	0.390	14800
	4	1	5	1170	750	520	383	293	231	187	155	130	111	96	83	62	0.480	22500
	4¾	1¼	6	1650	1060	734	539	414	326	264	218	184	157	135	118	75	0.570	31800
	5¾	1¼	7	2420	1550	1070	790	606	478	388	320	269	229	198	172	87	0.690	46500
	6¾	1¼	8	3340	2140	1480	1090	835	659	534	441	371	316	272	237	100	0.810	64100
	7¾	1¼	9	4400	2820	1950	1430	1100	869	704	581	489	417	360	313	113	0.930	84500
	8¾	1¼	10	5610	3590	2490	1830	1400	1110	897	741	623	531	459	399	125	1.050	107800

TABLE 8.—USE FOR CONTINUOUS RECTANGULAR BEAMS

Safe Loading and Reinforcement for Beams 1 In. in Width

Based on $M=\frac{wl^2}{12}$, $n=15$
$f_s=16{,}000$ lb. per square inch.
$f_c=650$ lb. per square inch.

For $\left(M=\frac{wl^2}{10}\right)$ deduct $\left(16\frac{2}{3}\%\right)$, $\left(M=\frac{wl^2}{8}\right)$ deduct $\left(33\frac{1}{3}\%\right)$ from safe loads, using same steel area.

Depth to steel (d)	Total safe load (w) per linear foot for beam one inch wide including weight of beam (See footnotes) — Span in feet (l)																			Steel area in a beam one inch wide (a_s)	Safe moment of resistance (M)
in.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	sq. in.	in.-lb.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
6.0	107	79	60	48	39	32	27	23	20	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.046	3870
6.5	126	93	71	56	45	38	31	27	23	20	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.050	4540
7.0	146	107	82	65	52	43	36	31	27	23	21	.....	.....	.....	.....	.....	.....	.....	.....	0.054	5260
7.5	168	123	94	75	60	50	42	36	31	27	24	21	.....	.....	.....	.....	.....	.....	.....	0.058	6040
8.0	191	140	107	85	69	57	48	41	35	31	27	24	21	.....	.....	.....	.....	.....	.....	0.062	6870
8.5	216	158	121	96	78	64	54	46	40	34	30	27	24	21	.....	.....	.....	.....	.....	0.065	7760
9.0	242	178	136	107	87	72	60	51	44	39	34	30	27	24	22	.....	.....	.....	.....	0.069	8700
9.5	269	198	151	120	97	80	67	57	49	43	38	33	30	27	24	22	.....	.....	.....	0.073	9690
10.0	298	219	168	133	107	89	75	64	55	48	42	37	33	30	27	24	22	.....	.....	0.077	10740
10.5	329	242	185	146	118	98	82	70	60	53	46	41	36	33	30	27	24	22	.....	0.081	11840
11.0	361	265	203	160	130	107	90	77	66	58	51	45	40	36	32	29	27	25	22	0.085	13000
11.5	394	290	222	175	142	117	98	84	72	63	55	49	44	39	35	32	29	27	25	0.089	14200

X

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
12.0	430	316	242	191	155	128	107	92	79	69	60	53	48	43	39	35	32	29	27	0.092	15470
12.5	466	342	262	207	168	138	116	99	86	74	65	58	52	46	42	38	35	32	29	0.096	16780
13.0	504	370	284	224	182	150	126	107	93	81	71	63	56	50	45	41	37	34	31	0.100	18150
13.5	544	399	306	242	196	161	136	116	100	87	76	68	60	54	49	44	40	37	34	0.104	19570
14.0	585	430	329	260	210	174	146	125	107	93	82	73	65	58	53	48	43	40	36	0.108	21050
14.5	628	461	353	279	226	187	157	133	115	100	88	78	70	62	56	51	46	43	39	0.112	22580
15.0	671	493	378	298	242	200	168	143	123	107	94	83	74	67	60	55	50	46	42	0.115	24160
15.5	716	526	403	318	258	213	179	152	131	114	100	89	79	71	64	58	53	49	45	0.119	25800
16.0	764	561	430	339	275	227	191	163	140	122	107	95	85	76	69	62	57	52	48	0.123	27490
16.5	812	596	457	361	292	241	203	173	149	130	114	101	90	81	73	66	60	55	51	0.127	29240
17.0	863	634	485	384	310	257	216	183	158	138	121	107	96	86	78	70	64	59	54	0.131	31040
17.5	914	672	514	406	329	272	228	195	168	146	128	114	101	91	82	74	68	62	57	0.135	32890
18.0	967	710	544	430	348	288	242	206	177	155	136	120	107	96	87	79	72	66	60	0.139	34800
20.0	1193	877	671	530	430	355	298	254	219	191	168	148	132	119	107	97	89	81	74	0.154	42960
22.0	1444	1060	812	642	520	430	361	308	265	231	203	180	160	144	140	118	107	99	90	0.169	51980
24.0	1718	1262	968	764	619	512	430	366	316	275	242	214	191	171	155	140	127	117	107	0.185	61860
26.0	....	1481	1134	897	726	600	504	430	370	323	284	251	224	201	181	164	150	138	126	0.200	72600
28.0	....	1717	1314	1039	842	696	584	498	429	374	328	291	260	233	210	190	173	160	146	0.216	84200
30.0	....	....	1511	1193	967	799	671	572	494	428	378	334	297	267	242	219	199	183	168	0.231	96660

Total depth of beam in inches	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Weight of beam 1 in. wide per linear foot.	7.3	8.3	9.4	10.4	11.5	12.5	13.5	14.6	15.6	16.7	17.7	18.7	19.8	20.8	21.9	22.9	24.0	25.0	26.0	27.1	28.1	29.2	30.2	31.2

For safe load of any width of beam multiply by width in inches.
For area of cross-section of steel for any width of beam multiply column (21) by the width in inches.
The shearing resistance of the concrete at an average value of (7/8) (40) = 35 lb. per square inch is sufficient for all loads to the right of the zigzag line $X-X'$, considering the maximum shear $1/2wl$.

TABLE 9, PART 1.—USE FOR T-BEAMS

$n=15.$ $f_s=$ or $<16{,}000$ lb. per square inch. $f_c=$ or <650 lb. per square inch.

[1. The dimensions given, to find the safe resisting moment or the stresses in the steel and concrete under a given load.

2. When flange of T-beam forms a portion of floor slab already designed—to determine suitable web dimensions and steel area.]

$$k=\frac{pn+1/2\left(\frac{t}{d}\right)^2}{pn+\frac{t}{d}}$$

$$j=\frac{6-6\frac{t}{d}+2\left(\frac{t}{d}\right)^2+\left(\frac{t}{d}\right)^3\left(\frac{1}{2pn}\right)}{6-3\frac{t}{d}}$$

$$M_s=f_s a_s jd=f_s pjbd^2,\text{ or }\frac{M_s}{bd^2}=f_s pj$$

$$M_c=f_c\left(1-\frac{t}{2kd}\right)bt.jd,\text{ or }\frac{M_c}{bd^2}=f_c\left(1-\frac{t}{2kd}\right)\frac{t}{d}j$$

$$\frac{f_s}{f_c}=\frac{n(1-k)}{k}$$

M_r = safe resisting moment = lesser of M_c and M_s

	$p=0.002$						$p=0.004$					
Values of p for neutral axis at lower edge of slab	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=$ 16,000 lb. per sq. in.	$\frac{M_r}{bd^2}$	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=$ 16,000 lb. per sq. in.	$\frac{M_r}{bd^2}$
	.10	0.269	0.954	26500	392	30.5	.10	0.406	0.954	14200	730	54.3
	.11	0.257	0.950	28200	369	30.4	.11	0.388	0.948	15400	675	58.2
	.12	0.248	0.946	29500	352	30.3	.12	0.373	0.944	16400	634	60.4
	.13	0.240	0.943	30800	337	30.2	.13	0.360	0.940	17300	600	60.2
	.14	0.234	0.940	31900	326	30.1	.14	0.349	0.936	18200	572	59.9
	.15	0.229	0.937	32800	316	30.0	.15	0.339	0.932	19000	546	59.7
	.16	0.225	0.935	33600	310	29.9	.16	0.331	0.928	19700	528	59.4
	.17	0.222	0.933	34200	304	29.9	.17	0.323	0.925	20400	509	59.2
	.18	0.220	0.931	34600	301	29.8	.18	0.317	0.921	21000	495	59.0
	.19	0.218	0.929	35000	297	29.7	.19	0.312	0.918	21500	484	58.8
	.20	0.217	0.928	35200	296	29.7	.20	0.308	0.915	21900	475	58.6
	.21	0.217	0.928	35200	296	29.7	.21	0.304	0.914	22300	465	58.5
	.22	0.217	0.927	35200	296	29.7	.22	0.300	0.912	22700	457	58.4
	Neutral axis in flange for greater values of $\frac{t}{d}$											
	Values below this line correspond to values of p in first column											
.0023	.23	0.230	0.924	32600	319	34.0	.23	0.298	0.909	22900	452	58.2
.0025	.24	0.240	0.918	30800	337	36.8	.24	0.296	0.907	23200	448	58.0
.0028	.25	0.250	0.916	29300	356	41.1	.25	0.294	0.906	23400	444	58.0
.0030	.26	0.260	0.913	27800	375	43.8	.26	0.293	0.904	23500	442	57.9
.0033	.27	0.270	0.911	26500	395	48.2	.27	0.292	0.903	23600	440	57.8
.0036	.28	0.280	0.908	25000	415	52.3	.28	0.291	0.903	23800	438	57.8
.0039	.29	0.290	0.904	23800	436	56.5	.29	0.291	0.903	23800	438	57.8
							Neutral axis in flange for greater values of $\frac{t}{d}$					

TABLE 9, PART 2.—USE FOR T-BEAMS.

$n=15$. $f_s=$ or $<$ 16,000 lb. per square inch. $f_c=$ or $<$ 650 lb. per square inch

	$p=0.004$						$p=0.006$					
Values of p for neutral axis at lower edge of slab	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=16,000$ lb. per sq. in.	$\frac{M_r}{bd^2}$	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=16,000$ lb. per sq. in.	$\frac{M_r}{bd^2}$
	.10	0.406	0.954	14200	730	54.3	.10	0.500	0.952	9750	1067	55.7
	.11	0.388	0.948	15400	675	58.2	.11	0.480	0.947	10600	984	59.9
	.12	0.373	0.944	16400	634	60.4	.12	0.462	0.943	11300	917	64.0
	.13	0.360	0.940	17300	600	60.2	.13	0.447	0.939	12000	862	67.8
	.14	0.349	0.936	18200	572	59.9	.14	0.434	0.934	12700	818	71.3
	.15	0.339	0.932	19000	546	59.7	.15	0.422	0.930	13400	779	74.5
	.16	0.331	0.928	19700	528	59.4	.16	0.411	0.926	14000	745	77.5
	.17	0.323	0.925	20400	509	59.2	.17	0.402	0.923	14500	716	80.4
	.18	0.317	0.921	21000	495	59.0	.18	0.394	0.919	15000	693	83.0
	.19	0.312	0.918	21500	484	58.8	.19	0.386	0.915	15500	670	85.6
	.20	0.308	0.915	21900	475	58.6	.20	0.380	0.912	15900	654	87.4
	.21	0.304	0.914	22300	465	58.5	.21	0.373	0.909	16400	635	87.3
	.22	0.300	0.912	22700	457	58.4	.22	0.368	0.906	16700	621	87.0
	.23	0.298	0.909	22900	452	58.2	.23	0.364	0.903	17000	610	86.7
	.24	0.296	0.907	23200	448	58.0	.24	0.360	0.900	17300	600	86.4
	.25	0.294	0.906	23400	444	58.0	.25	0.357	0.897	17500	592	86.1
	.26	0.293	0.904	23500	442	57.9	.26	0.354	0.895	17800	584	85.9
	.27	0.292	0.903	23600	440	57.8	.27	0.351	0.893	18000	577	85.7
	.28	0.291	0.903	23800	438	57.8	.28	0.349	0.891	18200	572	85.5
	.29	0.291	0.903	23800	438	57.8	.29	0.348	0.889	18200	569	85.3
	Neutral axis in flange for greater values of $\frac{t}{d}$											
	Values below this line correspond to values of p in first column											
.0043	.30	0.300	0.901	22700	457	62.1	.30	0.346	0.888	18400	565	85.2
.0046	.31	0.310	0.897	21700	479	66.1	.31	0.345	0.887	18500	562	85.2
.0050	.32	0.320	0.893	20700	502	71.5	.32	0.344	0.886	18600	559	85.1
.0054	.33	0.330	0.890	19800	525	76.9	.33	0.344	0.886	18600	559	85.1
.0058	.34	0.340	0.886	18900	550	82.3	.34	0.344	0.885	18600	559	85.0
							Neutral axis in flange for greater values of $\frac{t}{d}$					

TABLE 9, PART 3.—USE FOR T-BEAMS

$n=15$. $f_s=$ or $<16{,}000$ lb. per square inch. $f_c=$ or <650 lb. per square inch.

	$p=0.006$						$p=0.008$					
	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=$ 16,000 lb. per sq. in.	$\frac{M_r}{bd^2}$	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=$ 16,000 lb. per sq. in.	$\frac{M_r}{bd^2}$
	.10	0.500	0.952	9750	1067	55.7	.10	0.568	0.952	7420	1400	56.4
	.11	0.480	0.947	10600	984	59.9	.11	0.547	0.947	8070	1287	60.9
	.12	0.462	0.943	11300	917	64.0	.12	0.530	0.943	8650	1200	65.2
	.13	0.447	0.939	12000	862	67.8	.13	0.514	0.938	9230	1125	69.2
	.14	0.434	0.934	12700	818	71.3	.14	0.499	0.934	9800	1060	73.1
	.15	0.422	0.930	13400	779	74.5	.15	0.486	0.930	10300	1008	76.7
	.16	0.411	0.926	14000	745	77.5	.16	0.474	0.925	10800	960	79.9
	.17	0.402	0.923	14500	716	80.4	.17	0.463	0.921	11300	920	83.0
	.18	0.394	0.919	15000	693	83.0	.18	0.453	0.917	11800	884	85.9
	.19	0.386	0.915	15500	670	85.6	.19	0.445	0.914	12100	855	88.8
	.20	0.380	0.912	15900	654	87.4	.20	0.437	0.910	12500	827	91.2
	.21	0.373	0.909	16400	635	87.3	.21	0.430	0.906	12900	805	93.5
	.22	0.368	0.906	16700	621	87.0	.22	0.424	0.903	13200	784	95.6
	.23	0.364	0.903	17000	610	86.7	.23	0.418	0.899	13600	766	97.4
	.24	0.360	0.900	17300	600	86.4	.24	0.413	0.896	13800	750	99.2
	.25	0.357	0.897	17500	592	86.1	.25	0.409	0.893	14100	738	100.7
	.26	0.354	0.895	17800	584	85.9	.26	0.405	0.890	14300	726	102.1
	.27	0.351	0.893	18000	577	85.7	.27	0.401	0.888	14500	715	103.3
	.28	0.349	0.891	18200	572	85.5	.28	0.398	0.885	14700	705	104.4
	.29	0.348	0.889	18200	569	85.3	.29	0.395	0.883	14900	696	105.4
	.30	0.346	0.888	18400	565	85.2	.30	0.393	0.881	15100	690	106.2
	.31	0.345	0.887	18500	562	85.2	.31	0.391	0.879	15200	685	107.0
	.32	0.344	0.886	18600	559	85.1	.32	0.389	0.877	15300	678	107.4
	.33	0.344	0.886	18600	559	85.1	.33	0.388	0.876	15400	676	108.0
	.34	0.344	0.885	18600	559	85.0	.34	0.386	0.874	15500	670	108.2
	Neutral axis in flange for greater values of $\frac{t}{d}$											
Values of p for neutral axis at lower edge of slab.	Values below this line correspond to values of p in first column.											
0.0063	.35	0.350	0.883	18100	575	89.0	.35	0.386	0.873	15500	670	108.6
0.0068	.36	0.360	0.879	17300	601	95.6	.36	0.385	0.873	15500	668	108.7
0.0072	.37	0.370	0.878	16600	627	101.2	.37	0.383	0.873	15700	662	108.7
0.0078	.38	0.380	0.873	15900	654	107.9	.38	0.380	0.873	15900	654	107.9
							Neutral axis in flange for greater values of $\frac{t}{d}$					

TABLE 9, PART 4.—USE FOR T-BEAMS

$n=15$. $f_s =$ or $<16{,}000$ lb. per square inch. $f_c =$ or <650 lb. per square inch.

Values of p for neutral axis at lower edge of slab.	$p=0.008$ $\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=16{,}000$ lb. per sq. in.	$\frac{M_r}{bd^2}$	$p=0.010$ $\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=16{,}000$ lb. per sq. in.	$\frac{M_r}{bd^2}$
	.10	0.568	0.952	7420	1400	56.4	.10	0.620	0.951	5980	1740	56.8
	.11	0.547	0.947	8070	1287	60.9	.11	0.600	0.947	6500	1600	61.5
	.12	0.530	0.943	8650	1200	65.2	.12	0.582	0.942	7000	1484	65.9
	.13	0.514	0.938	9230	1125	69.2	.13	0.566	0.938	7470	1390	70.1
	.14	0.499	0.934	9800	1060	73.1	.14	0.551	0.933	7940	1310	74.1
	.15	0.486	0.930	10300	1008	76.7	.15	0.538	0.929	8360	1240	77.9
	.16	0.474	0.925	10800	960	79.9	.16	0.525	0.925	8820	1179	81.6
	.17	0.463	0.921	11300	920	83.0	.17	0.513	0.921	9260	1121	84.9
	.18	0.453	0.917	11800	884	85.9	.18	0.502	0.916	9670	1075	88.0
	.19	0.445	0.914	12100	855	88.8	.19	0.494	0.913	10000	1040	91.0
	.20	0.437	0.910	12500	827	91.2	.20	0.486	0.909	10300	1010	93.8
	.21	0.430	0.906	12900	805	93.5	.21	0.477	0.905	10700	973	96.4
	.22	0.424	0.903	13200	784	95.6	.22	0.470	0.901	11000	944	97.7
	.23	0.418	0.899	13600	766	97.4	.23	0.464	0.898	11300	922	101.0
	.24	0.413	0.896	13800	750	99.2	.24	0.459	0.894	11500	903	103.1
	.25	0.409	0.893	14100	738	100.7	.25	0.453	0.891	11800	883	104.8
	.26	0.405	0.890	14300	726	102.1	.26	0.448	0.888	12000	866	106.5
	.27	0.401	0.888	14500	715	103.3	.27	0.444	0.885	12200	851	108.1
	.28	0.398	0.885	14700	705	104.4	.28	0.440	0.882	12400	839	109.5
	.29	0.395	0.883	14900	696	105.4	.29	0.436	0.879	12600	825	110.5
	.30	0.393	0.881	15100	690	106.2	.30	0.433	0.876	12800	815	111.7
	.31	0.391	0.879	15200	685	107.0	.31	0.430	0.874	12900	805	112.7
	.32	0.389	0.877	15300	678	107.4	.32	0.428	0.872	13000	799	113.5
	.33	0.388	0.876	15400	676	108.0	.33	0.426	0.870	13100	792	114.3
	.34	0.386	0.874	15500	670	108.2	.34	0.424	0.868	13200	784	114.9
	.35	0.386	0.873	15500	670	108.6	.35	0.422	0.866	13300	779	115.2
	.36	0.385	0.873	15500	668	108.7	.36	0.421	0.865	13400	776	115.8
	.37	0.383	0.873	15700	662	108.7	.37	0.420	0.864	13400	773	116.4
	.38	0.380	0.873	15900	654	107.9	.38	0.419	0.862	13500	769	116.3
	Neutral axis in flange for greater values of $\frac{t}{d}$											
Values of p for neutral axis at lower edge of slab.	Values below this line correspond to values of p in first column											
.0083	.39	0.390	0.870	15200	682	110.4	.39	0.419	0.862	13500	769	117.0
.0089	.40	0.400	0.867	14600	712	112.8	.40	0.418	0.861	13600	766	117.0
.0095	.41	0.410	0.864	14000	742	115.2	.41	0.418	0.861	13600	766	117.0
.0100	.42	0.420	0.861	13500	774	117.5	.42	0.418	0.861	13600	766	117.0
							Neutral axis in flange for greater values of $\frac{t}{d}$					

TABLE 9, PART 5.—USE FOR T-BEAMS

$n=15$. $f_s=$ or $<16{,}000$ lb. per square inch. $f_c=$ or <650 lb. per square inch.

Values of p for neutral axis at lower edge of slab.	$p=0.010$						$p=0.012$					
	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=$ 16,000 lb. per sq. in.	$\frac{M_r}{bd^2}$	$\frac{t}{d}$	k	j	Maximum fiber stress in steel corresponding to $f_c=650$ lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s=$ 16,000 lb. per sq. in.	$\frac{M_r}{bd^2}$
	.10	0.620	0.951	5980	1740	56.8	.10	0.660	0.951	5020	2070	57.1
	.11	0.600	0.947	6500	1600	61.5	.11	0.641	0.947	5450	1905	61.9
	.12	0.582	0.942	7000	1484	65.9	.12	0.624	0.942	5880	1770	66.4
	.13	0.566	0.938	7470	1390	70.1	.13	0.608	0.938	6290	1650	70.8
	.14	0.551	0.933	7940	1310	74.1	.14	0.592	0.933	6720	1545	74.9
	.15	0.538	0.929	8360	1240	77.9	.15	0.579	0.929	7100	1465	78.8
	.16	0.525	0.925	8820	1179	81.6	.16	0.566	0.924	7480	1390	82.5
	.17	0.513	0.921	9260	1121	84.9	.17	0.555	0.920	7810	1330	86.1
	.18	0.502	0.916	9670	1075	88.0	.18	0.544	0.916	8180	1270	89.5
	.19	0.494	0.913	10000	1040	91.0	.19	0.535	0.912	8470	1225	92.7
	.20	0.486	0.909	10300	1010	93.8	.20	0.527	0.908	8750	1190	95.6
	.21	0.477	0.905	10700	973	96.4	.21	0.518	0.904	9080	1145	98.5
	.22	0.470	0.901	11000	944	97.7	.22	0.511	0.900	9310	1115	101.0
	.23	0.464	0.898	11300	922	101.0	.23	0.504	0.896	9600	1080	103.4
	.24	0.459	0.894	11500	903	103.1	.24	0.497	0.893	9870	1050	105.7
	.25	0.453	0.891	11800	883	104.8	.25	0.490	0.889	10100	1025	107.6
	.26	0.448	0.888	12000	866	106.5	.26	0.486	0.886	10300	1010	109.6
	.27	0.444	0.885	12200	851	108.1	.27	0.481	0.883	10500	990	111.4
	.28	0.440	0.882	12400	839	109.5	.28	0.476	0.879	10700	969	112.9
	.29	0.436	0.879	12600	825	110.5	.29	0.472	0.876	10900	953	114.4
	.30	0.433	0.876	12800	815	111.7	.30	0.469	0.874	11000	941	115.9
	.31	0.430	0.874	12900	805	112.7	.31	0.465	0.871	11200	927	116.9
	.32	0.428	0.872	13000	799	113.5	.32	0.462	0.868	11300	916	118.1
	.33	0.426	0.870	13100	792	114.3	.33	0.460	0.866	11400	909	119.1
	.34	0.424	0.868	13200	784	114.9	.34	0.457	0.863	11600	898	119.8
	.35	0.422	0.866	13300	779	115.2	.35	0.455	0.861	11700	890	120.5
	.36	0.421	0.865	13400	776	115.8	.36	0.453	0.860	11800	883	121.3
	.37	0.420	0.864	13400	773	116.4	.37	0.452	0.858	11800	880	121.7
	.38	0.419	0.862	13500	769	116.3	.38	0.450	0.856	11900	872	122.2
	.39	0.419	0.862	13500	769	117.0	.39	0.449	0.855	11900	869	122.7
	.40	0.418	0.861	13600	766	117.0	.40	0.448	0.854	12000	865	123.0
	.41	0.418	0.861	13600	766	117.0	.41	0.447	0.853	12000	861	123.0
	.42	0.418	0.861	13600	766	117.0	.42	0.447	0.852	12000	861	123.0
	Neutral axis in flange for greater values of $\frac{t}{d}$											
Values of p for neutral axis at lower edge of slab.	Values below this line correspond to values of p in first column.											
.0108	.43	0.430	0.857	12900	805	119.8	.43	0.447	0.852	12000	861	123.0
.0115	.44	0.440	0.855	12400	839	122.1	.44	0.446	0.851	12100	859	123.0
							Neutral axis in flange for greater values of $\frac{t}{d}$					

TABLE 10.—USE FOR T-BEAMS

[3. Loading and working stresses given—to determine suitable proportions for entire beam.]

$f_c = 650$ lb. per square inch. $f_s = 16{,}000$ lb. per square inch. $n = 15$.

$$k = \frac{1}{1+\frac{f_s}{nf_c}} = 0.379 \qquad \frac{z}{t} = \frac{3k - 2\frac{t}{d}}{2k - \frac{t}{d}} \cdot \frac{1}{3} \qquad jd = d - z$$

$\frac{t}{d}$	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21
$\frac{z}{t}$	0.475	0.471	0.468	0.465	0.462	0.458	0.455	0.451	0.448	0.444	0.440	0.436

$\frac{t}{d}$	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33
$\frac{z}{t}$	0.431	0.427	0.422	0.417	0.412	0.407	0.402	0.396	0.391	0.384	0.378	0.371

$\frac{t}{d}$	0.34	0.35	0.36	0.37	0.38	Neutral axis in flange when $\frac{t}{d} = k = 0.379$
$\frac{z}{t}$	0.364	0.356	0.349	0.341	0.332	

TABLE 11, PART 1.—USE FOR RECTANGULAR BEAMS WITH STEEL IN TOP AND BOTTOM

$n=15$ $\qquad k=\sqrt{2n\left(p+p'\frac{d'}{d}\right)+n^2(p+p')^2}-n(p+p')$

$M_c=bd^2f_cL$ and $f_c=\frac{M}{bd^2L}$, in which $L=\frac{k}{2}\left(1-\frac{k}{3}\right)+\frac{np'}{k}\left(k-\frac{d'}{d}\right)\left(1-\frac{d'}{d}\right)$

$M_s=bd^2f_sK$ and $f_s=\frac{M}{bd^2K}$, in which $K=p\left(1-\frac{d'}{d}\right)-\frac{k^2}{2n(1-k)}\left(\frac{k}{3}-\frac{d'}{d}\right)$

$p'=0.25\,p$						$p'=0.5\,p$					
	p	p'	k	L	K		p	p'	k	L	K
$\frac{d'}{d}=0.05$	0.005	0.00125	0.307	0.153	0.0045	$\frac{d'}{d}=0.05$	0.005	0.0025	0.296	0.163	0.0046
	0.01	0.0025	0.394	0.202	0.0088		0.01	0.005	0.373	0.225	0.0090
	0.015	0.00375	0.451	0.239	0.0130		0.015	0.0075	0.421	0.275	0.0134
	0.02	0.005	0.490	0.269	0.0172		0.02	0.01	0.454	0.320	0.0178
	0.025	0.00625	0.521	0.295	0.0214		0.025	0.0125	0.479	0.361	0.0222
	0.03	0.0075	0.546	0.321	0.0256		0.03	0.015	0.499	0.400	0.0266
$\frac{d'}{d}=0.10$	0.005	0.00125	0.309	0.150	0.0045	$\frac{d'}{d}=0.10$	0.005	0.0025	0.300	0.158	0.0045
	0.01	0.0025	0.398	0.198	0.0087		0.01	0.005	0.381	0.216	0.0088
	0.015	0.00375	0.454	0.232	0.0125		0.015	0.0075	0.428	0.261	0.0131
	0.02	0.005	0.495	0.260	0.0170		0.02	0.01	0.462	0.302	0.0174
	0.025	0.00625	0.525	0.285	0.0205		0.025	0.0125	0.488	0.338	0.0215
	0.03	0.0075	0.551	0.308	0.0251		0.03	0.015	0.509	0.374	0.0258
$\frac{d'}{d}=0.15$	0.005	0.00125	0.312	0.148	0.0044	$\frac{d'}{d}=0.15$	0.005	0.0025	0.305	0.153	0.0044
	0.01	0.0025	0.402	0.194	0.0086		0.01	0.005	0.386	0.207	0.0087
	0.015	0.00375	0.458	0.226	0.0127		0.015	0.0075	0.436	0.249	0.0128
	0.02	0.005	0.499	0.253	0.0167		0.02	0.01	0.471	0.285	0.0169
	0.025	0.00625	0.530	0.275	0.0207		0.025	0.0125	0.498	0.319	0.0210
	0.03	0.0075	0.555	0.296	0.0247		0.03	0.015	0.518	0.350	0.0251
$\frac{d'}{d}=0.20$	0.005	0.00125	0.314	0.146	0.0044	$\frac{d'}{d}=0.20$	0.005	0.0025	0.309	0.149	0.0044
	0.01	0.0025	0.404	0.190	0.0086		0.01	0.005	0.392	0.199	0.0086
	0.015	0.00375	0.462	0.221	0.0126		0.015	0.0075	0.442	0.238	0.0126
	0.02	0.005	0.503	0.245	0.0165		0.02	0.01	0.479	0.271	0.0166
	0.025	0.00625	0.534	0.266	0.0204		0.025	0.0125	0.506	0.301	0.0206
	0.03	0.0075	0.560	0.285	0.0243		0.03	0.015	0.527	0.329	0.0245
$\frac{d'}{d}=0.25$	0.005	0.00125	0.316	0.144	0.0045	$\frac{d'}{d}=0.25$	0.005	0.0025	0.314	0.146	0.0044
	0.01	0.0025	0.408	0.187	0.0086		0.01	0.005	0.398	0.194	0.0085
	0.015	0.00375	0.465	0.216	0.0125		0.015	0.0075	0.450	0.229	0.0125
	0.02	0.005	0.507	0.239	0.0164		0.02	0.01	0.486	0.258	0.0164
	0.025	0.00625	0.539	0.259	0.0202		0.025	0.0125	0.514	0.287	0.0202
	0.03	0.00725	0.565	0.275	0.0240		0.03	0.015	0.537	0.311	0.0240

TABLE 11, PART 2.—USE FOR RECTANGULAR BEAMS WITH STEEL IN TOP AND BOTTOM

	$p'=p$						$p'=1.5\,p$				
	p	p'	k	L	K		p	p'	k	L	K
$\frac{d'}{d}=0.05$	0.005	0.005	0.274	0.183	0.0046	$\frac{d'}{d}=0.05$	0.005	0.0075	0.256	0.203	0.0046
	0.01	0.01	0.336	0.271	0.0092		0.01	0.015	0.305	0.316	0.0093
	0.015	0.015	0.372	0.348	0.0138		0.015	0.0225	0.332	0.420	0.0140
	0.02	0.02	0.395	0.420	0.0184		0.02	0.03	0.349	0.521	0.0186
	0.025	0.025	0.412	0.491	0.0230		0.025	0.0375	0.361	0.619	0.0232
	0.03	0.03	0.425	0.561	0.0275		0.03	0.045	0.369	0.716	0.0280
$\frac{d'}{d}=0.10$	0.005	0.005	0.284	0.172	0.0045	$\frac{d'}{d}=0.10$	0.005	0.0075	0.268	0.186	0.0045
	0.01	0.01	0.349	0.250	0.0089		0.01	0.015	0.322	0.284	0.0090
	0.015	0.015	0.386	0.318	0.0133		0.015	0.0225	0.351	0.374	0.0134
	0.02	0.02	0.410	0.381	0.0177		0.02	0.03	0.369	0.458	0.0178
	0.025	0.025	0.428	0.442	0.0221		0.025	0.0375	0.382	0.541	0.0222
	0.03	0.03	0.442	0.503	0.0265		0.03	0.045	0.392	0.623	0.0267
$\frac{d'}{d}=0.15$	0.005	0.005	0.292	0.163	0.0044	$\frac{d'}{d}=0.15$	0.005	0.0075	0.280	0.171	0.0045
	0.01	0.01	0.360	0.233	0.0087		0.01	0.015	0.338	0.256	0.0087
	0.015	0.015	0.399	0.292	0.0129		0.015	0.0225	0.369	0.333	0.0129
	0.02	0.02	0.425	0.347	0.0171		0.02	0.03	0.389	0.404	0.0171
	0.025	0.025	0.444	0.400	0.0213		0.025	0.0375	0.403	0.475	0.0213
	0.03	0.03	0.458	0.451	0.0255		0.03	0.045	0.414	0.544	0.0256
$\frac{d'}{d}=0.20$	0.005	0.005	0.299	0.154	0.0044	$\frac{d'}{d}=0.20$	0.005	0.0075	0.292	0.160	0.0044
	0.01	0.01	0.371	0.218	0.0086		0.01	0.015	0.353	0.234	0.0086
	0.015	0.015	0.411	0.270	0.0126		0.015	0.0225	0.386	0.298	0.0126
	0.02	0.02	0.439	0.318	0.0166		0.02	0.03	0.409	0.360	0.0166
	0.025	0.025	0.460	0.364	0.0206		0.025	0.0375	0.424	0.420	0.0206
	0.03	0.03	0.475	0.408	0.0246		0.03	0.045	0.436	0.478	0.0246
$\frac{d'}{d}=0.25$	0.005	0.005	0.308	0.149	0.0045	$\frac{d'}{d}=0.25$	0.005	0.0075	0.304	0.152	0.0044
	0.01	0.01	0.382	0.206	0.0085		0.01	0.015	0.369	0.216	0.0084
	0.015	0.015	0.425	0.252	0.0124		0.015	0.0225	0.404	0.271	0.0123
	0.02	0.02	0.454	0.294	0.0162		0.02	0.03	0.428	0.325	0.0161
	0.025	0.025	0.475	0.333	0.0200		0.025	0.0375	0.444	0.373	0.0199
	0.03	0.03	0.491	0.371	0.0238		0.03	0.045	0.457	0.423	0.0237

TABLE 12.—USE FOR COLUMNS

$\frac{P}{P'} = 1 + (n-1)p$ P = total strength of reinforced column, for stress f_c.
P' = total strength of plain column, for stress f_c.

p	$n=10$	$n=12$	$n=15$	$n=20$
	Values of $\frac{P}{P'}$			
0.005	1.045	1.055	1.070	1.095
0.006	1.054	1.066	1.084	1.114
0.007	1.063	1.077	1.098	1.133
0.008	1.072	1.088	1.112	1.152
0.009	1.081	1.099	1.126	1.171
0.010	1.090	1.110	1.140	1.190
0.011	1.099	1.121	1.154	1.209
0.012	1.108	1.132	1.168	1.228
0.013	1.117	1.143	1.182	1.247
0.014	1.126	1.154	1.196	1.266
0.015	1.135	1.165	1.210	1.285
0.016	1.144	1.176	1.224	1.304
0.017	1.153	1.187	1.238	1.323
0.018	1.162	1.198	1.252	1.342
C.019	1.171	1.209	1.266	1.361
0.020	1.180	1.220	1.280	1.380
0.021	1.189	1.231	1.294	1.399
0.022	1.198	1.242	1.308	1.418
0.023	1.207	1.253	1.322	1.437
0.024	1.216	1.264	1.336	1.456
0.025	1.225	1.275	1.350	1.475
0.026	1.234	1.286	1.364	1.494
0.027	1.243	1.297	1.378	1.513
0.028	1.252	1.308	1.392	1.532
0.029	1.261	1.319	1.406	1.551
0.030	1.270	1.330	1.420	1.570
0.031	1.279	1.341	1.434	1.589
0.032	1.288	1.352	1.448	1.608
0.033	1.297	1.363	1.462	1.627
0.034	1.306	1.374	1.476	1.646
0.035	1.315	1.385	1.490	1.665
0.036	1.324	1.396	1.504	1.684
0.037	1.333	1.407	1.518	1.703
0.038	1.342	1.418	1.532	1.722
0.039	1.351	1.429	1.546	1.741
0.040	1.360	1.440	1.560	1.760

TABLE 13.—NUMBER OF RODS AND SECTIONAL AREA IN SQUARE INCHES FOR BEAM AND COLUMN REINFORCEMENT

Size of rod		4	5	6	7	8	9	10	11	12	13	14	15	16
$\frac{1}{2}''$	round	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75	2.94	3.14
	square	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00
$\frac{9}{16}''$	round	0.99	1.24	1.49	1.74	1.99	2.24	2.48	2.73	2.98	3.23	3.48	3.73	3.98
	square	1.27	1.58	1.90	2.21	2.53	2.85	3.16	3.48	3.80	4.11	4.43	4.75	5.06
$\frac{5}{8}''$	round	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30	4.60	4.91
	square	1.56	1.95	2.34	2.73	3.12	3.52	3.91	4.30	4.69	5.08	5.47	5.86	6.25
$\frac{11}{16}''$	round	1.48	1.86	2.23	2.60	2.97	3.34	3.71	4.08	4.45	4.83	5.20	5.57	5.94
	square	1.89	2.36	2.84	3.31	3.78	4.25	4.73	5.20	5.67	6.15	6.62	7.09	7.56
$\frac{3}{4}''$	round	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19	6.63	7.07
	square	2.25	2.81	3.38	3.94	4.50	5.06	5.62	6.19	6.75	7.31	7.88	8.44	9.00
$\frac{13}{16}''$	round	2.07	2.59	3.11	3.63	4.15	4.67	5.18	5.70	6.22	6.74	7.26	7.78	8.30
	square	2.64	3.30	3.96	4.62	5.28	5.94	6.60	7.26	7.92	8.58	9.24	9.90	10.56
$\frac{7}{8}''$	round	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42	9.02	9.62
	square	3.06	3.83	4.59	5.36	6.12	6.89	7.66	8.42	9.19	9.95	10.72	11.48	12.25
$\frac{15}{16}''$	round	2.76	3.45	4.14	4.83	5.52	6.21	6.90	7.59	8.28	8.97	9.66	10.35	11.04
	square	3.52	4.39	5.27	6.15	7.03	7.91	8.79	9.67	10.55	11.43	12.30	13.18	14.06
1″	round	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00	11.78	12.57
	square	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00
$1\frac{1}{8}''$	round	3.98	4.97	5.96	6.96	7.95	8.95	9.94	10.94	11.93	12.92	13.92	14.91	15.90
	square	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72	18.98	20.25
$1\frac{1}{4}''$	round	4.91	6.14	7.36	8.59	9.82	11.04	12.27	13.50	14.73	15.95	17.18	18.41	19.64
	square	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87	23.44	25.00
$1\frac{3}{8}''$	round	5.94	7.42	8.91	10.39	11.88	13.36	14.85	16.33	17.82	19.30	20.79	22.27	23.76
	square	7.56	9.45	11.34	13.23	15.12	17.01	18.91	20.80	22.69	24.58	26.47	28.36	30.25
$1\frac{1}{2}''$	round	7.07	8.84	10.60	12.37	14.14	15.90	17.67	19.44	21.21	22.97	24.74	26.51	28.27
	square	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50	33.75	36.00
$1\frac{5}{8}''$	round	8.30	10.37	12.44	14.52	16.59	18.67	20.74	22.81	24.89	26.96	29.03	31.11	33.18
	square	10.56	13.20	15.84	18.48	21.12	23.77	26.41	29.05	31.69	34.33	36.97	39.61	42.25
$1\frac{3}{4}''$	round	9.62	12.03	14.43	16.84	19.24	21.65	24.05	26.46	28.86	31.27	33.67	36.08	38.48
	square	12.25	15.31	18.37	21.44	24.50	27.56	30.62	33.69	36.75	39.81	42.87	45.94	49.00
$1\frac{7}{8}''$	round	11.04	13.81	16.57	19.33	22.09	24.85	27.61	30.37	33.13	35.90	38.66	41.42	44.18
	square	14.06	17.58	21.09	24.61	28.12	31.64	35.16	38.67	42.19	45.70	49.22	52.73	56.25
2″	round	12.57	15.71	18.85	21.99	25.13	28.27	31.42	34.56	37.70	40.84	43.98	47.12	50.27
	square	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00	60.00	64.00
$2\frac{1}{4}''$	round	15.90	19.88	23.86	27.83	31.81	35.78	39.76	43.74	47.71	51.69	55.67	59.64	63.62
	square	20.25	25.31	30.38	35.44	40.50	45.56	50.62	55.69	60.75	65.81	70.87	75.94	81.00
$2\frac{1}{2}''$	round	19.63	24.54	29.45	34.36	39.27	44.18	49.09	54.00	58.90	63.81	68.72	73.63	78.54
	square	25.00	31.25	37.50	43.75	50.00	56.25	63.50	69.75	75.00	81.25	87.50	93.75	100.00

TABLE 14.—HOOPED COLUMN REINFORCEMENT

Diameter of enclosed concrete (inches)	Pitch (inches)	Sectional area of hooping (square inches)	Length of hooping in 1 ft. in height (inches)
6	1¼	0.0187	181
	1½ max.	0.0225	151
7	1½	0.0262	176
	1¾ max.	0.0306	151
8	1¾	0.0350	172
	2 max.	0.0400	151
9	2	0.0450	170
	2¼ max.	0.0506	151
10	2¼	0.0562	167
	2½ max.	0.0625	151
11	2½	0.0687	166
	2¾ max.	0.0756	151
12	2¾	0.0825	165
	3 max.	0.0900	151
13	3	0.0975	163
	3¼ max.	0.1056	151
14	3	0.1050	176
	3½ max.	0.1225	151
15	3¼	0.1219	174
	3¾ max.	0.1406	151
16	3½	0.1400	172
	4 max.	0.1600	151
17	3¾	0.1594	171
	4¼ max.	0.1806	151
18	4	0.1800	170
	4½ max.	0.2025	151
19	4¼	0.2019	169
	4¾ max.	0.2256	151
20	4½	0.2250	168
	5 max.	0.2500	151
21	4½	0.2362	176
	5¼ max.	0.2756	151
22	4¾	0.2612	175
	5½ max.	0.3025	151
23	5	0.2875	173
	5¾ max.	0.3306	151
24	5¼	0.3150	172
	6 max.	0.3600	151
25	5½	0.3437	172
	6¼ max.	0.3906	151
26	5¾	0.3737	170
	6½ max.	0.4225	151
27	6	0.4050	170
	6¾ max.	0.4556	151
28	6¼	0.4375	169
	7 max.	0.4900	151
29	6¼	0.4531	175
	7¼ max.	0.5256	151
30	6½	0.4875	174
	7½ max.	0.5625	151

TABLE 15.—MAXIMUM DIAMETER OF ROUND OR SQUARE STIRRUPS

(From Taylor and Thompson's "Concrete, Plain and Reinforced"[1])

$$i = 2.4\frac{u}{f_s}d$$

Table gives values of $2.4\frac{u}{f_s}$ for different values of tension and bond.

Allowable unit bond stress (u)	Allowable unit tension in stirrups (f_s) (lb. per sq. in.)				
Lb. per sq. in.	12,000	14,000	15,000	16,000	20,000
80	0.016	0.014	0.013	0.012	0.010
100	0.020	0.017	0.016	0.015	0.012
120	0.024	0.020	0.019	0.018	0.014
150	0.030	0.026	0.024	0.022	0.018

TABLE 16.—MINIMUM LENGTH OF EMBEDMENT OF INCLINED RODS

(From Taylor and Thompson's "Concrete, Plain and Reinforced"[1])

(Round or square)

$$l' = \frac{f_s}{4u} \text{ diameters.}$$

Table gives values of $\frac{f_s}{4u}$ for different values of tension and bond.

Allowable unit bond stress (u)	Allowable unit stress in inclined rod (f_s) (lb. per sq. in.)				
Lb. per sq. in.	12,000	14,000	15,000	16,000	20,000
80	37	44	47	50	62
100	30	35	38	40	50
120	25	29	31	33	41
150	20	23	25	27	33

Transverse Spacing of Reinforcement.

Least width of beam should be the greater of the two values determined from the following formulas:

$$b = [2.5(n-1)+4]d_1$$
$$b = a_g(n+1)+nd_1$$

in which b = least width of beam.
d_1 = thickness of the rods.
n = maximum number of rods which occurs in a horizontal layer.
a_g = maximum size of aggregate in inches.

[1] From Taylor and Thompson's "Concrete, Plain and Reinforced," 2nd edition, page 454. Copyright, 1905, 1909, by Frederick W. Taylor.

Depth of Concrete Below Rods.

SLABS

Depth to steel (d)	Depth below center of steel
$3\frac{1}{4}$ in. and under	$\frac{3}{4}$ in.
Between $3\frac{1}{4}$ in. and $4\frac{3}{4}$ in.	1 in.
$4\frac{3}{4}$ in. and over	$1\frac{1}{4}$ in.

BEAMS AND GIRDERS

Depth to steel (d)	Depth in the clear below steel
10 in. and under	1 in.
Between 10 in. and 20 in.	$1\frac{1}{2}$ in.
20 in. and over.	2 in.

Formulas for Shear.

$$v=\frac{V}{bjd} \qquad v_o=\frac{V}{bd}$$

Formula for Bond of Tension Rods.

$$u=\frac{V}{\Sigma ojd}$$

Formulas for Vertical Stirrups.

$$s=\frac{3a_sf_sjd}{2V} \quad a_s=\frac{2}{3}\frac{Vs}{f_sjd} \quad x_1=\frac{l}{2}-\frac{v'bjd}{w}\text{(uniform loading only)}$$

$$i= \text{ or } < 2.4\frac{u}{f}d$$

Formulas for Inclined Rods.

$$l'=\frac{f_s}{4u}\text{diameters} \quad x_2= \text{ or } <\frac{l}{2}\left(1-\sqrt{\frac{8m_2}{em}}\right)\text{(uniform loading only)}$$

$$a_s=\frac{2}{3}\frac{0.7Vs}{f_sjd}\text{(45 degrees inclination)}$$

CHAPTER VIII

SLAB, BEAM, AND COLUMN DIAGRAMS

73. Illustrative Problems.—In this article the same illustrative problems will be worked out as in Art. 72. It is thought by so doing a good comparison can be made between tables and diagrams as to their advantages and limitations. The working stresses recommended by the Joint Committee will be employed throughout.

1. Design a beam to span 40 ft. and to support 600 lb. per foot (including weight of beam). Beam is assumed to be simply supported.

In Diagram 1, the intersection of the curves $f_c=650$ and $f_s=16{,}000$ is first found. Tracing down, p is found to be 0.0077, and tracing horizontally $K\ \left(\frac{M}{bd^2}\right)$ is found to be 107.3

$$M=\frac{wl^2}{8}=\frac{(600)(40)^2(12)}{8}=1{,}440{,}000 \text{ in.-lb.}$$

$$bd^2=\frac{1{,}440{,}000}{107.3}=13{,}400$$

$b=18$ in. and $d=27\ 1/2$ in. will be satisfactory. Area of cross-section, $bd=(18)(27.5)=495$ sq. in.

$$a_s=(495)(0.0077)=3.81 \text{ sq. in.}$$

We shall select four 1 1/8-in. round rods $=3.98$ sq. in.

Diagram 1 may be also employed to determine the safe resisting moment of a given beam and the greatest unit stresses in the steel and concrete due to a given bending moment.

To determine the safe resisting moment of a given beam, the value of p should be computed. After finding this value on the lower margin, trace vertically, stopping at the first of the two curves $f_c=650$ and $f_s=16{,}000$ (assuming the allowable stresses as recommended by the Joint Committee). Now trace horizontally to either side margin and the value of K is found. Then, $M=Kbd^2$. Consider a beam of the above dimensions to have 1 per cent of steel. Tracing vertically from this value on the lower margin, the 650 is the first curve to be reached and at a value of $K=117.0$ $M=(117)(18)(27.5)^2=1{,}593{,}000$ in.-lb.

To determine the greatest unit stresses in the steel and concrete of a given beam due to a given bending moment, the value of p should be computed as before. Also, K should be computed from the formula $K=\frac{M}{bd^2}$. With these values of p and K, find the intersection of the vertical and horizontal lines through these values respectively, and from the adjacent steel and concrete curves the values of f_c and f_s may be estimated. Consider a beam of

the above dimensions and with 0.7 per cent of steel, to be subjected to a bending moment of 1,200,000 in.-lb., or $K=\frac{1,200,000}{(18)(27.50)^2}=88.2$ The intersection of the vertical and horizontal lines through these values respectively, gives $f_c=550$ and $f_s=14,400$. This procedure is followed in reviewing beam design.

Diagram 1 may be also employed to find minimum allowable depth of beam for a given percentage of steel and various assumed widths, also to find the amount of steel for a beam with given loading. The preceding discussion and the discussion under Problem 1 of the preceding article should make clear the method of procedure.

2. Design a beam to span 10 ft. and to support a load of 4900 lb. per foot. Beam is assumed to be simply supported.

We shall use Diagram 7 in the design of this beam. The weight of beam will be assumed at 400 lb. per foot.

$$M=\frac{wl^2}{8}=\frac{(5300)(10)^2(12)}{8}=795,000 \text{ in.-lb.}$$

Assuming a width of beam of 14 in., the bending moment to use for one inch in width is

$$\frac{795,000}{14}=56,800 \text{ in.-lb.}$$

Selecting this value on the left-hand margin of Diagram 7, Part 3, and following the horizontal line to the right, a depth (d) of 23 in. will give maximum efficiency. This is shown by the fact that the horizontal line mentioned above meets a line half way between $d=22$ in. and $d=24$ in. much closer to the break in the curves than it does any other line representing depth. The area of steel required, shown by the curved lines crossing the d lines, is found to be 0.178 sq. in. per inch width or $(0.178)(14)=2.49$ sq. in. for the beam. We shall select ten 9/16-in. round rods = 2.485 sq. in. The spacing of the rods at the center of beam is shown in Fig. 56. This beam as designed contains two rows of steel and the computed weight per foot is

$$\frac{(26)(14)(150)}{144}=379 \text{ lb.}$$

The assumed and calculated weights do not differ materially and the beam as designed will be considered satisfactory.

It should be noted that Diagram 7 may be employed for all cases cited under Problem 1 except the case of finding the greatest unit stresses in the steel and concrete due to a given bending moment. The student should have no difficulty in determining the method of procedure.

Diagram 1 may also be used in the above design.

3. What safe load per square foot (including dead weight) can be supported by a slab 6 in. deep ($d=4\ 3/4$ in.) and 10-ft. span reinforced with 1/2-in. round rods placed 8 in. apart? The slab is simply supported and reinforced in only one direction.

$$p=\frac{0.1963}{(8)(4.75)}=0.0052$$

Referring to Diagram 6, Part 2, and tracing vertically from this value of p on the lower margin to an intersection with the curve of $d=4\ 3/4$ in., and

then tracing horizontally to the left-hand margin, a bending moment of 20,100 in.-lb. is found.

Select this value of the bending moment on the left-hand margin of Diagram 5 and trace horizontally to the right to an intersection with a vertical line through 10, denoting span length. The safe load, based on $M=\frac{wl^2}{10}$ can now be estimated directly by means of the curved lines and is found to be 168 lb. per square foot.

$$(168)(0.80)=134\ 1/2 \text{ lb. per square foot, safe load for slab simply supported.}$$

4. Design a slab to span 6 ft. and to carry a live load of 250 lb. per square foot. Slab is to be fully continuous and reinforced in only one direction.

Assume the weight of slab at 50 lb. per square foot. Total load for slab is thus 300 lb. per square foot.

From Diagram 5 for this span length and load per square foot, a bending moment of 11,000 in.-lb. is found, based on $\frac{wl^2}{12}$. Diagram 6, Part 1, shows that a depth (d) of 3 in. will be ample—total depth 3 3/4 in. Also, $a_s=0.275$ sq. in.

From Diagram 3, we may use 3/8-in. round rods spaced 4 3/4 in. on centers.

5. Design the center cross-section of a T-beam in a floor system; the beam is to have a span of 12 ft. and be fully continuous. Maximum shear (live plus dead) is closely equal to 12,200 lb. Maximum moment (live plus dead) $=356{,}300$ in.-lb. Supported slab is 6 in. thick.

Diagram 8 cannot be employed to solve for the resisting moment of a given beam but is useful in designing. Formula (7), Art. 59, may be put in the following form

$$\frac{M}{bd^2}=f_c\left(1-\frac{t}{2kd}\right)\frac{t}{d}j$$

k and j in this equation are functions of f_c and f_s, and hence the variables are f_c, f_s, and the ratio $\frac{t}{d}$. The curves at the left in Diagram 8 are plotted from this equation with a fixed value of $f_s=16{,}000$ lb. per square inch. Values of f_c may be determined for various values of $\frac{M}{bd^2}$ and $\frac{t}{d}$, or values of $\frac{M}{bd^2}$ may be determined for various values of f_c and $\frac{t}{d}$. It must not be overlooked, however, that this diagram will apply only when the amount of steel is such that $f_s=16{,}000$ lb. per sq. in. This amount of steel may be easily determined when the corresponding j is found from the curves at the right of the diagram. Suppose $\frac{M}{bd^2}=80$ and $\frac{t}{d}=0.2$, then the intersection of horizontal and vertical lines through these values respectively shows f_c to equal 600, and then tracing from this intersection horizontally to the right until the vertical line is reached indicating $f_c=600$ (at the right-hand side of the diagram), we find j equal to 0.91. Finally, $a_s=\frac{M}{f_sjd}$, in which $j=0.91$, $f_s=16{,}000$, and M and d are known. Diagram 8 will now be employed in working out the problem stated at the beginning of this discussion.

The breadth of the flange is controlled by one-fourth the span, or 36 in. Assuming a depth (d) of 16 in.

$$\frac{M}{bd^2}=\frac{356,300}{(36)(16)^2}=38.7$$

For this value of $\frac{M}{bd^2}$ and for $\frac{t}{d}=\frac{6.0}{16.0}=0.375$, we find from the diagram that this beam falls under Case I; that is, the neutral axis is in the flange.

Diagram 1 may be used for T-beams under Case I when the problem falls within the limits of the curves. In the problem at hand, a horizontal line through the value 38.7 for K would intersect the oblique line $f_s=16{,}000$ some distance below the diagram. It should be noted that the corresponding value of f_c is considerably below 400.

Diagram 7, Part 2, may be used to determine the amount of steel required. The bending moment to use for one inch width of beam is $\frac{356,300}{36}=9900$ in.-lb. The intersection of a horizontal line through this value and the curve for $d=16$ in. shows p to be 0.0028, or $a_s=(0.0028)(36)(16)=1.6$ sq. in. This diagram also shows that the stress in the concrete is far below the allowable.

6. The flange of a T-beam is 24 in. wide and 4 in. thick. The beam is to sustain a bending moment of 480,000 in.-lb. What depth of beam and amount of steel are necessary?

We will try $d=18$ in.

$$\frac{M}{bd^2}=\frac{480,000}{(24)(18)^2}=61.6$$

For this value of $\frac{M}{bd^2}$ and for $\frac{t}{d}=\frac{4}{18}=0.222$, we find from Diagram 8, $f_c=$ 485 lb. per square inch and $j=0.910$. Then,

$$a_s=\frac{480,000}{(16,000)(0.910)(18)}=1.84 \text{ sq. in.}$$

The stress in the concrete of 485 is permissible and the beam as designed is satisfactory.

Suppose 2.0 sq. in. of steel were inserted in a beam of the above dimensions, and suppose that the safe resisting moment is desired. Diagram 9 must be used for this case.

$$p=\frac{2.0}{(24)(18)}=0.0046$$

$\frac{t}{d}=0.222$ as before. Tracing vertically from this value on the lower margin to a value of $p=.0046$ and then tracing horizontally to the left margin, we find a value of $k=0.32$ Now tracing horizontally to the right until the vertical line is reached indicating $p=.0046$ (at the right-hand side of the diagram), we find j equal to 0.91

$$M_s=f_sa_sjd=(16,000)(2.0)(0.91)(18)=525,000 \text{ in.-lb.}$$

$$f_c=\frac{f_sk}{n(1-k)}=\frac{(16,000)(0.32)}{(15)(1-0.32)}=502 \text{ lb.}$$

f_c is less than 650; hence, the resisting moment depends upon the steel, or $M_s=525{,}000$ in.-lb.

7. Design a T-beam with span of 40 ft. Assume dead load = 1400 lb. per foot. Live load = 3000 lb. per foot. The beam is to be simply supported at the ends and the flange is to be proportioned as well as the web; that is, the flange does not form a part of a floor system already determined.

From Art. 61, $b' = 18$ in. and $d = 47$ in. are suitable dimensions and a thickness of flange of 12 in. is tried. The total bending moment on the beam is 10,560,000 in.-lb.

$$\frac{t}{d} = \frac{12}{47} = 0.256$$

By means of Diagram 8, we find that with $f_c = 650$ and $\frac{t}{d} = 0.256$, the value of $\frac{M}{bd^2} = 99.2$ Then,

$$b = \frac{10{,}560{,}000}{(99.2)(47)^2} = 48 \text{ in.}$$

Also, from the diagram, $j = 0.892$. Then, from Formula (7), Art. 59,

$$a_s = \frac{10{,}560{,}000}{(16{,}000)(.892)(47)} = 15.7 \text{ sq. in.}$$

The detailed design of this beam has been given at the end of Art. 61.

8. A continuous T-beam, uniformly loaded, has a bending moment at the center of each span of 358,000 in.-lb. Negative bending moment at the supports and the positive bending moment at the center of span are figured by the formula, $M = \frac{wl^2}{12}$. The tensile steel at the center of span consists of four 3/4-in. round rods. $b' = 9$ in. $d = 15.5$ in. Design the supports.

Diagrams 10 and 11 have been prepared to solve problems involving the determination of stresses for rectangular beams with steel in top and bottom. The formulas used in constructing these diagrams are radically different from those previously employed and an explanation of them will now be given.

Compressive reinforcement needs to be used only when the compressive concrete, if unreinforced, would be stressed too high. The question then arises of how much compressive reinforcement is needed to reduce the stress in the concrete to within the working limit. Let the following notation be used:

With *no* compressive reinforcement . . . . f_c, f_s, k, j.
With compressive reinforcement . . . . f_c', f_s', k', j'.

Formula 2, Art. 33, may be written as follows:

$$f_c = \frac{f_s k}{n(1-k)}$$

We also have

$$f_s = \frac{M}{a_s jd}$$

$$\therefore f_c = \frac{f_s k}{n(1-k)} = \frac{Mk}{jda_s n(1-k)} \tag{1}$$

Referring to Formula 1, Art. 62, and knowing that the fiber stress in the tensile steel of double-reinforced beams may be expressed by $f_s = \frac{M}{a_s jd}$

(using the proper value of j) the same as in rectangular and T-beams, we obtain

$$f'_c = \frac{f'_s k'}{n(1-k')} \text{ and } f'_s = \frac{M}{a_s j' d}$$

$$\therefore f'_c = \frac{f'_s k'}{n(1-k')} = \frac{Mk'}{j' d a_s n(1-k')} \tag{2}$$

From these equations (1) and (2), the *relative* reduction in f_c due to the addition of compressive steel is found to be

$$\frac{f_c - f'_c}{f_c} = 1 - \frac{j}{j'} \cdot \frac{k'}{k} \cdot \frac{1-k}{1-k'} \tag{3}$$

Since j and k depend on p, and j' and k' on p', equation (3) may be employed to show the relative reduction of f_c for different percentages of tensile and compressive steel due to the addition of the steel in the compression side of the beam. Diagram 10 gives concrete curves for various values of $\frac{d'}{d}$. It was found when constructing the diagram that, in some instances, one concrete curve may be employed with sufficient accuracy to represent two and sometimes three percentages of tensile steel.

Adding compressive steel also reduces the stress in the tensile steel. Using the same notation as above

$$f_s = \frac{M}{a_s j d} \qquad f'_s = \frac{M}{a_s j' d}$$

$$\therefore \frac{f_s - f'_s}{f_s} = 1 - \frac{j}{j'}$$

The tensile steel curves, Diagram 11, give this relative reduction for different percentages of tensile and compressive steel. Diagrams 10 and 11 will now be employed in working out the problem stated at the beginning of this discussion.

Two of the tensile rods on each side of the supports will be bent up and made to lap over the top of the supports, while the other two rods on each side will be continued straight and lapped over supports at the bottom of beam.

The ratios of steel in tension and compression are the same, and are respectively:

$$p = p' = \frac{1.77}{(9)(15.5)} = 0.013$$

We will assume $\frac{d'}{d} = 0.1$ as before.

The stress in the steel and concrete at the support must first be found with *no* compressive reinforcement. Diagram 1 may be employed when the values sought come within the limits of the curves. In this problem it will be necessary to employ Diagram 2 and do some computing. For $p = 0.013$, $j = 0.847$ and $k = 0.452$ Then

$$f_s = \frac{358{,}000}{(0.013)(0.847)(9)(15.5)^2} = 15{,}050 \text{ lb. per square inch.}$$

$$f_c = \frac{(2)(15{,}050)(0.013)}{0.452} = 865 \text{ lb. per square inch.}$$

The question now arises,—is the stress in the concrete brought down to a value 750 lb. per square inch (or less) by the introduction of 1.3 per cent of compressive steel. Also, the corresponding value of f_s should usually be determined.

Using Diagram 10 for $\frac{d'}{d}=0.10$ and for $p'=0.013$ and $p=0.013$, the relative reduction in the stress in the concrete is found to be 33.3 per cent, or the resulting stress equals $865-(.333)(865)=577$ lb. per square inch. Using Diagram 11, the relative reduction in the stress in the tensile steel equals 4.4 per cent; that is, the maximum tension in the steel is 14,390 lb. per square inch. The stresses in the concrete and steel are within the allowable and no haunch or additional steel are necessary.

9. The effective area of a column is 144 sq. in.; load to be carried is 80,000 lb.; and working stress on the concrete is 450 lb. per square inch. What percentage of longitudinal bars without hooping will be required? Take $n=15$.

The safe strength of a plain concrete column would be

$$144\times450=64{,}800 \text{ lb.}$$

Hence,

$$\frac{P}{P'}=\frac{80}{64.8}=1.235$$

From Diagram 12, for $n=15$, and $p=0.017$

$$\frac{P}{P'}=1.238$$

Thus, 1.7 per cent of steel is required, and

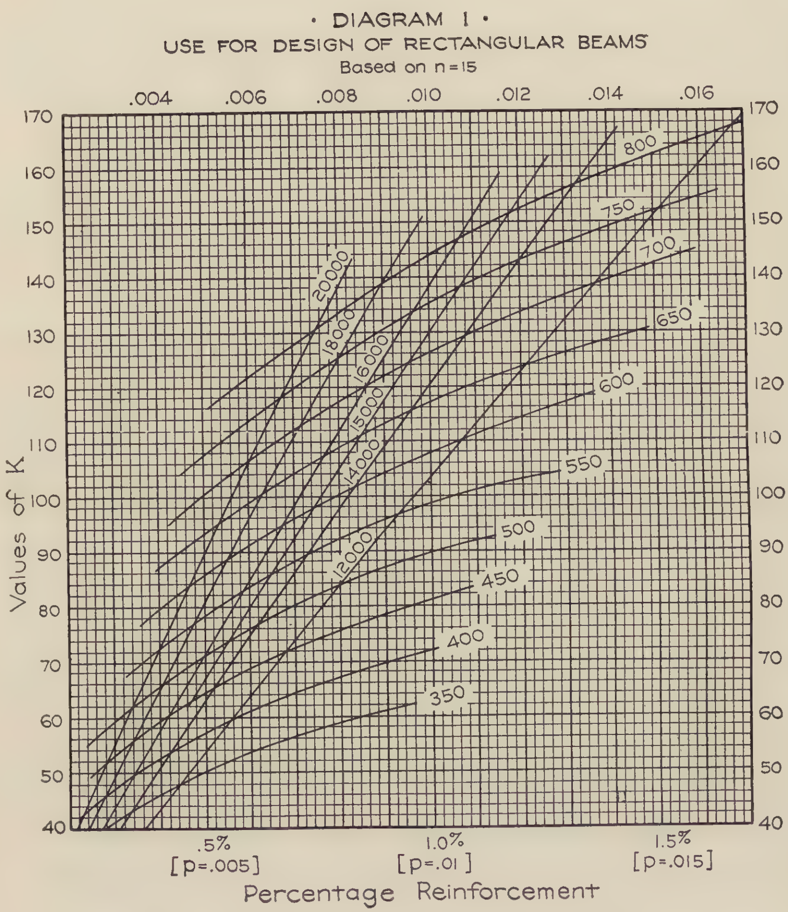
$$A_s=(144)(0.017)=2.45 \text{ sq. in.}$$

Problems

Unit stresses recommended by Joint Committee are to be used in all the following problems. Solve by using Diagrams.

75. Determine the cross-section and number of 1-in. round rods required for a rectangular girder to span 20 ft. and to sustain a live load of 2000 lb. per foot. The bending moment is to be figured by the formula $M=\frac{wl^2}{10}$. Design this girder by both Diagrams 1 and 7.
76. Design a slab to span 4.5 ft. and to carry a live load of 300 lb. per square foot. Slab is to be fully continuous and reinforced in only one direction.
77. Design the center cross-section of a T-beam, fully continuous, having a span of 16 ft. Maximum shear (not including dead weight of stem) is 13,000 lb. Maximum moment (not including dead weight of stem) $=628{,}000$ in.-lb. Supported slab is 5 in. thick. The depth of beam (d) is fixed at 18 in.
78. Solve Problem 77 considering the slab 3 3/4 in. thick.
79. A continuous T-beam, uniformly loaded, has a bending moment at the center of span of 1,400,000 in.-lb. Negative bending moment at the

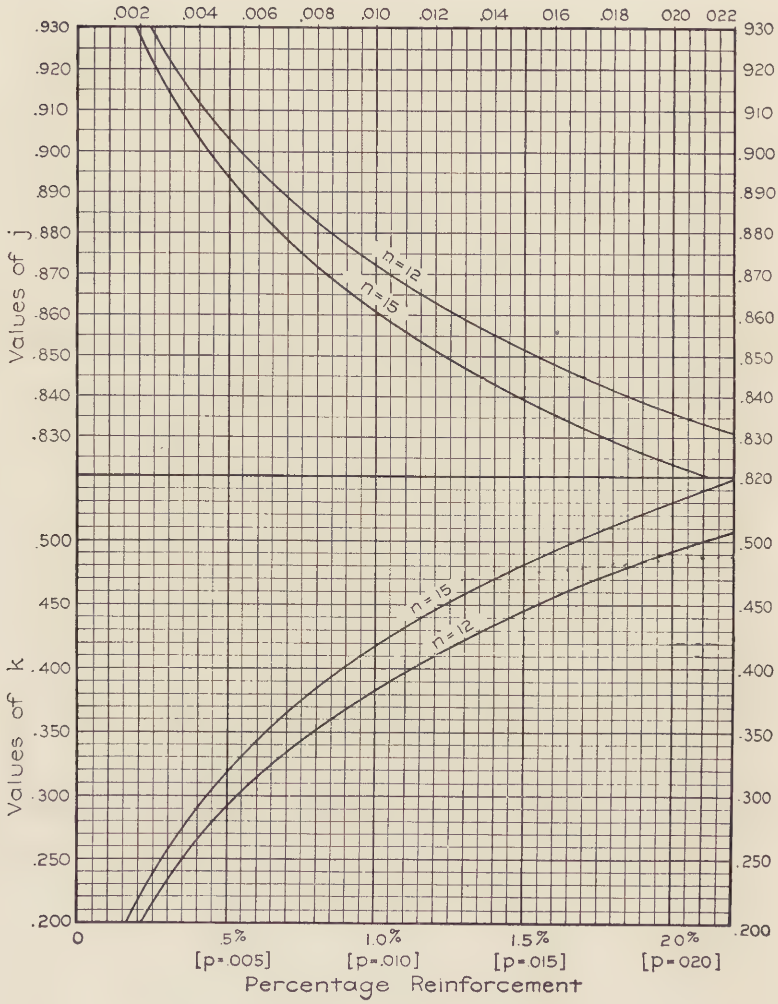
supports and the positive bending moment at the center of span are figured by the formula $\frac{wl^2}{12}$. The tensile steel at the center of span consists of four 1-in. round bars. $b'=10$ in. $d=25$ in. Design the supports assuming $\frac{d'}{d}=0.15$. Assume two of the rods to be bent up and lap over the top of the supports. Consider the other two rods to lap at the bottom of beam.



· DIAGRAM I ·
USE FOR DESIGN OF RECTANGULAR BEAMS
Based on n=15
.004
.006
.008
.010
.012
.014
.016
170
160
150
140
130
120
110
100
90
80
70
60
50
40
Values of K
800
750
700
650
600
550
500
450
400
350
20000
18000
16000
15000
14000
12000
.5%
[p=.005]
1.0%
[p=.01]
1.5%
[p=.015]
Percentage Reinforcement

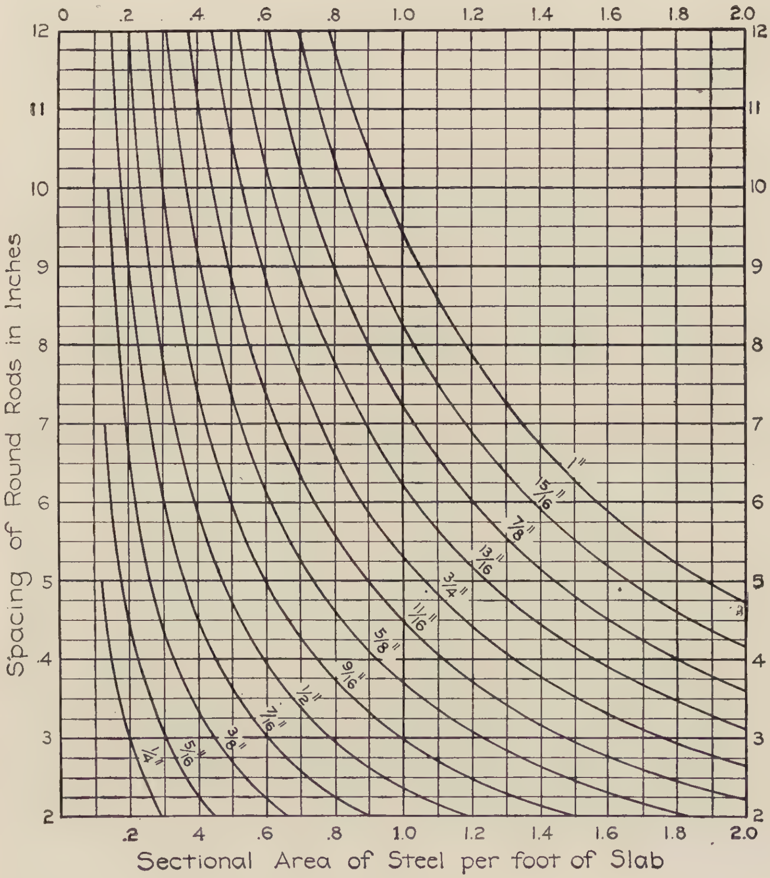
· DIAGRAM 2 ·

· CURVES FOR j AND k FOR RECTANGULAR BEAMS



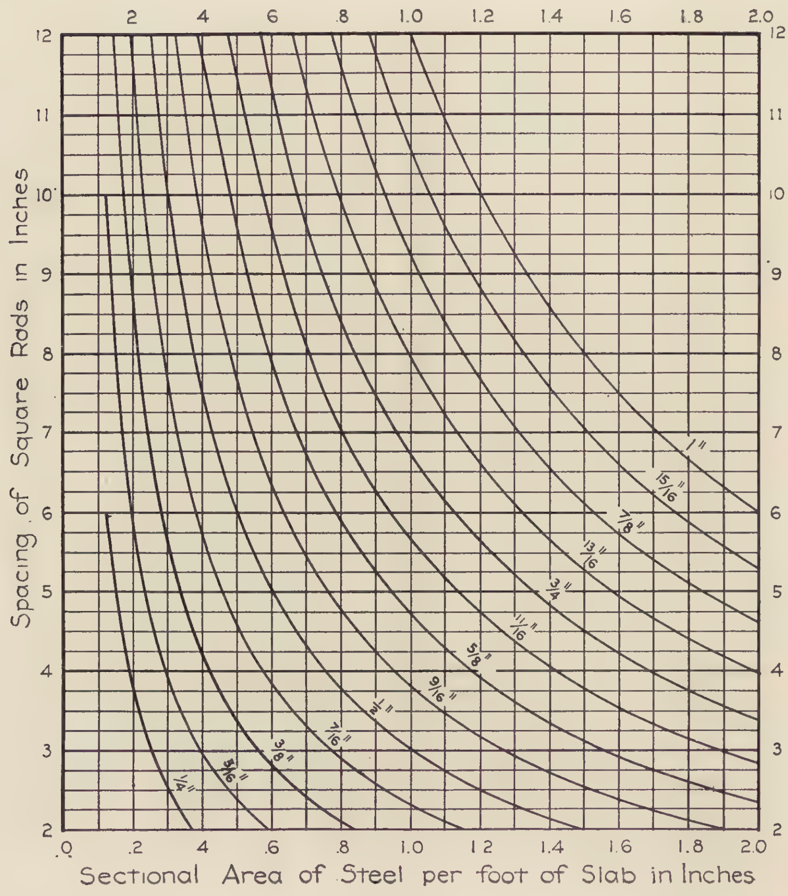
· DIAGRAM 3 ·

SPACING OF ROUND RODS IN SLABS



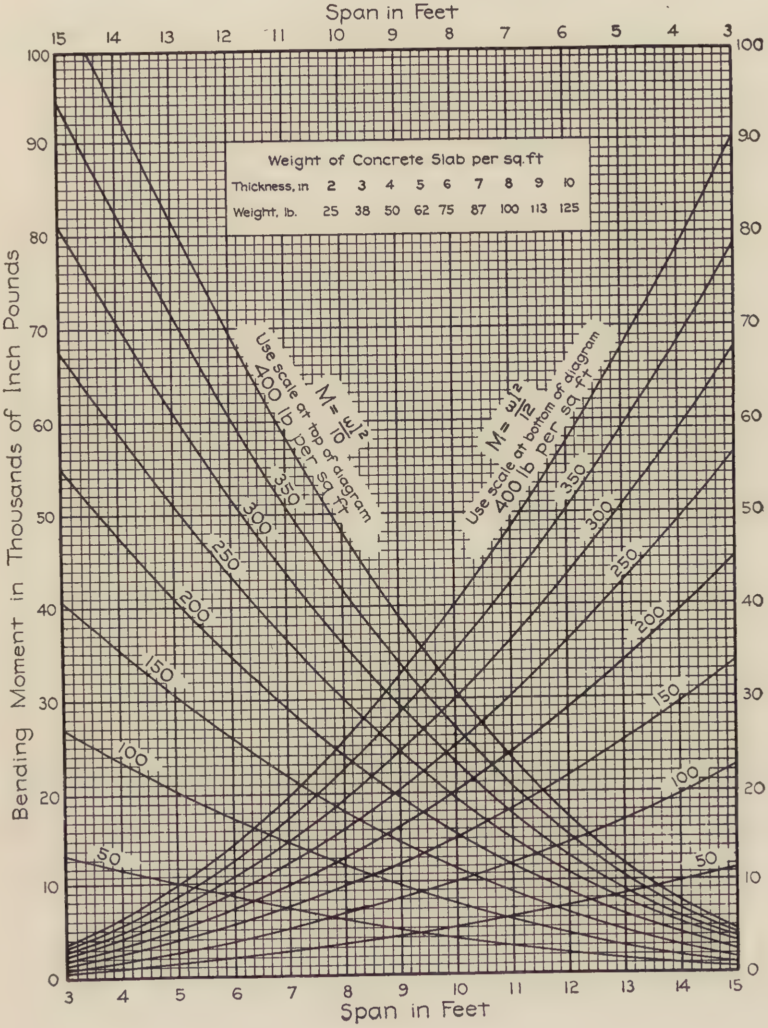
· DIAGRAM 4 ·

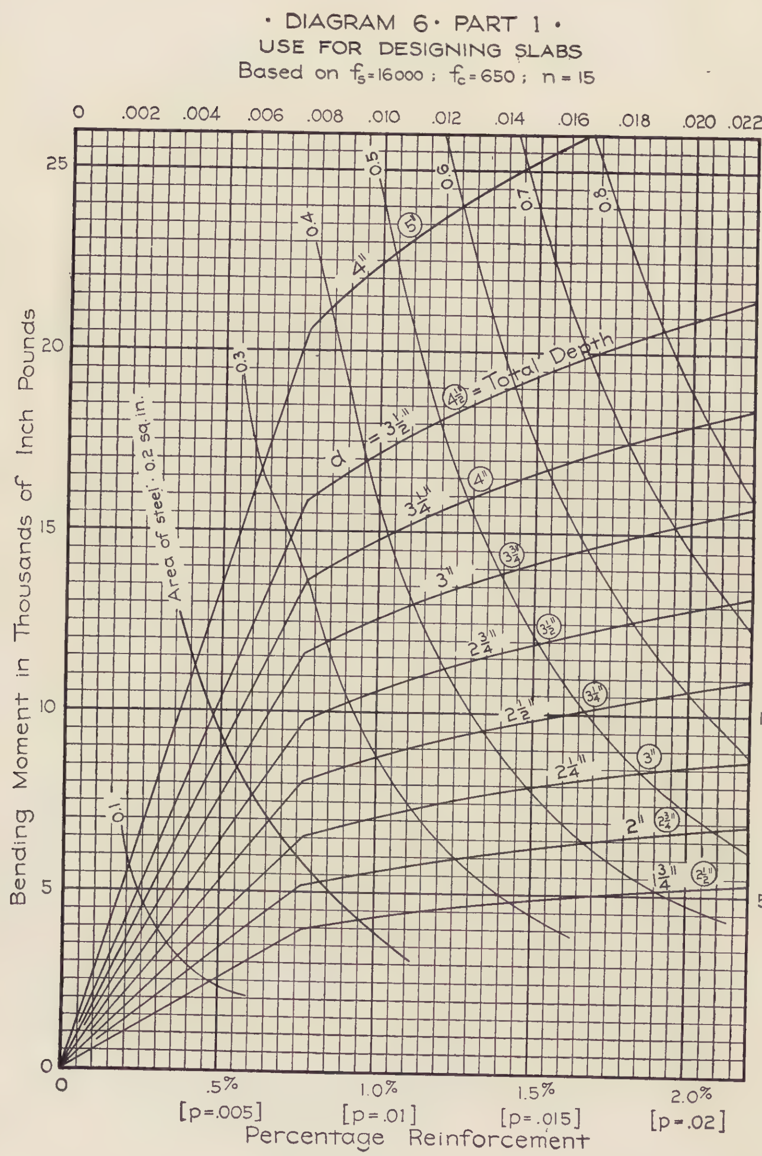
SPACING OF SQUARE RODS IN SLABS



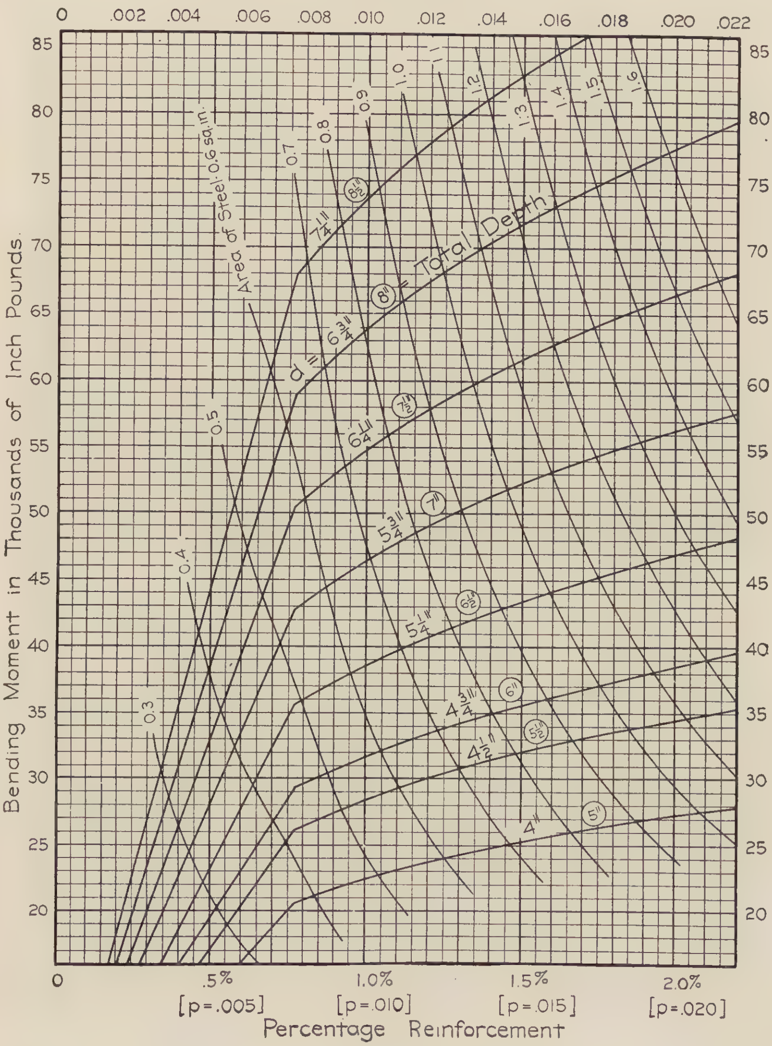
· DIAGRAM 5 ·

BENDING MOMENTS FOR UNIFORMLY DISTRIBUTED LOADS




· DIAGRAM 6 · PART 1 ·
USE FOR DESIGNING SLABS
Based on f_s=16000 ; f_c=650 ; n = 15
0 .002 .004 .006 .008 .010 .012 .014 .016 .018 .020 .022
Bending Moment in Thousands of Inch Pounds
Percentage Reinforcement
0 .5% 1.0% 1.5% 2.0%
[p=.005] [p=.01] [p=.015] [p=.02]
Area of steel: 0.2 sq. in.
d = 3½" (4½") = Total Depth

· DIAGRAM 6 · PART 2 ·
USE FOR DESIGNING SLABS.
Based on $f_s = 16000$; $f_c = 650$; $n = 15$



· DIAGRAM 7 · PART 1

USE FOR RECTANGULAR BEAMS

DEPTH AND AREA OF STEEL FOR BEAMS ONE INCH IN WIDTH

Based on $f_s = 16000$; $f_c = 650$, $n = 15$

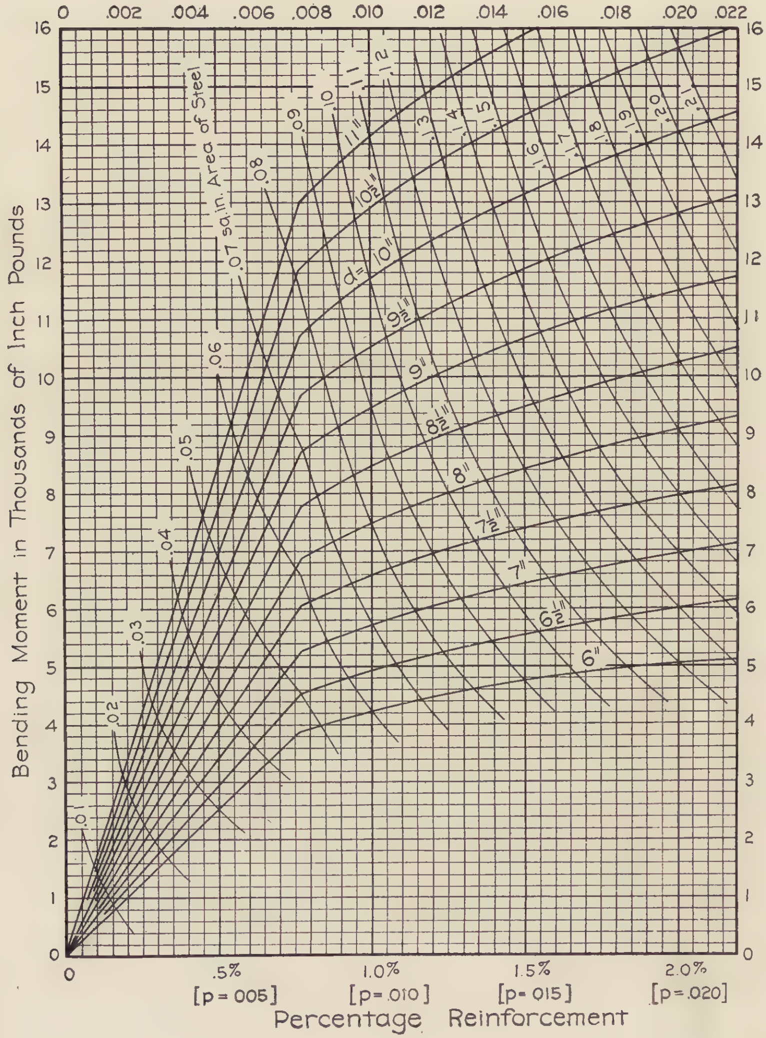


DIAGRAM 7 · PART 2

USE FOR RECTANGULAR BEAMS

DEPTH AND AREA OF STEEL FOR BEAMS ONE INCH IN WIDTH

Based on $f_s = 16000$; $f_c = 650$, $n = 15$

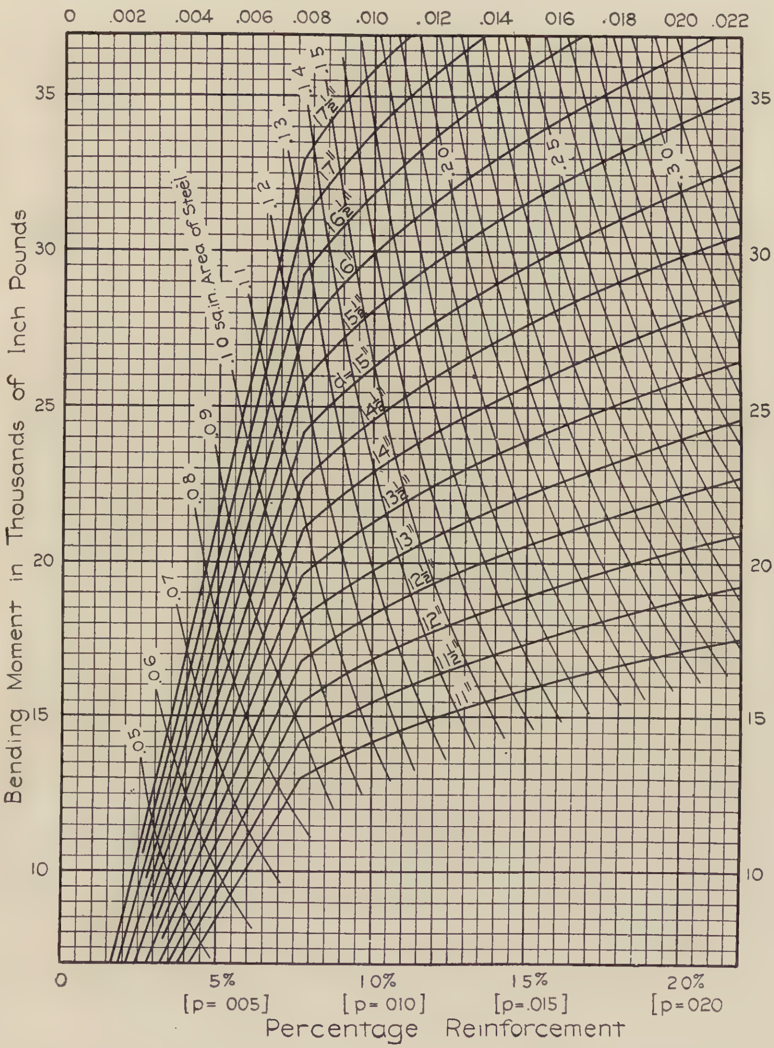


DIAGRAM 7 PART 3

USE FOR RECTANGULAR BEAMS

DEPTH AND AREA OF STEEL FOR BEAMS ONE INCH IN WIDTH

Based on $f_s=16000$, $f_c=650$, $n=15$

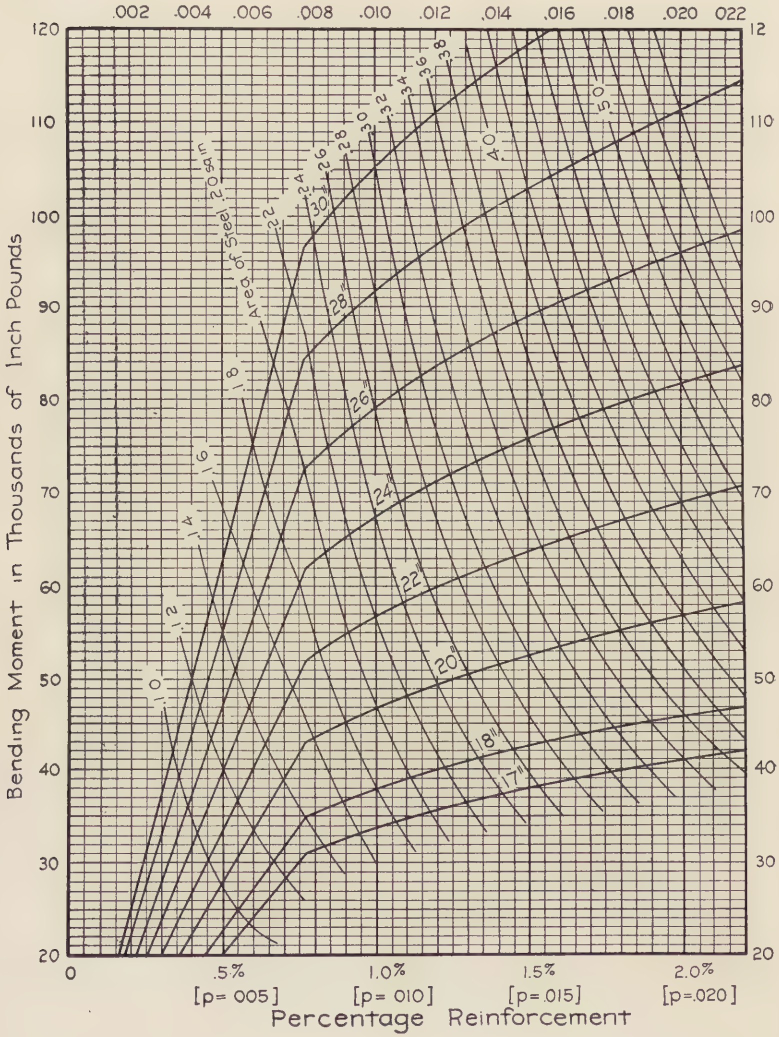
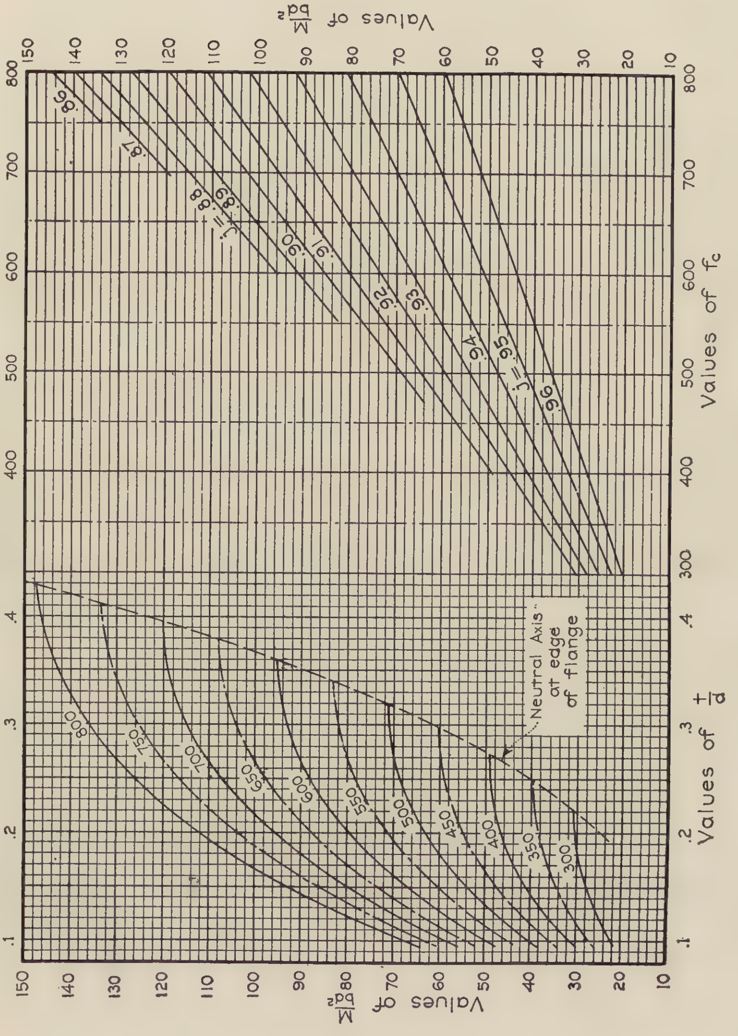
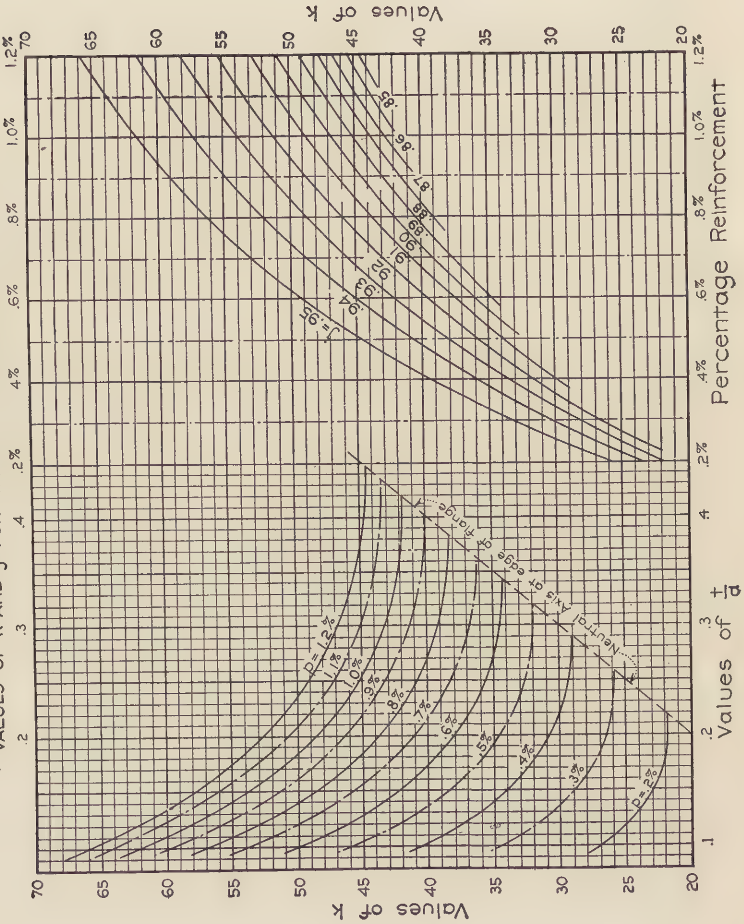


DIAGRAM 8 · USE FOR T- BEAMS ·
Based on $f_s = 16000$; $n = 15$



· DIAGRAM 9 · USE FOR T-BEAMS ·
· VALUES OF k AND j FOR VARIOUS PERCENTAGES OF STEEL ·

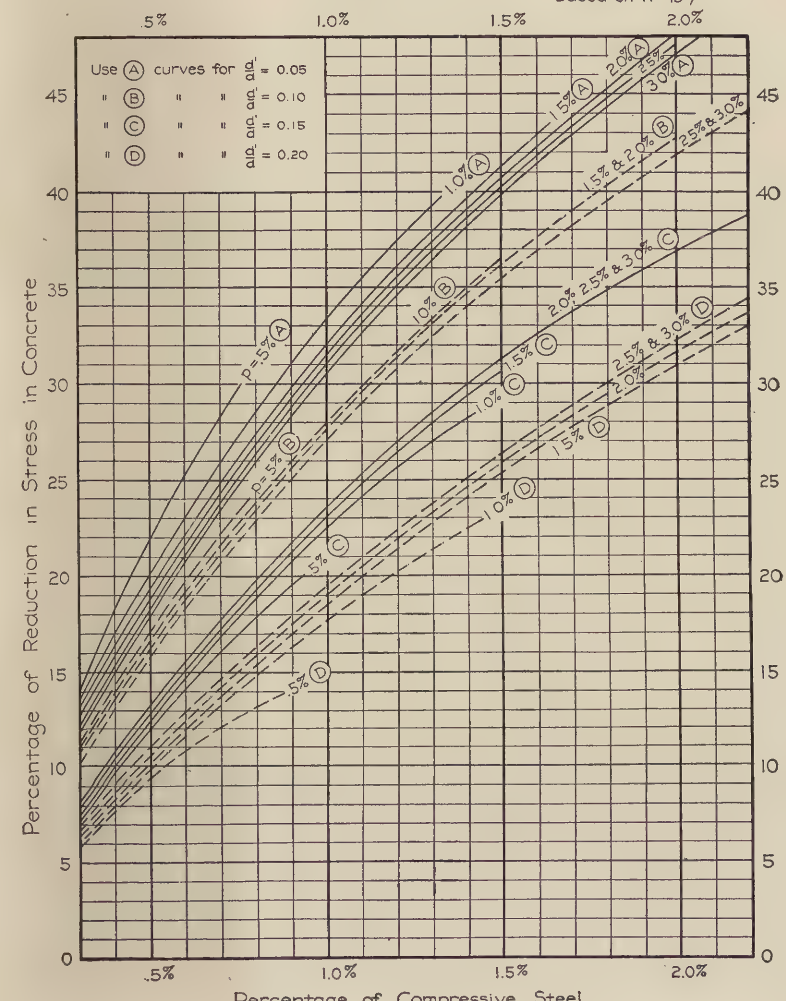


• DIAGRAM 10 •

USE FOR RECTANGULAR BEAMS WITH STEEL IN TOP AND BOTTOM

CONCRETE CURVES

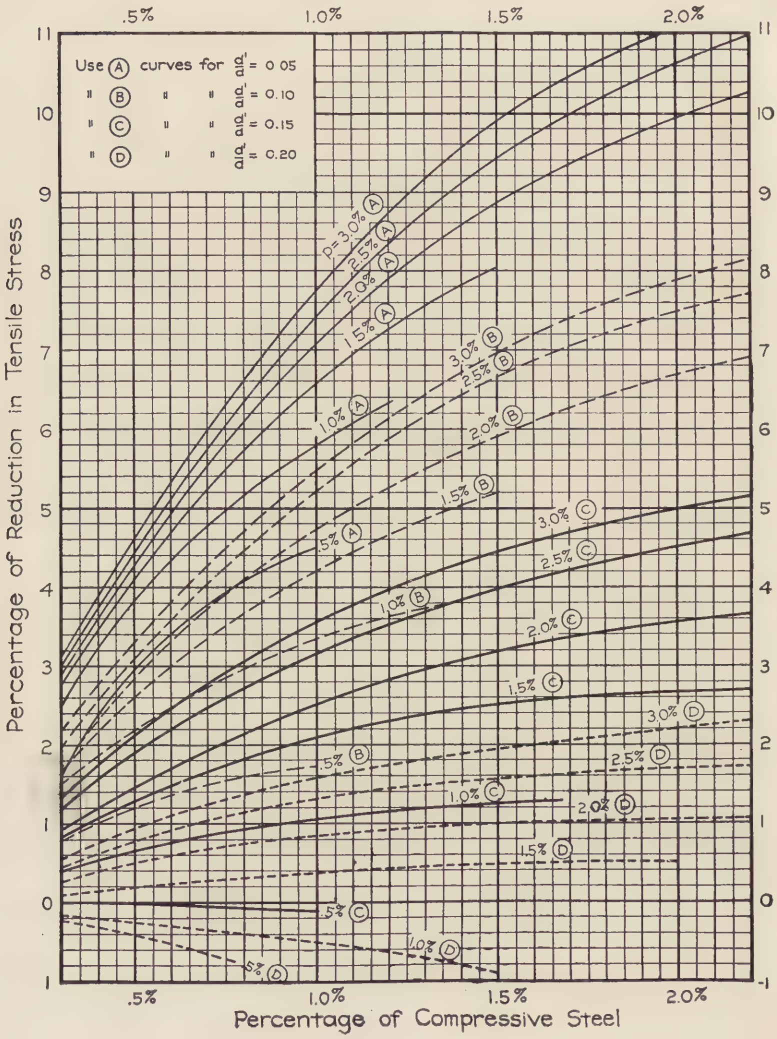
Based on n = 15,



· DIAGRAM 11 ·

USE FOR RECTANGULAR BEAMS WITH STEEL IN TOP AND BOTTOM

TENSILE STEEL CURVES Based on n = 15



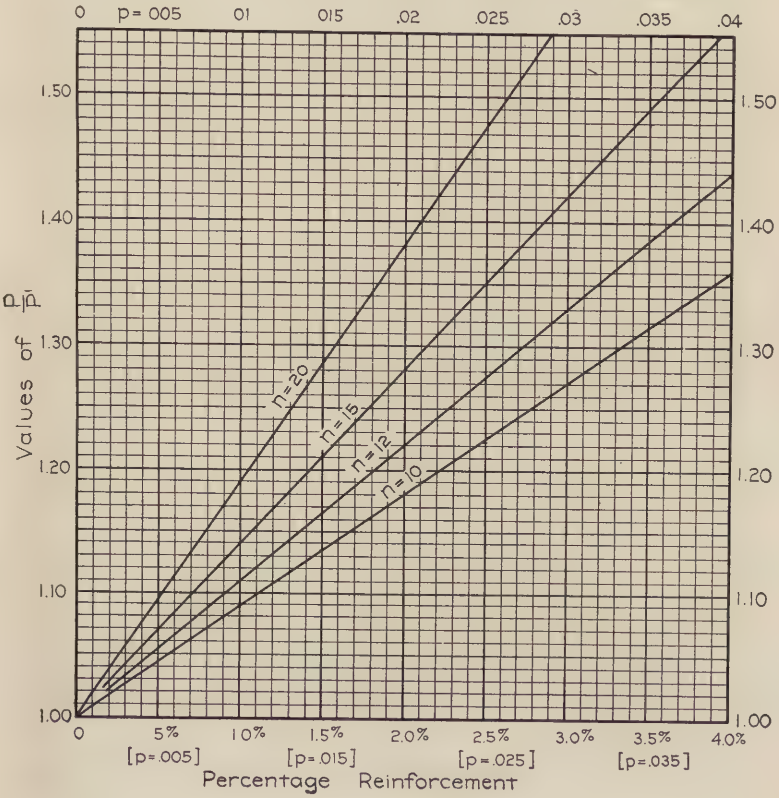
· DIAGRAM 12 ·

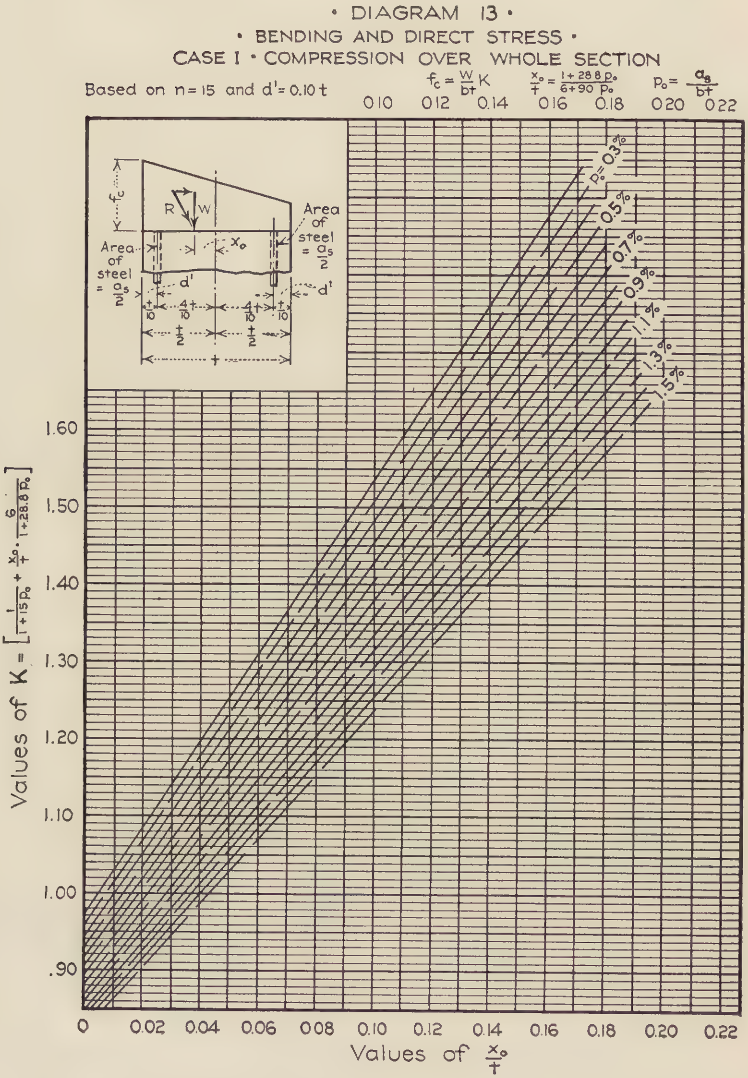
USE FOR COLUMNS

$\frac{P}{P'} = 1 + (n-1)p$

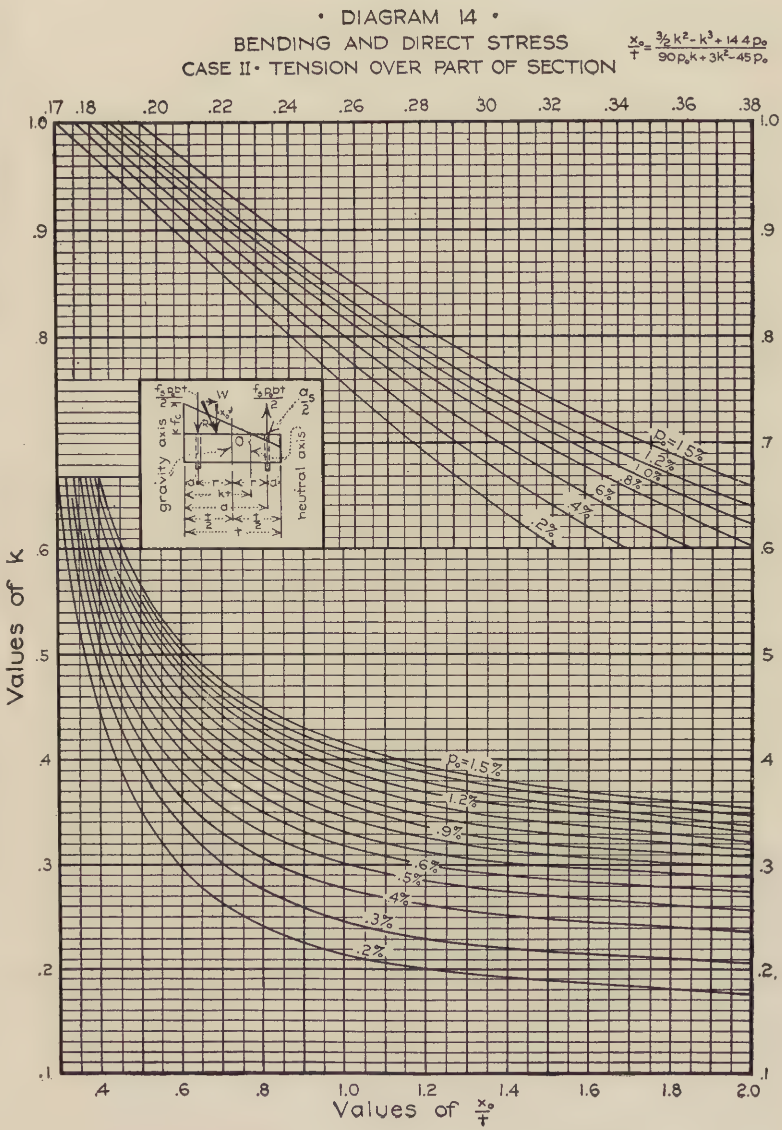
P = Total strength of reinforced column for stress f_c

P' = Total strength of plain column for stress f_c





[1] From Taylor and Thompson's, "Concrete, Plain and Reinforced," 2nd edition, page 569.


• DIAGRAM 14 •
BENDING AND DIRECT STRESS
CASE II • TENSION OVER PART OF SECTION
$\frac{x_o}{t} = \frac{3/2\,k^2 - k^3 + 14.4\,p_o}{90\,p_o k + 3k^2 - 45\,p_o}$
Values of k
Values of $\frac{x_o}{t}$
gravity axis
neutral axis
p_o=1.5%
1.2%
.9%
.6%
.5%
.4%
.3%
.2%
1.0%
.8%

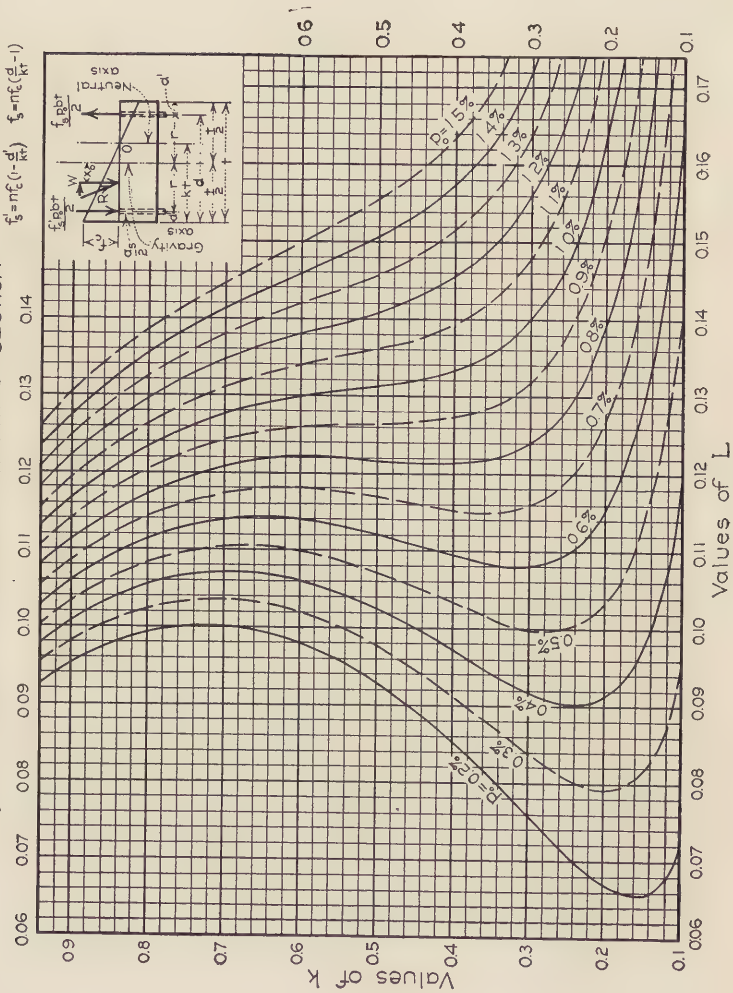


DIAGRAM 15 · BENDING AND DIRECT STRESS
· CASE II · TENSION OVER PART OF SECTION ·

$f_c = \frac{M}{Lbt^2}$ $L = \frac{12p_0}{5k} + \frac{k}{12}(3-2k)$

$f'_s = nf_c\left(1 - \frac{d'}{kt}\right)$ $f_s = nf_c\left(\frac{d}{kt} - 1\right)$

CHAPTER IX

BENDING AND DIRECT STRESS

74. Theory in General.—Consider any given section of a beam or column and locate its center of gravity. Now, if the resultant of the forces on one side of the section does not pass through this center of gravity, bending will result, and the distribution of stress normal to the section will not be uniform. Bending may result from lateral pressure; or it may be due to the eccentric application of a force in the direction of the axis; or it may be due to the two combined.

If the structure considered is a beam and is acted upon by forces which are all normal to its length, then the stresses resulting are due to simple bending and the formulas already deduced may be employed. If, however, any of the forces acting throughout the length of a beam be inclined, or if additional forces be applied at the ends, then our beam formulas for simple bending will not apply. Likewise, in columns, if the load be eccentrically applied or if lateral pressure be exerted, both bending and direct stresses will result and the ordinary column formulas cannot be used.

The same combination of stresses occurs also in arch rings and may occur in special cases. The formulas to be derived can be employed in any type of reinforced concrete structure provided the normal component of the resultant thrust on the given section acts with a lever arm about the center of gravity of the section. In long beams and columns, the deflection resulting from flexure should be given consideration when determining the eccentricity of the axial and inclined forces.

Let us first consider structures of homogeneous material, as structures of plain concrete. The distribution of pressure on any section due to a resultant pressure acting at different points will be explained. Consider a section represented in projection by EF, Fig. 84. When the resultant, R, acts at the center of gravity, O, the intensity of stress is uniform over the section and is equal to the vertical component of R divided by the area of section, or $\frac{W}{A}$. If R acts at any other point, as N, and if

the projection of the section is taken such that the distance x_o represents the true lever arm of W about the center of gravity, then the force W is equivalent to an equal W at O and a couple whose moment is Wx_o. The intensity of the uniformly varying stress due to this bending moment at a distance x from O is (by the common flexure formula for homogeneous beams) $\frac{Wx_ox}{I}$, in which I is the moment of inertia of the section about an axis

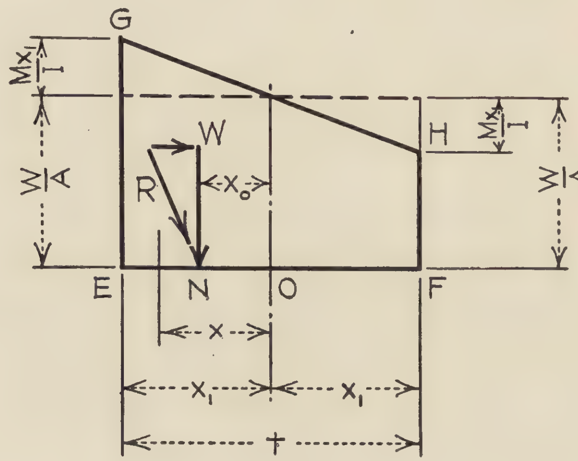


Fig. 84.

through O at right angles to the plane of the paper. At the edges E and F this intensity $=\frac{Wx_ox_1}{I}$. Regarding compressive and tensile stresses as positive and negative respectively, the intensity of stress at edge E is

$$f_c=\frac{W}{A}+\frac{Wx_ox_1}{I}$$

At edge F it is

$$f'_c=\frac{W}{A}-\frac{Wx_ox_1}{I}$$

If the stress f'_c comes out minus, the value obtained is the maximum tension as shown in Fig. 85. In plain concrete construction a greater tension than about 50 lb. per square inch should not be allowed or else cracks may be expected on the tension side.

When we come to reinforced concrete, which is composed of two materials (concrete and steel) with different values of E,

then the steel area at any given cross-section may be replaced by an area of concrete equal to n times the area of the steel, placed in the plane of the steel reinforcement. This section may be called the transformed section, or section of concrete

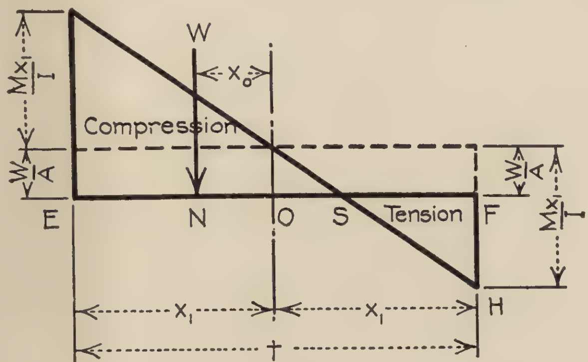


FIG. 85.

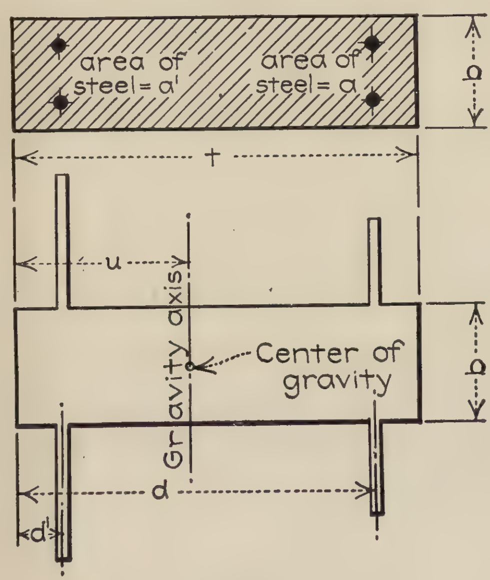


FIG. 86.

theoretically equivalent in resistance to the actual section. Under this heading rectangular sections only will be considered and Fig. 86 represents a transformed section as referred to above.

Thus, if a_c is the area of the concrete, and a_s is the area of the steel $= a + a'$; then the equivalent area

$$A = a_c + na_s = bt + n(a + a')$$

If I_c is the moment of inertia of the concrete about the gravity axis, and I_s is the moment of inertia of the steel about the same axis, then

$$I = I_c + nI_s$$

and

$$\frac{(f_c)}{(f'_c)} = \frac{W}{a_c + na_s} \frac{(+)}{(-)} \frac{Wx_o x_1}{I_c + nI_s}$$

If we denote p and p' by $\frac{a}{bt}$ and $\frac{a'}{bt}$ respectively, then the distance from the face most highly stressed to the center of gravity of the transformed section is (by moments)

$$u = \frac{bt\frac{(t)}{2} + nad + na'd'}{A}$$

$$= \frac{\frac{bt^2}{2} + nad + na'd'}{bt + n(a + a')}$$

$$= \frac{t/2 + npd + np'd'}{1 + np + np'}$$

$$I_c = 1/3bu^3 + 1/3b(t-u)^3 = \frac{b}{3}\left[u^3 + (t-u)^3\right]$$

$$I_s = a(d-u)^2 + a'(u-d')^2$$

$$I = I_c + nI_s = \frac{b}{3}\left[u^3 + (t-u)^3\right] + na(d-u)^2 + na'(u-d')^2$$

If the reinforcement is symmetrical, then $u = \frac{t}{2}$ and

$$I = 1/12bt^3 + 2na(1/2t - d')^2 = 1/12bt^3 + 2npbt(1/2t - d')^2$$

Since, $A = bt + n(a + a') = bt + nbt(p + p')$

$$\frac{(f_c)}{(f'_c)} = \frac{W}{bt + nbt(p + p')} \quad \frac{(+)}{(-)} \quad \frac{Wx_o\frac{t}{2}}{1/12bt^3 + 2npbt(1/2t - d')^2}$$

75. Case I. Compression Over the Whole Section.—The formulas above developed apply when the stress is either compression over the entire section, or when there is compression over a portion of the section with a tension over the remainder not exceeding the allowable tensile stress in the concrete. The formulas we shall use will assume rectangular sections with

symmetrical reinforcement and are given in the following form for convenience:

$$r=\frac{t}{2}-d'$$

$$\begin{matrix}(f_c)\\(f'_c)\end{matrix}=\frac{W}{bt}\left[\frac{1}{1+n(p+p')}\begin{matrix}(+)\\(-)\end{matrix}\frac{6x_ot}{t^2+24npr^2}\right]\quad\begin{matrix}(1)\\(2)\end{matrix}$$

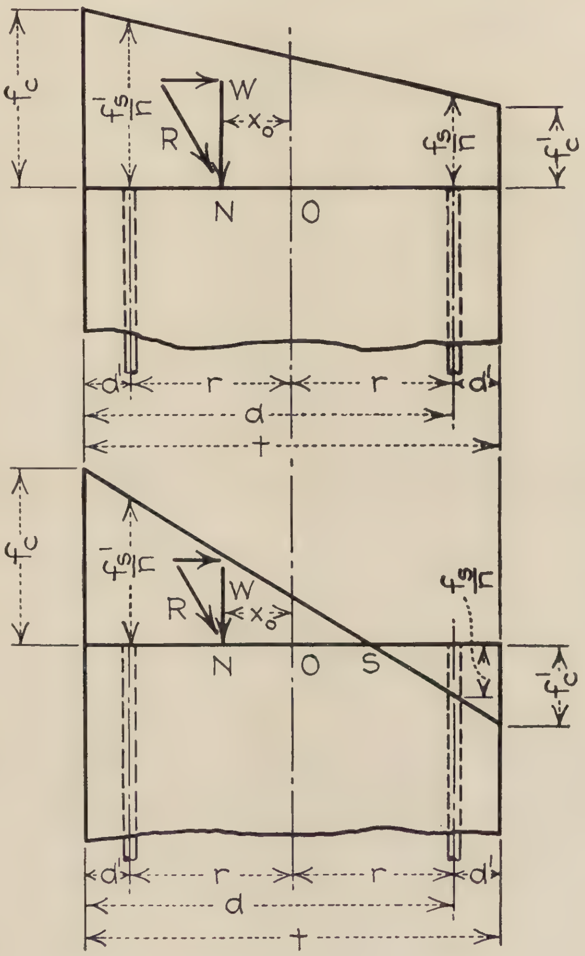


FIG. 87.

By referring to Fig. 87 it will be clear that the stress in the steel is always less than $n\times f_c$; thus, if f_c is kept within its allowable value, the steel is sure to be safely stressed.

Equation 2 gives a means of determining the eccentricity of the resultant force, or x_o, for which there can be neither tension nor compression at the surface opposite to that near which the thrust acts. To obtain the value of x_o which gives a zero value to f'_c equate the two terms within the brackets, and solve.

$$\frac{1}{1+n(p+p')}=\frac{6x_o t}{t^2+24npr^2}$$

or

$$x_o=\frac{t^2+24npr^2}{1+n(p+p')}\cdot\frac{1}{6t} \tag{3}$$

If n is assumed to be 15, and, if the steel is embedded in the concrete 1/10 of the total depth from each surface so that $2r=\frac{4}{5}t$, formula (3) may be written (since $p_o=p+p'$)

$$\frac{x_o}{t}=\frac{1+28.8p_o}{6+90p_o} \tag{4}$$

If the values $n=15$ and $2r=\frac{4}{5}t$ are substituted in formula (1), this formula becomes

$$f_c=\frac{W}{bt}\left[\frac{1}{1+15p_o}+\frac{x_o}{t}\cdot\frac{6}{1+28.8p_o}\right] \tag{5}$$

or if the expression in the brackets is denoted by K,

$$f_c=\frac{WK}{bt} \tag{6}$$

Diagram 13 gives values of K for various values of p_o and $\frac{x_o}{t}$. The termination of the curves are determined by equation (4). For greater values of $\frac{x_o}{t}$, Case I does not apply; that is, there is tension in the concrete and Case II must be employed.

76. Case II. Tension Over Part of Section.—It will be on the safe side and convenient as regards the construction of working diagrams to consider that, when any tension exists in the concrete, the steel carries all the tensile stresses. In this case there are three unit stresses to be determined; namely, maximum unit compression in concrete f_c, maximum unit compression in steel f'_s, and maximum unit tension in steel f_s. The general formulas previously developed are not applicable to this case and the following method may be used.

Referring to Fig. 88, it follows that

$$f'_s = nf_c\left(1 - \frac{d'}{kt}\right) \tag{7}$$

and

$$f_s = nf_c\left(\frac{d}{kt} - 1\right) \tag{8}$$

Since the resultant fiber stress equals W,

$$W = \frac{f'_s p_o bt}{2} + \frac{f_c bkt}{2} - \frac{f_s p_o bt}{2}$$

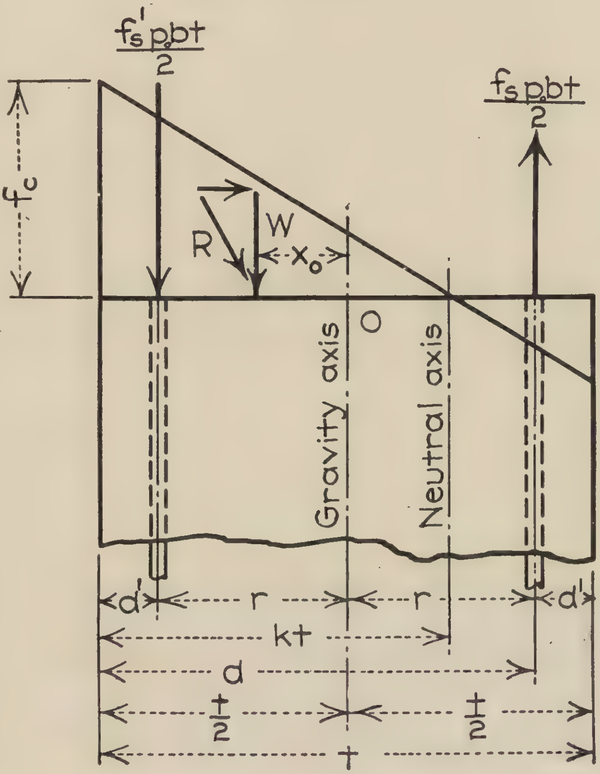


FIG. 88.

Eliminating f'_s and f_s by means of equations (7) and (8)

$$W = \frac{f_c bt}{2} \cdot \frac{k^2 + 2np_o k - np_o}{k}$$

$$= \frac{f_c bt}{2} \cdot \frac{k^2 + 2nkp_o - np_o}{k} \tag{9}$$

The moment of the stresses about the gravity axis, eliminating f'_s and f_s as before, is

$$M = f_c bt^2\left[\frac{np_o r^2}{kt^2} + \frac{k}{12}(3-2k)\right] \tag{10}$$

or, if the quantity within the brackets is designated by L, then

$$M = f_c bt^2 L, \text{ or } f_c = \frac{M}{Lbt^2} \tag{11}$$

The position of the neutral axis must be determined before equation (11) can be used. Since $Wx_o = M$ we may multiply equation (9) by x_o and equate it to equation (10). Proceeding in this manner the following equation results

$$k^3 - 3\left(1/2 - \frac{x_o}{t}\right)k^2 + 6np_o k\frac{x_o}{t} = 3np_o\left(\frac{x_o}{t} + 2\frac{r^2}{t^2}\right) \tag{12}$$

By solving this formula for $\frac{x_o}{t}$, using the values $n=15$ and $2r = \frac{4}{5}t$, we have

$$\frac{x_o}{t} = \frac{\frac{3}{2}k^2 - k^3 + 14.4p_o}{90p_o k + 3k^2 - 45p_o} \tag{13}$$

Diagram 14 based on equation (13) gives values of k for various values of p_o and $\frac{x_o}{t}$. L in equation (11), if solved for $n=15$, and $2r = \frac{4}{5}t$, is as follows:

$$L = \frac{12p_o}{5k} + \frac{k}{12}(3-2k) \tag{14}$$

Diagram 15 is based on this equation and gives values of L for various values of k and p_o.

The method of procedure in solving problems under Case II is as follows: (1) determine k from Diagram 14; (2) find L from Diagram 15; (3) solve equation (11) for f_c; (4) find unit stresses in the steel from formulas (7) and (8).

Illustrative Problem.—A beam is 9 in. wide and 20 in. deep. The reinforcement both above and below consists of 1 steel rod 1 in. in diameter embedded at a depth of 2 in. At a certain section, the normal component of the resultant force is 60,000lb., acting at a distance of 3.4 in. from the gravity axis. Assume $n=15$. Compute the maximum unit compressive stress in the concrete.

$$p_o = \frac{a+a'}{bt} = \frac{(2)(0.7854)}{(9)(20)} = 0.0087$$

$$\frac{x_o}{t} = \frac{3.4}{20} = 0.17$$

For these values of p_o and $\frac{x_o}{t}$, Diagram 13 gives $K=1.70$ and shows that the problem falls under Case I. Then by formula (6)

$$f_c=\frac{WK}{bt}=\frac{(60,000)(1.70)}{(9)(20)}=567 \text{ lb. per square inch.}$$

Illustrative Problem.—Change the eccentricity of the preceding problem to 6 in. and solve.

$$\frac{x_o}{t}=\frac{6}{20}=0.30$$

For $p_o=0.0087$ and $\frac{x_o}{t}=0.30$, Diagram 13 shows that $\frac{x_o}{t}$ is too great for the problem to come under Case I. The method of procedure for Case II must then be followed.

Diagram 14 gives $k=0.73$ for the values of p_o and $\frac{x_o}{t}$ given above. With $k=0.73$ and $p_o=0.0087$, Diagram 15 shows L to be 0.122. Solving equation (11)

$$f_c=\frac{M}{Lbt^2}=\frac{(60,000)(6)}{(0.122)(9)(20)^2}=820 \text{ lb. per square inch.}$$

Using the formula (8) gives

$$f_s=nf_c\left(\frac{d}{kt}-1\right)=(15)(820)\left(\frac{18}{0.73\times 20}-1\right)=2830 \text{ lb. per square inch.}$$

The stress f'_s may be found by formula 7 but is always less than $n\times f_c$.

Illustrative Problem.—An arch is 20 in. deep and is reinforced with 3 rods 3/4 in. in diameter to each foot of width, both above and below. If the rods are embedded to a depth of 2 in. and the normal component of the resultant thrust on a section is 100,000 lb. for 1-ft. width of arch with an eccentricity of 3.4 in., determine the maximum intensity of compressive stress on the concrete. Assume $n=15$.

$$p_o=\frac{(6)(0.4418)}{(12)(20)}=0.0110.$$

$$\frac{x_o}{t}=\frac{3.4}{20}=0.170$$

Diagram 13 gives $K=1.63$ and the problem comes under Case I. Then by formula (6)

$$f_c=\frac{WK}{bt}=\frac{(100,000)(1.63)}{(12)(20)}=679 \text{ lb. per square inch.}$$

Problems

80. A rectangular beam is 8 in. wide and 20 in. deep, and contains four 1/2 in. round rods both at the top and bottom. At a given section the normal component of the resultant force is 77,000 lb. and acts at a distance of 2 in. from the gravity axis. Assume $n=15$. (a) Compute the maximum unit compressive stress in the concrete. (b) How much steel reinforcement would be needed to make the stress in the concrete 600 lb. per square inch?
81. Change the eccentricity in part (a) of the preceding problem to 5 in. and solve.

INDEX